Computing Contrast on Conceptual Spaces

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“small” problem

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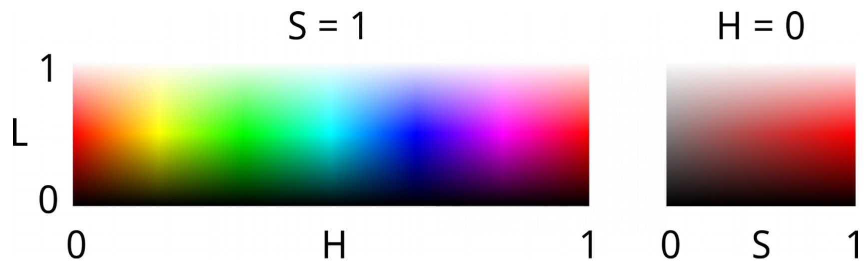
If ***red***, or ***green***, or ***brown*** correspond to regions in the color space...



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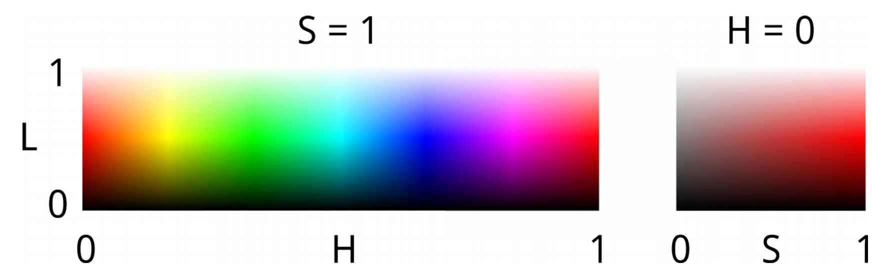

why do we say “**red dogs**” even if they are actually brown?

images after Google

Predicates resulting from contrast

Alternative hypothesis [Dessalles2015]:

Predicates are generated *on the fly* after an operation of ***contrast***.

$$c = o - p \rightsquigarrow \text{“red”}$$

contrastor

object ***(target)***

prototype ***(reference)***

Predication follows principles of *descriptive pertinence*:

objects are determined by distinctive features

Dessalles, J.-L. (2015). From Conceptual Spaces to Predicates. Applications of Conceptual Spaces: The Case for Geometric Knowledge Representation, 17–31.

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contrastor ***object (target)*** ***prototype (reference)***

These dogs are “red dogs”:

- not because their color is red (they are brown),
- because they are ***more red*** than the dog prototype

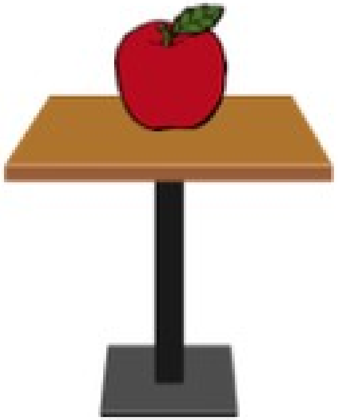
Predicates resulting from contrast

In logic, usually: *above(a, b)* $\leftrightarrow$ *below(b, a)*

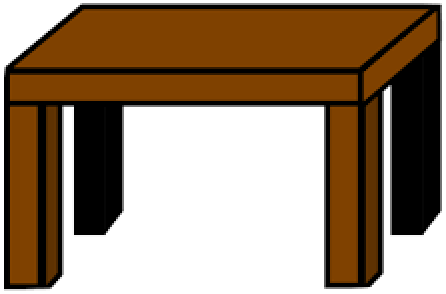
Predicates resulting from contrast

In logic, usually: *above(a, b)* ↔ *below(b, a)*

However, people don't say



“the table is
below the apple.”



“the board is
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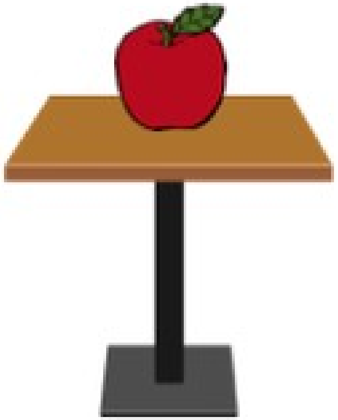

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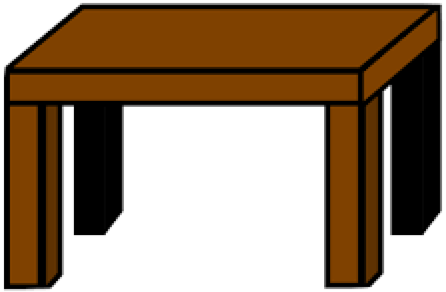


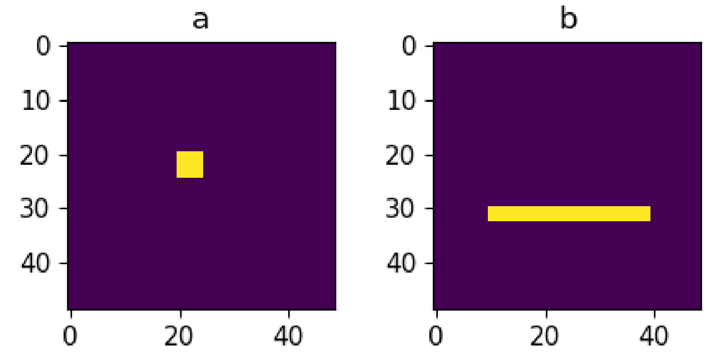
“the board is above the leg.”

If the contrastive hypothesis is correct, $c = a - b$ ↝ “above”

superior in *strength* to $c' = b - a$ ↝ “below”

Directional relationships

We considered an existing method [Bloch2006] used in image processing to compute directional relative positions of visual entities (e.g. of biomedical images).



objects

Bloch, I. (2006). Spatial reasoning under imprecision using fuzzy set theory, formal logics and mathematical morphology. International Journal of Approximate Reasoning, 41(2), 77–95.

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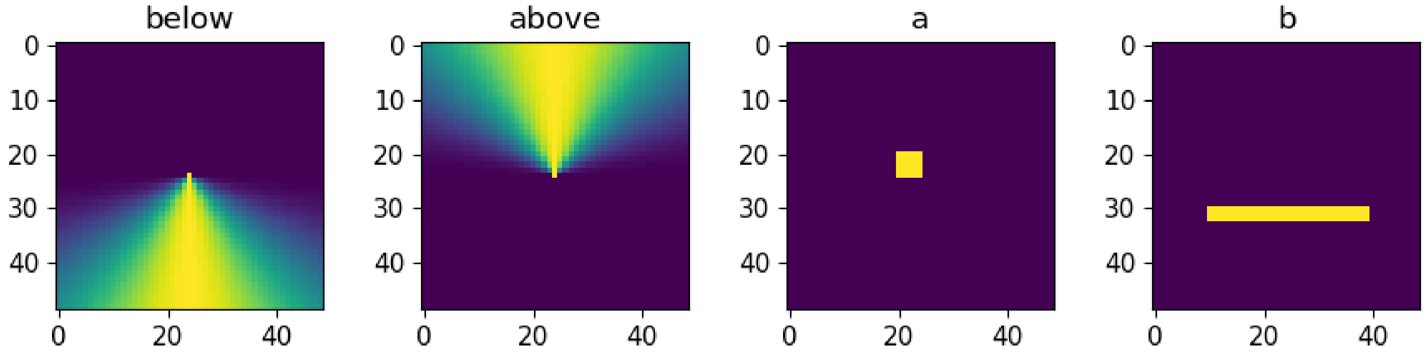

