Logic and Knowledge Representation

Programming as problem-solving, First steps in Prolog

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Logic and Knowledge Representation

This course is an introduction to *symbolic* techniques in Artificial Intelligence (AI), following an homonym course held by **Jean-Louis Dessalles**.

The course website:

http://aicourse.r2.enst.fr:4242/SCIA

Frontal teacher: **Giovanni Sileno**, gsileno@enst.fr

Do not hesitate to write me or ask questions in class!

Logic and Knowledge Representation

Being a new course, we will plausibly tweak on the fly the content, the exercises, and the workload.

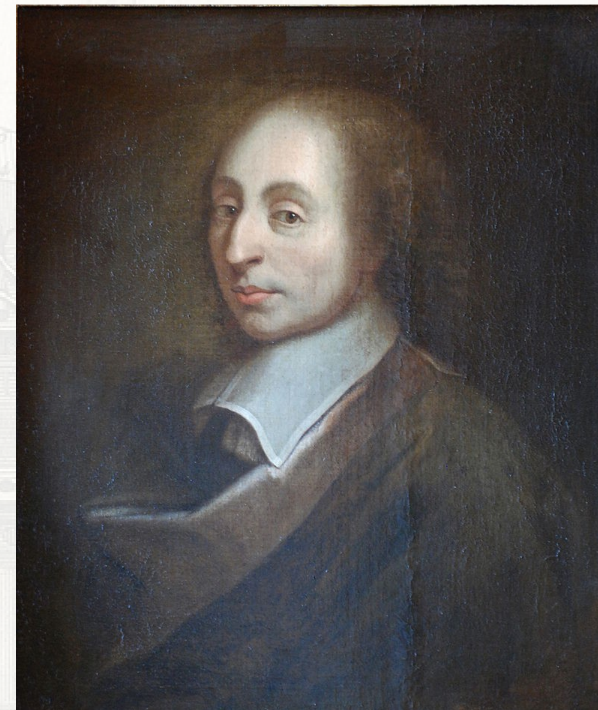
In principle, your grade will be a weighted average of:

- weekly exercises proposed on the course webpages
- a final exam
- a programming project

“Mechanical” computing

Pascal: Pascaline ~ 1650

Helping his father (tax accountant of Normandy, appointed by Richelieu), Pascal invented a machine for *mechanic calculation*, performing **addition** and **subtraction**.



Blaise Pascal

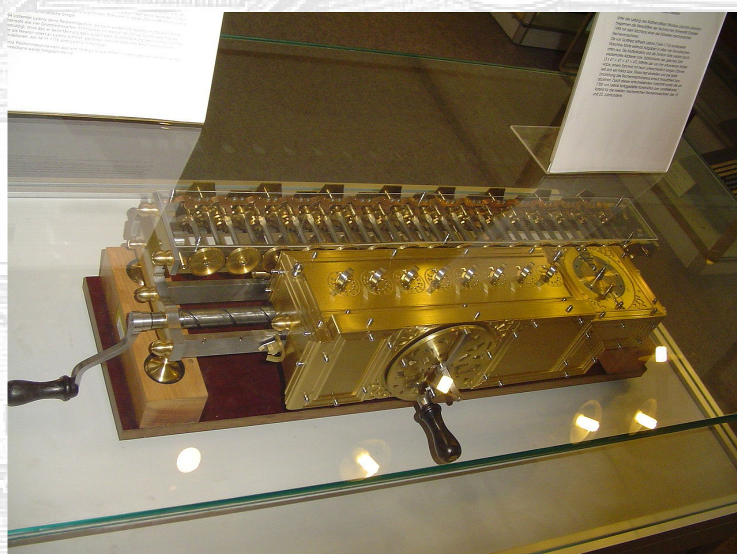
Schickard: Calculating Clock ~1625

Before him, Schickard had already invented an “artithmetic instrument”, but unfortunately he was not able to publicly present a full working copy.

Wilhelm Schickard

Leibniz: Stepped Reckoner ~1680

Influenced by the Pascaline, Leibniz proposed a mechanic calculator performing all four operations: **addition**, **subtraction**, **multiplication** and **division**.



Gottfried Wilhelm von Leibniz

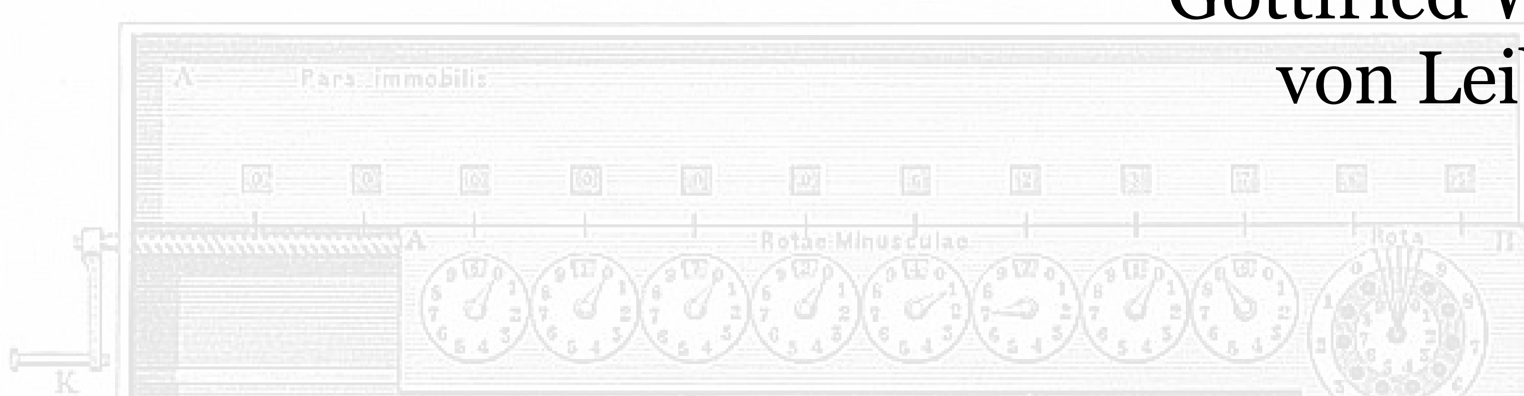
Leibniz: *Calculemus!* ~1686

Furthermore, Leibniz believed that calculation would be the key to settle all human conflicts and disagreements...

