Similarity and Contrast on Conceptual Spaces for Pertinent Description Generation

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Similarity is crucial to cognition

General (often implicit) hypothesis:

*similar stimulus* in *similar context*  ➔ *similar response*
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*similar* stimulus in *similar* context $\rightarrow$ *similar* response

$\uparrow$

$\sim$ fixing the task
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Practical uses: *description generation*

*proximate elements* can be used as *reference* to identify a certain *target* (*object*, *situation*, etc.)
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\[
\text{similar stimulus in similar context} \rightarrow \text{similar response}
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\[\sim \text{fixing the task}\]

Practical uses: \textit{description generation}

\textit{proximate elements} can be used as \textit{reference} to identify a certain \textit{target} (\textit{object}, \textit{situation}, etc.)

the caudate nucleus is an \textit{internal brain structure} which is very close to the lateral ventricles
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but how two stimuli are defined *similar* ?
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but how two stimuli are defined similar?

psychology

- similarity is a function of a mental \textit{distance} between conceptualizations [Shepard1962]

\[ \text{“psychological space” hypothesis} \]
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**machine learning**

- relies on some *metric* to compare inputs

- offers *pseudo-metric* learning methods
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geometrical model of cognition
Problems:

*geometry model of cognition*
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- similarity in human judgments does not satisfy fundamental geometric axioms [Tversky77]

  basis of feature-based models
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- reasoning via artificial devices (still?) relies on **symbolic** processing
  
  _e.g. through ontologies_
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  **but..** symbol grounding? predicate selection?
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Proposed solutions:

- enriching the metric model with additional elements (e.g. density [Krumhansl78])

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- approaching logical structures through geometric methods (e.g. [Distel2014])
Towards an alternative solution...

- conceptual spaces
- associationistic methods
- symbolic methods
Overview on conceptual spaces

- Conceptual spaces stem from (continuous) perceptive spaces.

- Natural properties emerge as convex regions over integral domains (e.g. color).

- Concepts are combinations of properties

- Prototypes can be seen as centroids of convex regions (properties or concepts).

  → convex regions can be seen as resulting from the competition between prototypes (forming a Voronoi Tessellation).
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If red, or green, or brown correspond to regions in the color space...

Why do we say “red dogs” even if they are actually brown?
Predicates resulting from contrast

Alternative hypothesis [Dessalles2015]:
predicates are generated *on the fly* after an operation of *contrast*.

\[ C = O - P \]

- **contrastor**
- **object** *(target)*
- **prototype** *(reference)*
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These dogs are “red dogs”:

- not because their color is red (they are brown),
- because they are more red with respect to the dog prototype

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Test:

- Colors of 9 common dog furs on the internet

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<tbody>
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<td>mean:</td>
<td>[ 0.10,</td>
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0.29 is the std dev of a uniform distribution on [0, 1]! we neglect the dimensions approaching it.
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<td>0.24,</td>
<td>0.92</td>
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<tr>
<td>P</td>
<td>[0.10,</td>
<td>*,</td>
<td>*</td>
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\[
O = [0.07, 0.24, 0.92] \quad P = [0.10, *, *]
\]

\[
C = O - P = [-0.16, 0.24, 0.92] \sim \text{“red”}
\]

Still in the gravitation of red, but not brown!
Predicates resulting from contrast

In logic, usually: $above(a, b) \leftrightarrow below(b, a)$
Predicates resulting from contrast

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However, we don't say
“the table is below the apple.”
“the trunk is below the crown.”
etc..
Predicates resulting from contrast

In logic, usually: \(\text{above}(a, b) \leftrightarrow \text{below}(b, a)\)

However, we don't say
“the table is below the apple.”
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etc..

If our hypothesis is correct, \(C = A - B \sim \text{“above”}\)

Before we handled points, here we have extended objects.
→ mathematical morphology methods
Predicates are generated on the fly

We considered an existing method [Bloch2006] used in image processing to compute directional relative positions of visual entities (e.g. of biomedical images).