models of relations
for a point centered
in the origin

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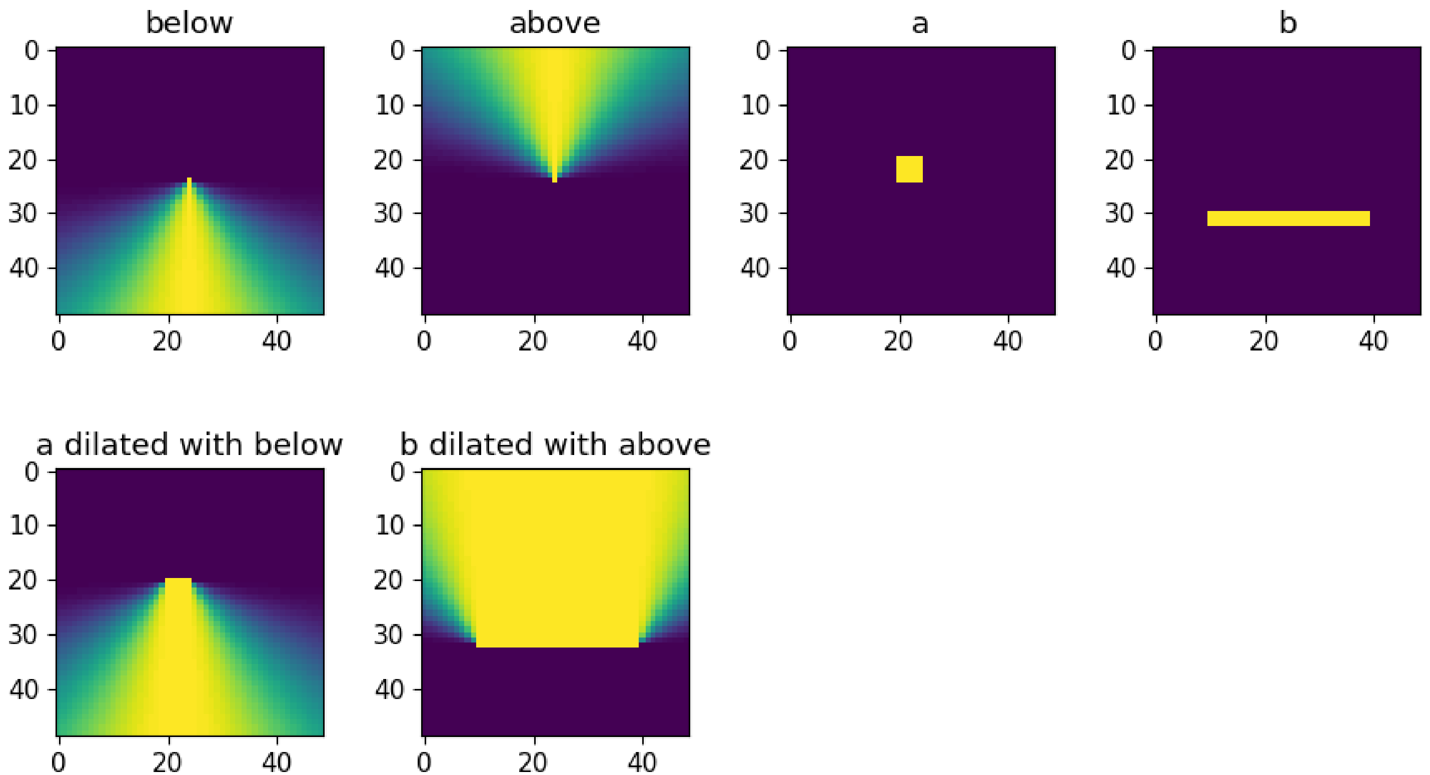

“below a” “above b”

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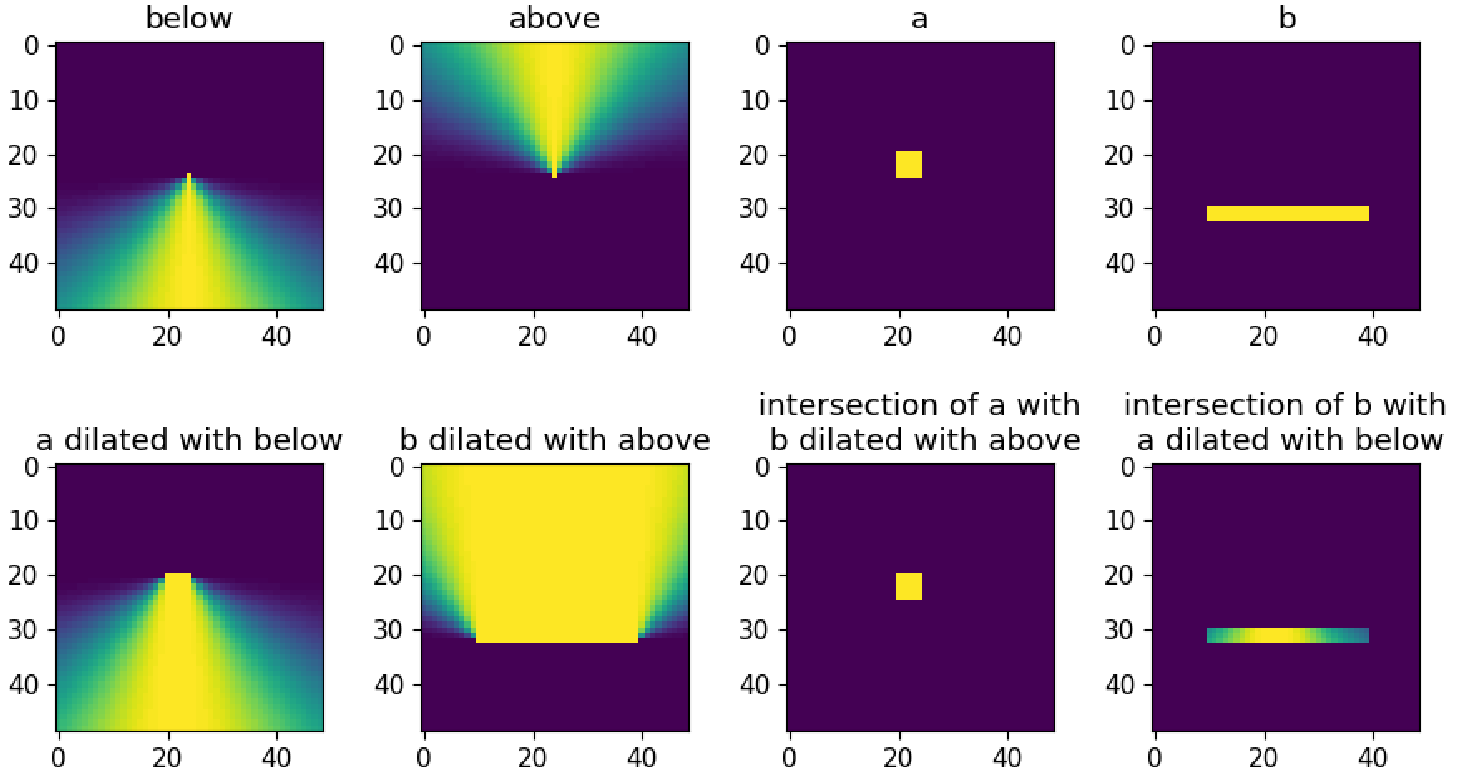

"below a"	"above b"	how much a is (in) "above b"	how much b is (in) "below a"

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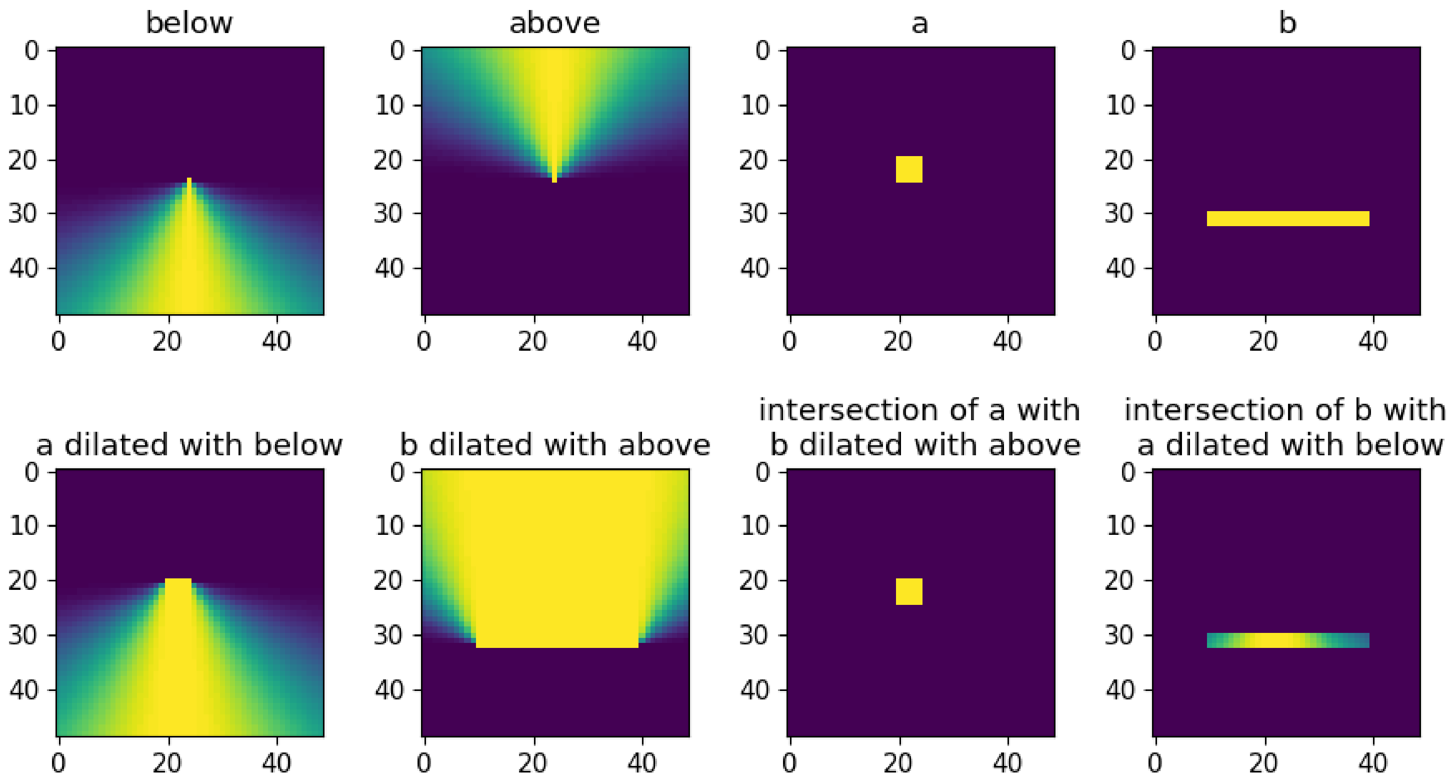

how much a is "above b"

operation scheme: $a \rightsquigarrow b + \text{“above”}$

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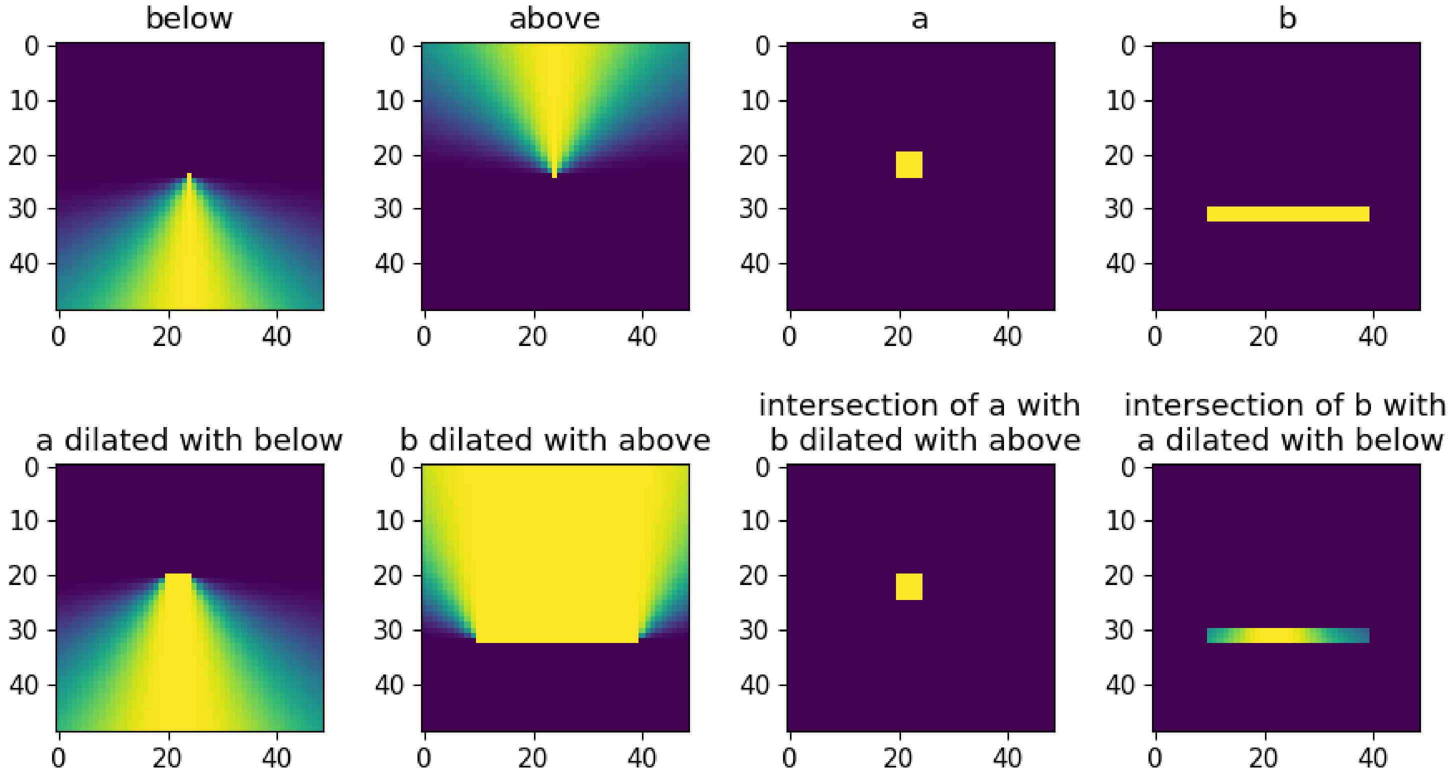

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inverse operation to contrast: ***merge***