![Diagram showing objects a and b with coordinates](image-url)
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models of relations
for a point centered in the origin
Predicates are generated on the fly

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“below a”  “above b”
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"below a"  "above b"  how much a is (in) "above b"  how much b is (in) "below a"
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\[
\text{operation scheme: } a \sim b + \text{“above”}
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Inverse operation to contrast: *merge*

Operation scheme: $a \sim b + \text{"above"}$
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**Operation scheme:**

- $a \sim b + \text{“above”}$

 how much $a$ is “above $b$”

**Inverse operation to contrast:** $\text{merge}$

**Alignment as overlap**
Predicates are generated on the fly

We considered an existing method [Bloch2006] used in image processing to compute directional relative positions of visual entities (e.g. of biomedical images).

**Operation scheme:**

\[ a \bowtie b + \text{“above”} \]

Inverse operation to contrast: **merge**

**Alignment as overlap**

**How much a is “above b”**

cf. with \( o - p \bowtie \text{“red”} \)
From contrast to concept similarity

- Up to now, for calculating contrast, we have used distances inherent to the integral dimensions. These distances may be interpreted as related to (local) 
  **dissimilarity**. *(no holistic distance)*
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- Up to now, for calculating contrast, we have used distances inherent to the integral dimensions. These distances may be interpreted as related to (local) dissimilarity. *(no holistic distance)*

- *But what about concept (i.e. multi-dimensional) similarity?*
From contrast to concept similarity

“she is strong.”

this person − prototype person \sim “strong”
From contrast to concept similarity

“she is strong.”

this person − prototype person $\sim$ “strong”

(metaphor as conceptual analogy)

“she is (like) a lion.”
From contrast to concept similarity

“she is strong.”

this person − prototype person \(\sim\) “strong”

“she is (like) a lion.”

double contrast

\textit{target}

this person − prototype person \(\sim\) “strong”, etc.

\textit{comparison ground}

\textit{reference}

prototype lion − prototype animal \(\sim\) “strong”, etc.
From contrast to concept similarity

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The reference activates certain **discriminating features**.
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The reference activates certain discriminating features.

Concept similarity is a sequential, multi-layered computation
Problems:

- similarity in human judgments does not satisfy **fundamental geometric axioms** [Tversky77]

  basis of feature-based models

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Proposed solutions:

- enriching the metric model with additional elements (e.g. density [Krumhansl78])

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1. Problems with symmetry

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**However,**

Tel Aviv is like New York

has a different meaning than:

New York is like Tel Aviv
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- *Distance between two points should be the same when inverting the terms of comparison.*

*However,*

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Our explanation: **changing of reference activates different features**
2. Problems with triangle inequality

\[ d(a, b) + d(b, c) \geq d(a, c) \]
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**However,**

Jamaica is similar to Cuba
Cuba is similar to Russia
Jamaica is *not* similar to Russia.
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However,

Jamaica is similar to Cuba
Cuba is similar to Russia
Jamaica is not similar to Russia.

Our explanation: different/no comparison grounds after contrast
3. Problems with minimality

\[ d(a, b) \geq d(a, a) = 0. \]

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Our explanation: *sequential nature of similarity assessment.*
4. **Diagnosticity effect**

- *The distance between two points in a set should not change when changing the set.*
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However,

- when people were asked for the country most similar to a reference amongst a given group of countries, they changed answers depending on the group.
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Our explanation: effect due to the change of group prototype.
Conclusions

- We propose a fundamental distinction between:
  - *perceptual* similarity
  - *contrastively analogical* similarity
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  – perceptual similarity
  – contrastively analogical similarity

• The two are commonly conflated:
  – by using MDS on people’s similarity judgments to elicit dimensions of psychological (conceptual) spaces
  – in similar dimensional reduction techniques used in ML
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- Future investigations: normalizing effects, contrast with regions, non-descriptive pertinence.