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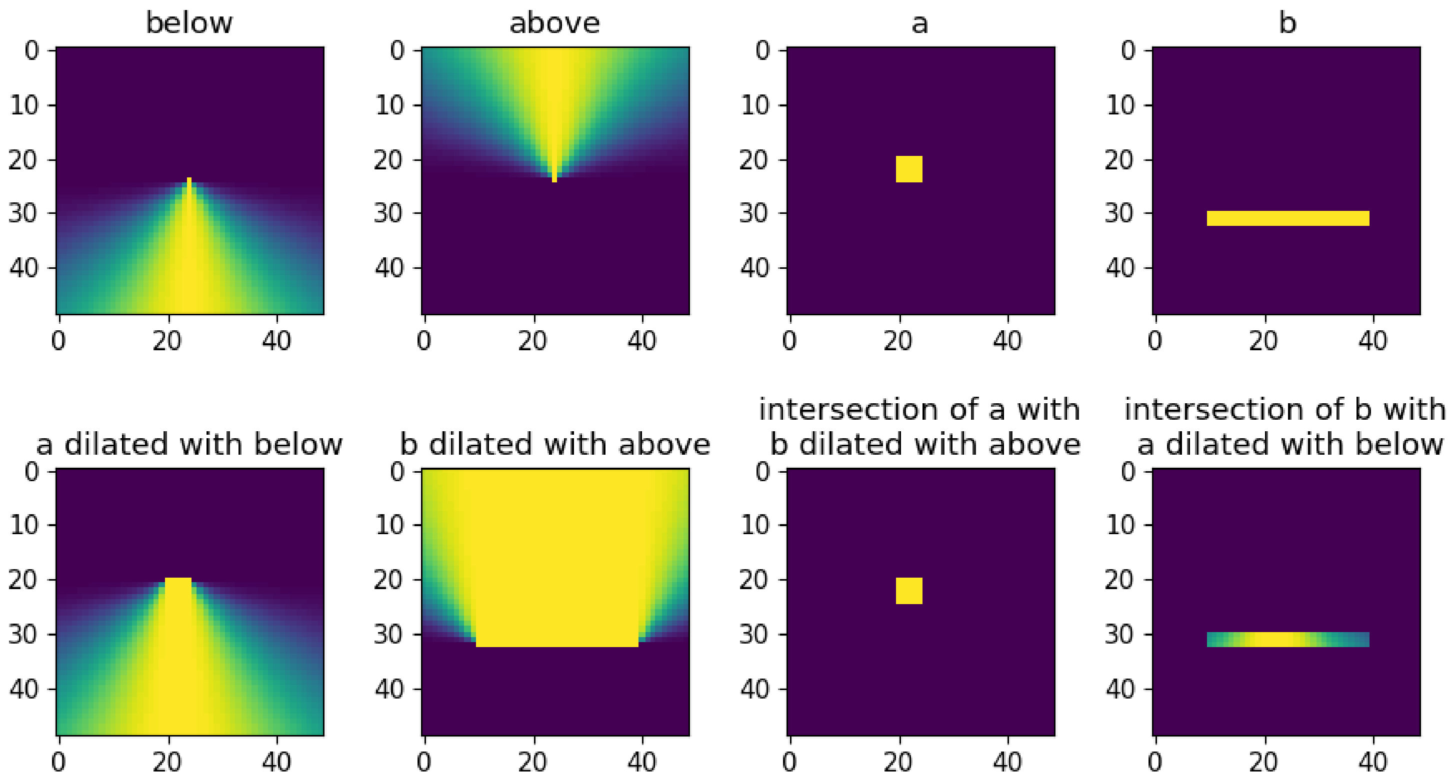

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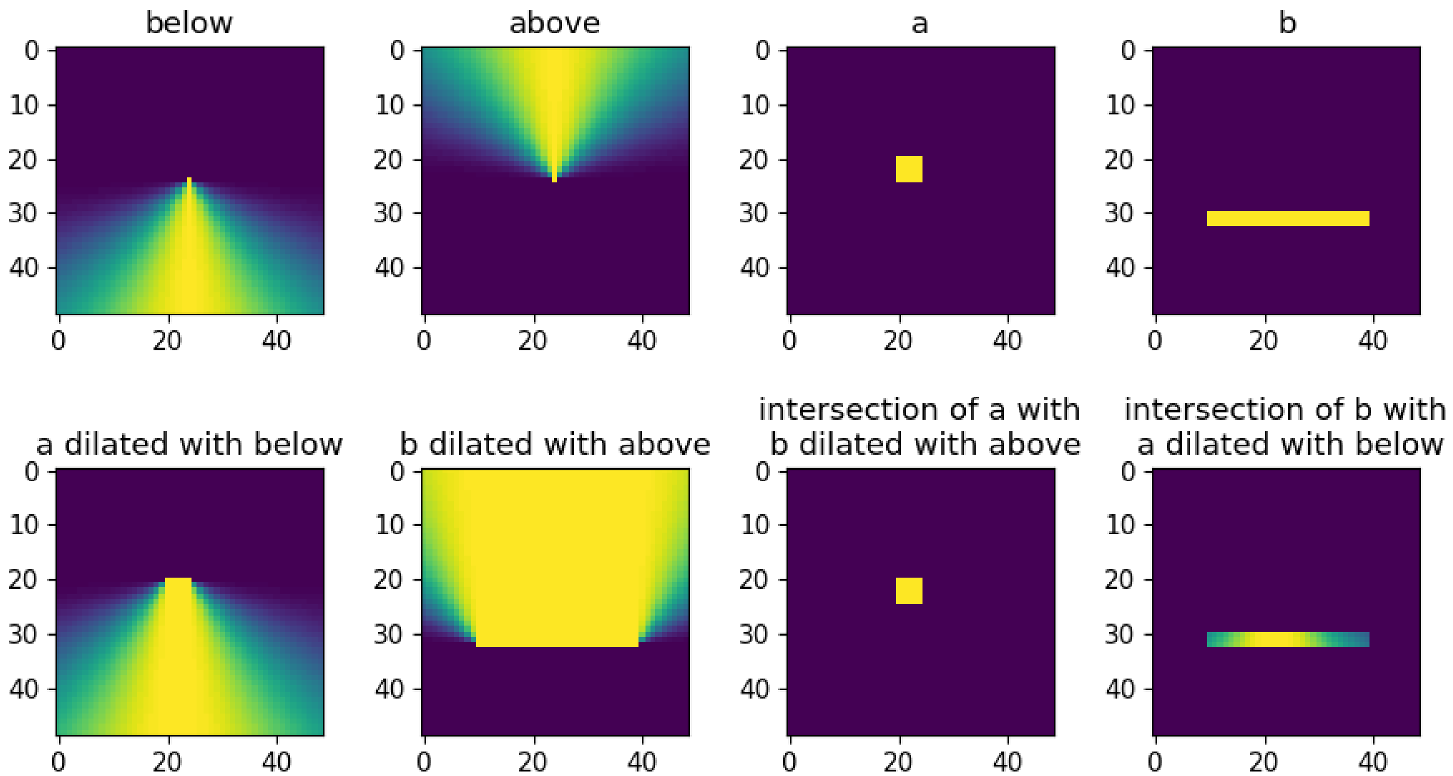

how much a is "above b"

inverse operation to contrast: ***merge***

operation scheme: *a* ↝ *b* + *"above"*

cf. with *o* - *p* ↝ *"red"*

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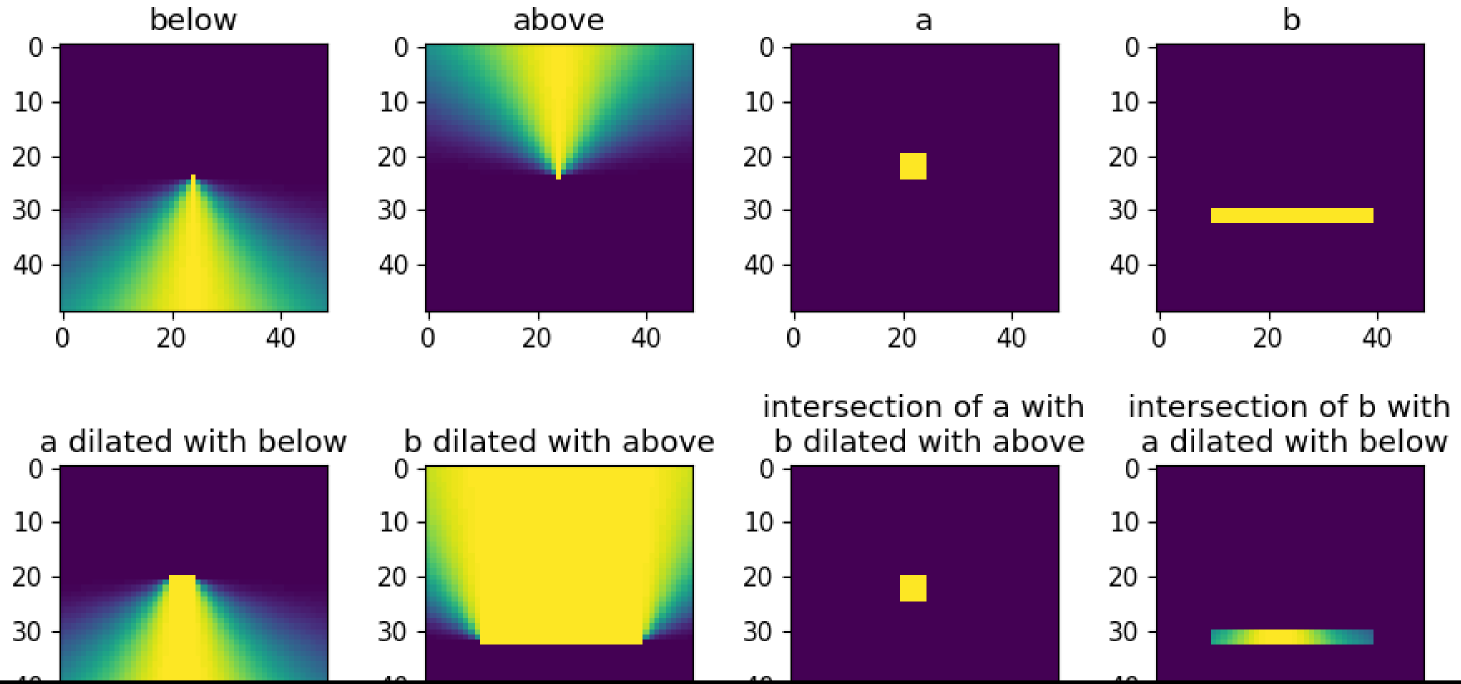

If we settle upon contrast, we can categorize its output for relations!

“above b”

operation scheme: $a \leadsto b$ + “*above*” *cf. with* o - $p \leadsto$ “*red*”

alignment as overlap

How does contrast work?

Computing contrast (1D)

- Consider coffees served in a bar. Intuitively, whether a coffee is qualified as being ***hot*** or ***cold*** depends mostly on what the speaker expects of coffees served at bars, rather than a specific absolute temperature.

$$c = o - p \rightsquigarrow \text{“hot”}$$

object
(target)

prototype
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- For simplicity, we represent objects on 1D (*temperature)* with **real coordinates**.

Computing contrast (1D)

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object (target)

prototype (reference)

- Because prototypes are defined together with a concept region, let us consider some regional information, for instance represented as an ***egg-yolk*** structure.

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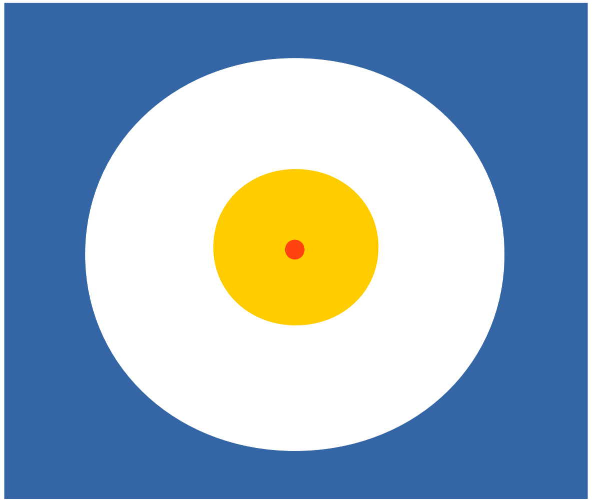

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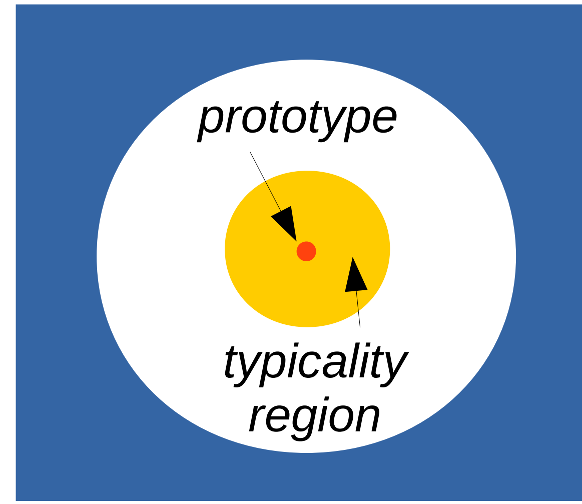

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  - internal boundary (*yolk*) $\boldsymbol{p \pm \sigma}$ for **typical** elements of that category of objects (e.g. coffee served at bar).

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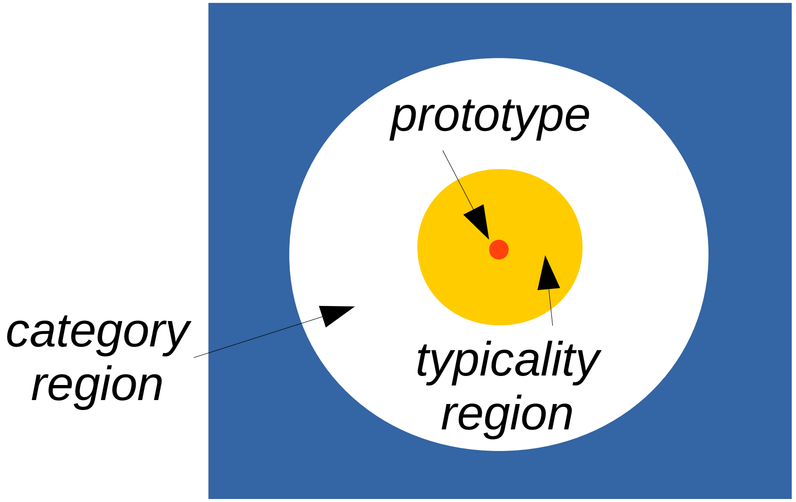
contrastor — ***object*** ***(target)*** — ***prototype*** ***(reference)***



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  - internal boundary (*yolk*) $\boldsymbol{p \pm \sigma}$ for **typical** elements of that category of objects (e.g. coffee served at bar).
  - external boundary (*egg*) $\boldsymbol{p \pm \rho}$ for **all** elements directly associated to that category of objects

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Computing contrast (1D)

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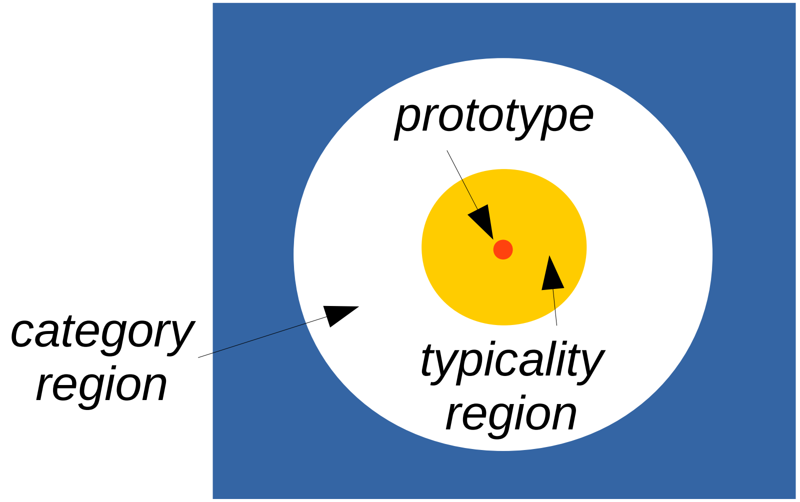
contrastor ***object (target)*** ***prototype (reference)***



- Two required functions:
  - ***centering*** of target with respect to typical region
  - ***scaling*** to neutralize effects of scale (e.g. "hot coffee" vs "hot planet")

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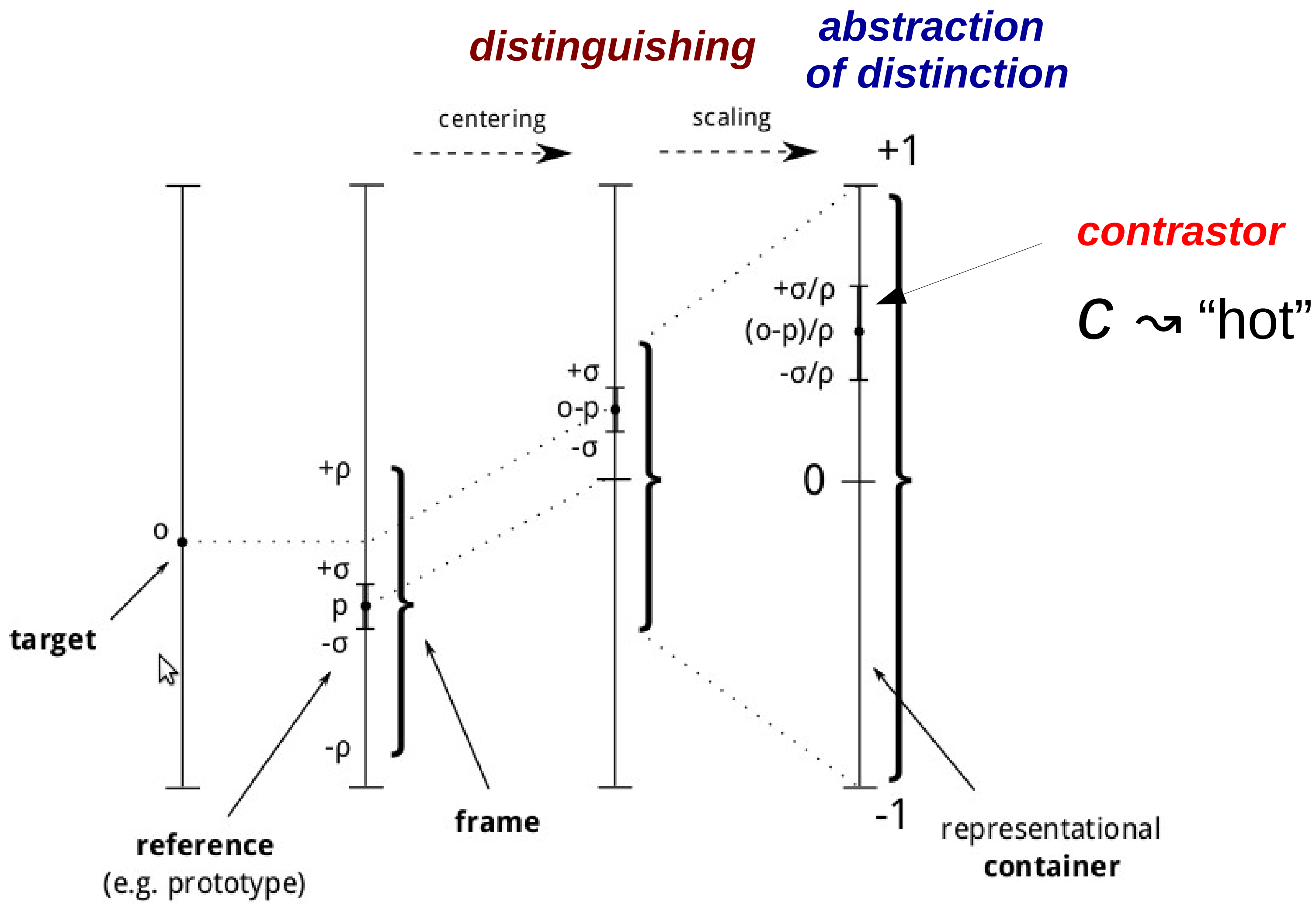
Computing contrast (1D)



$$C = \mathsf{contrast}(o, \langle p, \sigma, \rho \rangle) = \odot^{\sigma}_{o-p} * \frac{1}{\rho} = \odot^{\sigma/\rho}_{(o-p)/\rho}$$

Computing contrast (1D)

- As *contrastors* are extended objects, they might be compared to model categories represented as regions by measuring their *degree of overlap:*

contrastor

model region of property

$$\mathrm{strength}(r) = \frac{|C \cap M^{(r)}|}{|C|}$$

property label

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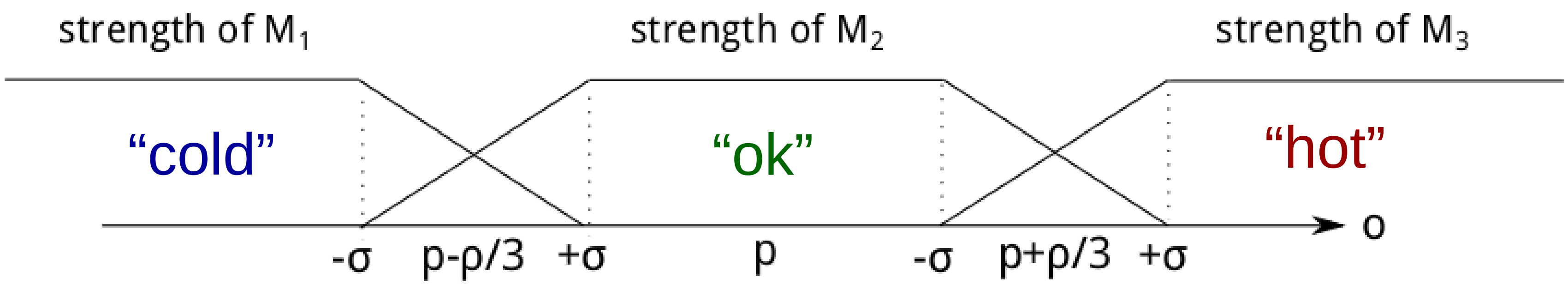
- Most distinctive property: $\arg\max_{r} |C \cap M^{(r)}|$

Computing contrast (1D)

- Applying the previous computation, we can easily derive the ***membership functions*** of some general relations with respect to the objects of that category.
- For instance, by dividing the representational container in 3 equal parts, we have:



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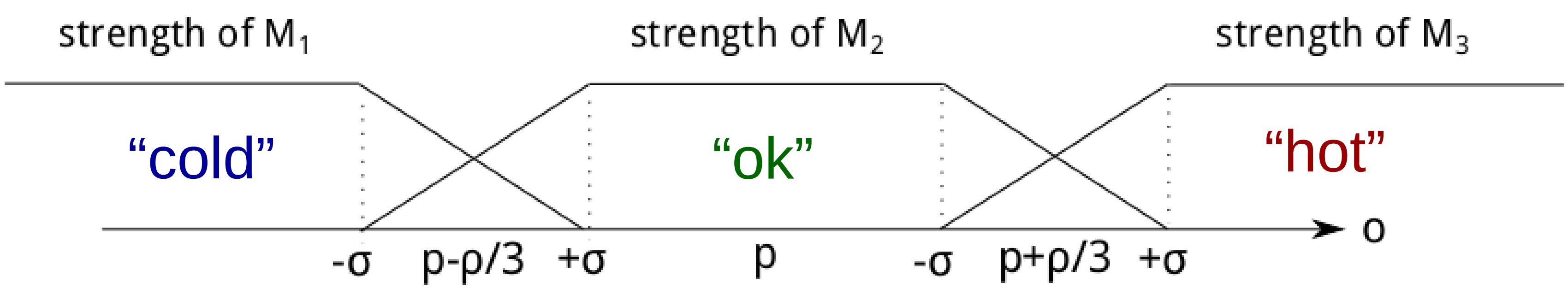
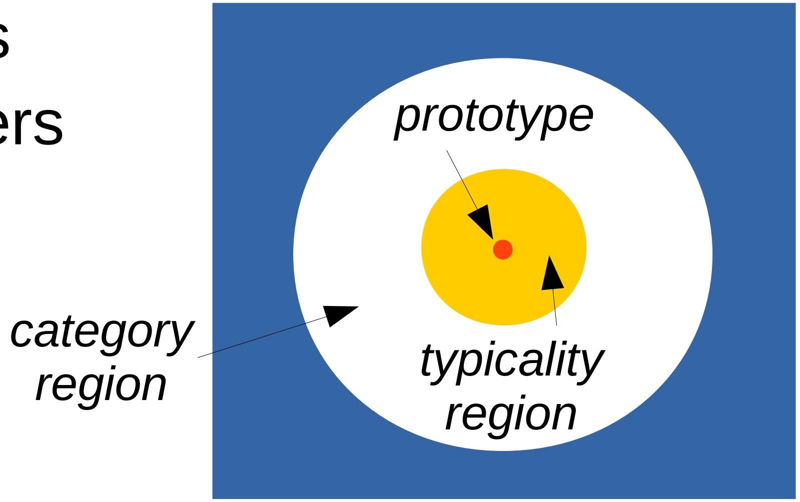


membership functions ***consequent*** to contrastive mechanisms

Adaptation of parameters

- Given a certain category of objects and a certain dimension, parameters are chosen such as that
  - σ captures the *most typical* exemplars
  - ρ covers *all* exemplars



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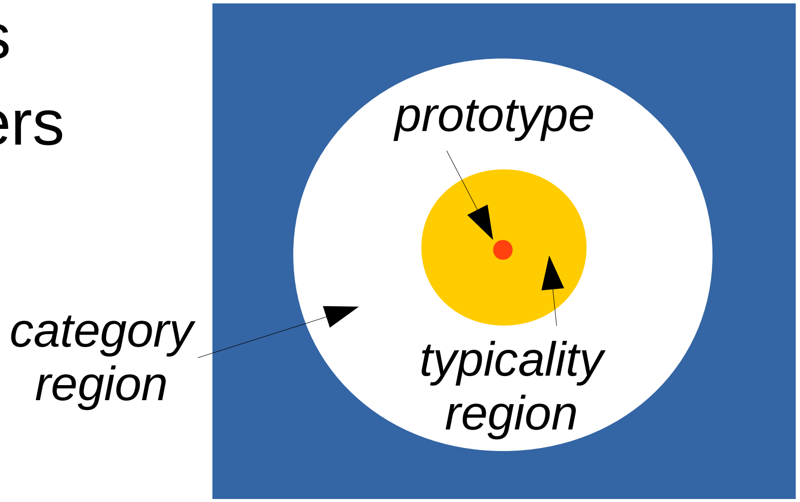

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  - ***relativization***: providing more contrastive exemplars, those which were highly contrastive before become less contrastive

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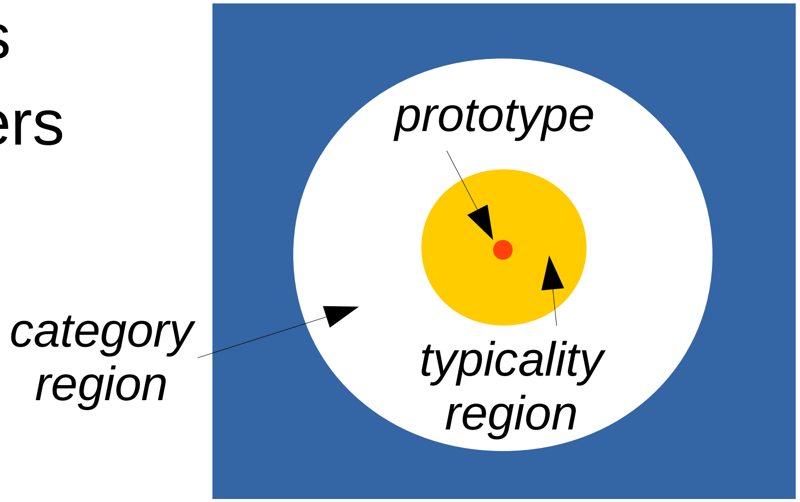

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  - if pruning of exemplars holds, ***hardening***: the concept region will recenter around the most recent elements.

Computing contrast (1D)

- The previous formulation might be extended to consider contrast between *two regions*, by utilizing ***discretization*** ($\lfloor \cdot \rceil$ denotes the approximation to the nearest integer):

$$C = \mathsf{contrast}_{\mathsf{R}}(\langle t, \tau \rangle, \langle r, \sigma \rangle, \langle f, \rho \rangle) \approx \mathsf{contrast}\left(\lfloor \frac{t}{2\tau} \rceil, \left\langle \lfloor \frac{r}{2\tau} \rceil, \lfloor \frac{\sigma}{\tau} \rceil, \lfloor \frac{\rho}{\tau} \rceil \right\rangle \right)$$

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- Simplification possible if the target region *much smaller* than the reference region...
- but in the other cases?
  - possible solution: <u>aggregation of contrastors obtained by point-wise contrast</u>

Computing contrast (>1D)

- Let us consider two 2D visual objects A and B (the two dimensions form a *Cartesian space*, and they are ***not*** perceptually independent).
- We can apply contrast iteratively for each point of A with respect to B, and then ***aggregate*** the resulting contrastors.

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accumulation set

point-wise distinguishing
based on vectorial difference

$$\mathcal{H}(A,B)(z) = \{a \in A, b \in B \mid a - b = z\}$$

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$$\mathcal{H}(A,B)(z) = \{a \in A, b \in B \mid a - b = z\}$$

counting

$$\mu_{A \ominus B}(z) = \frac{|\mathcal{H}(A,B)(z)|}{\max_w |\mathcal{H}(A,B)(w)|}$$

normalization

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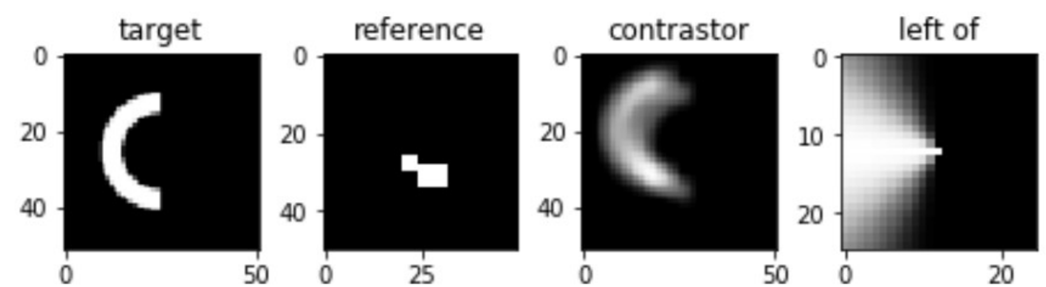

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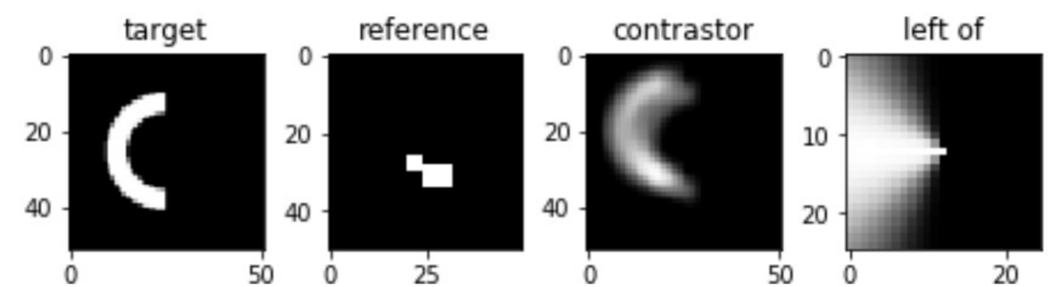

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Work in progress: use of ***erosion*** to compute contrast!

Computing contrast (>1D)

- If dimensions are ***perceptually independent***, we can apply contrast on each dimension separately:

$$C = (C_1, \ldots, C_n) = (\mathsf{contrast}(o_1, \langle p_1, \sigma_1, \rho_1 \rangle), \ldots, \mathsf{contrast}(o_n, \langle p_n, \sigma_n, \rho_n \rangle))$$

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- The result can be used to create a contrastive description of the object, i.e. its ***most distinguishing*** features.

- e.g. *apples* (as fruits):

  red, spherical, quite sugared

Computing contrast (>1D)

- Example: fruit domain from [Bechberger2017]:

concept	region			prototype (center)		
	hue	roundness	sweetness	hue	roundness	sweetness
pear	0.50–0.70	0.40–0.60	0.35–0.45	0.60	0.50	0.40
orange	0.80–0.90	0.90–1.00	0.60–0.70	0.80	0.95	0.65
lemon	0.70–0.80	0.45–0.55	0.00–0.10	0.75	0.50	0.05
granny smith	0.55–0.60	0.70–0.80	0.35–0.45	0.575	0.75	0.40
apple (green type)	0.50–0.80	0.65–0.80	0.35–0.50	0.65	0.725	0.425
apple (yellow type)	0.65–0.85	0.65–0.80	0.40–0.55	0.75	0.725	0.475
apple (red type)	0.70–1.00	0.65–0.80	0.45–0.60	0.85	0.725	0.525

Bechberger, L., Kuhnberger, K.U.: Measuring relations between concepts in conceptual spaces. Proceedings of SGAI 2017 LNAI 10630, pp. 87–100 (2017)

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concept	region			prototype (center/centroid)		
	hue	roundness	sweetness	hue	roundness	sweetness
apple	0.50–1.00	0.65–0.80	0.35–0.60	0.75/0.75	0.725/0.725	0.475/0.475
fruit	0.50–1.00	0.40–1.00	0.00–0.70	0.75/0.72	0.70/0.70	0.35/0.42

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granny smith	0.55–0.60	0.70–0.80	0.35–0.45	0.575	0.75	0.40
apple (green type)	0.50–0.80	0.65–0.80	0.35–0.50	0.65	0.725	0.425
apple (yellow type)	0.65–0.85	0.65–0.80	0.40–0.55	0.75	0.725	0.475
apple (red type)	0.70–1.00	0.65–0.80	0.45–0.60	0.85	0.725	0.525

concept	region			prototype (center/centroid)		
	hue	roundness	sweetness	hue	roundness	sweetness
apple	0.50–1.00	0.65–0.80	0.35–0.60	0.75/0.75	0.725/0.725	0.475/0.475
fruit	0.50–1.00	0.40–1.00	0.00–0.70	0.75/0.72	0.70/0.70	0.35/0.42

Container regions can be used as basis to extract distinctive features

Computing contrast (>1D)

- For instance, by contrasting each fruit concept with the aggregate “fruit” concept (using *discretization*, taking σ = 0.5ρ), we obtain the following contrastors (centers):

concept	hue	roundness	sweetness	*red*	*green*	*blue*
pear	−0.6	−0.7	0.1	−0.3	1.0	0.0
orange	0.4	0.8	0.4	1.0	0.0	−1.0
lemon	0.0	−0.7	−0.4	0.8	0.8	−1.0
apple	0.0	0.1	0.2	0.0	0.0	0.0




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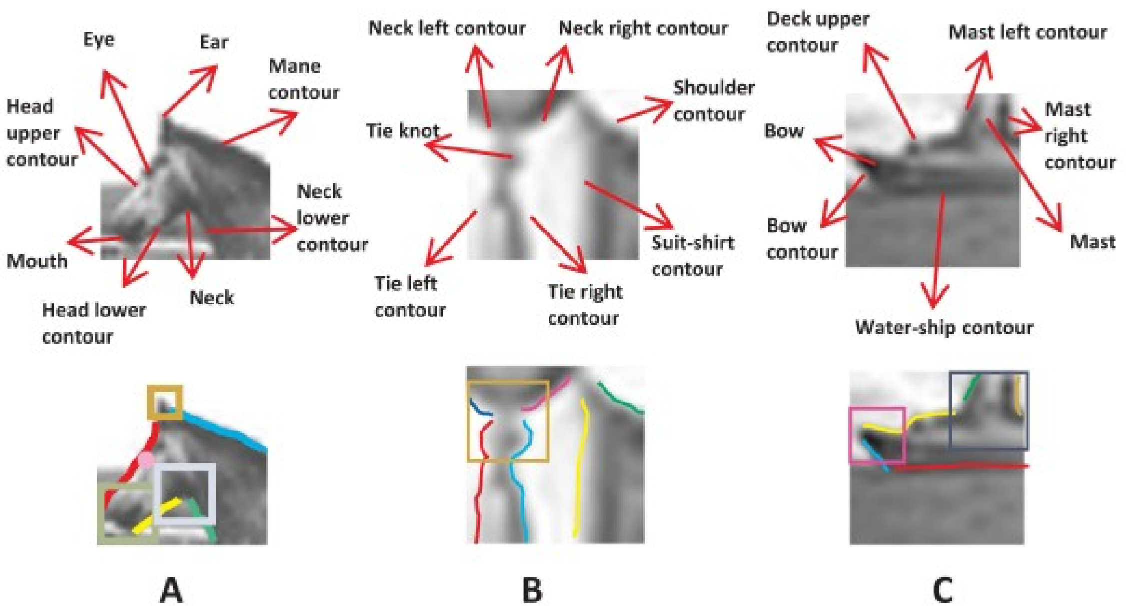
In principle, a similar output could provide the ***weights*** of features in forming a certain concept → basis for *object categorization*

Individuation and concept formation

- We believe discriminatory aspects might be crucial not only for ***individuation***, but also for the ***formation of concepts***.
  - this is aligned with recent empirical experiences [Ben-Yosef2018] showing the fundamental role of the spatial organization of visual elements in *object recognition* tasks.



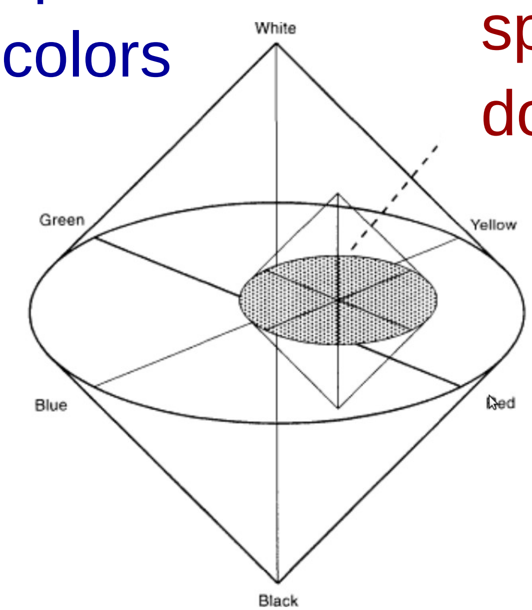
Ben-Yosef, G., Assif, L., Ullman, S.: Full interpretation of minimal images. Cognition 171, pp. 65–84 (2018)

Conclusion

- By referring to a **contrast** mechanism:
  - ***membership functions*** become derived objects,
  - references and frames provide a natural ***contextualization***,
  - ***modifier-head*** concept combinations are directly implemented (no need of *contrast classes*).

space of colors

space of dog colors



adapted from Gärdenfors, P. (2000). Conceptual Spaces: The Geometry of Thought. MIT Press.

Conclusion

- By referring to a **contrast** mechanism:
  - ***membership functions*** become derived objects,
  - references and frames provide a natural ***contextualization***,
  - ***modifier-head*** concept combinations are directly implemented (no need of *contrast classes*),
  - problems with geometric axioms in relation to ***similarity judgments*** (*symmetry*, *triangle inequality*, *minimality*, *diagnosticity effect*) are easily explained [Sileno2017]

Sileno, G., Bloch, I., Atif, J., & Dessalles, J.-L. (2017). Similarity and Contrast on Conceptual Spaces for Pertinent Description Generation. Proceedings of the 2017 KI conference, 10505 LNAI.

Conclusion

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  - ***membership functions*** become derived objects,
  - references and frames provide a natural ***contextualization***,
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- Future research track:
  - contrast is defined in duality with merge,
  - merge produces order relations between concepts
  - the resulting lattice is a space of concepts

Do conceptual spaces emerge from contrastive functions?