→ ***characteristics numbers*** instead of concepts.

Gottfried Wilhelm von Leibniz

2. Rechenmaschine von Leibniz (1673, Hannover).



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→ ***characteristics numbers***
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animal = 2
rational = 3
man = rational animal = 2*3 = 6

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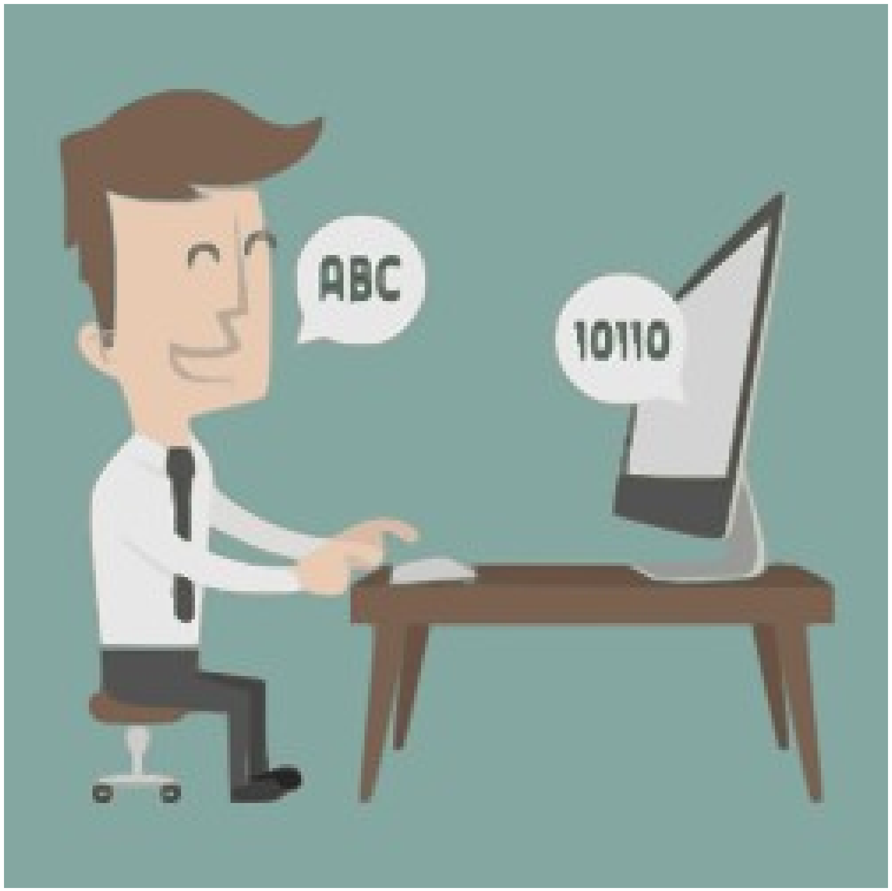
→ ***characteristics numbers***
instead of concepts.

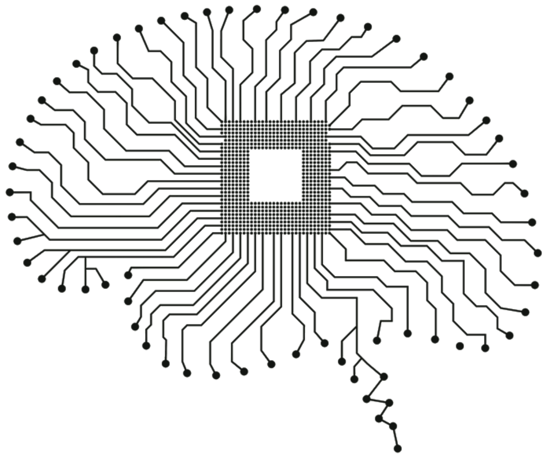
animal = 2
rational = 3
man = rational animal = 2*3 = 6
Is every man rational?
Yes, because 6 is divisible by 3.

Gottfried Wilhelm von Leibniz

Machines as symbol handlers



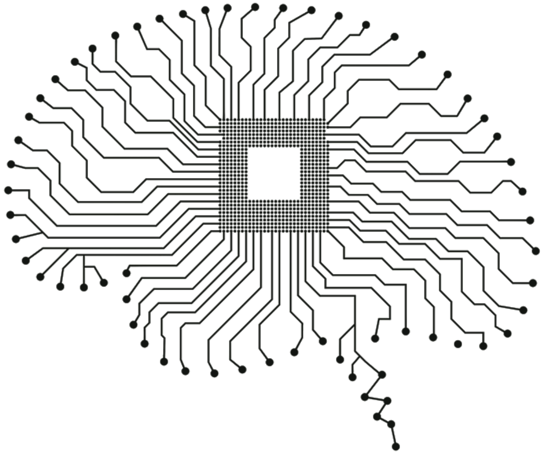
Starting from the Pascaline, computing machines respond to the need to displace tedious, repetitive (symbolic) work.



A physical symbol system has the *necessary* and *sufficient* means for general intelligent action

Allen Newell and Herbert A. Simon

Computer Science as Empirical Inquiry: Symbols and Search (1976)



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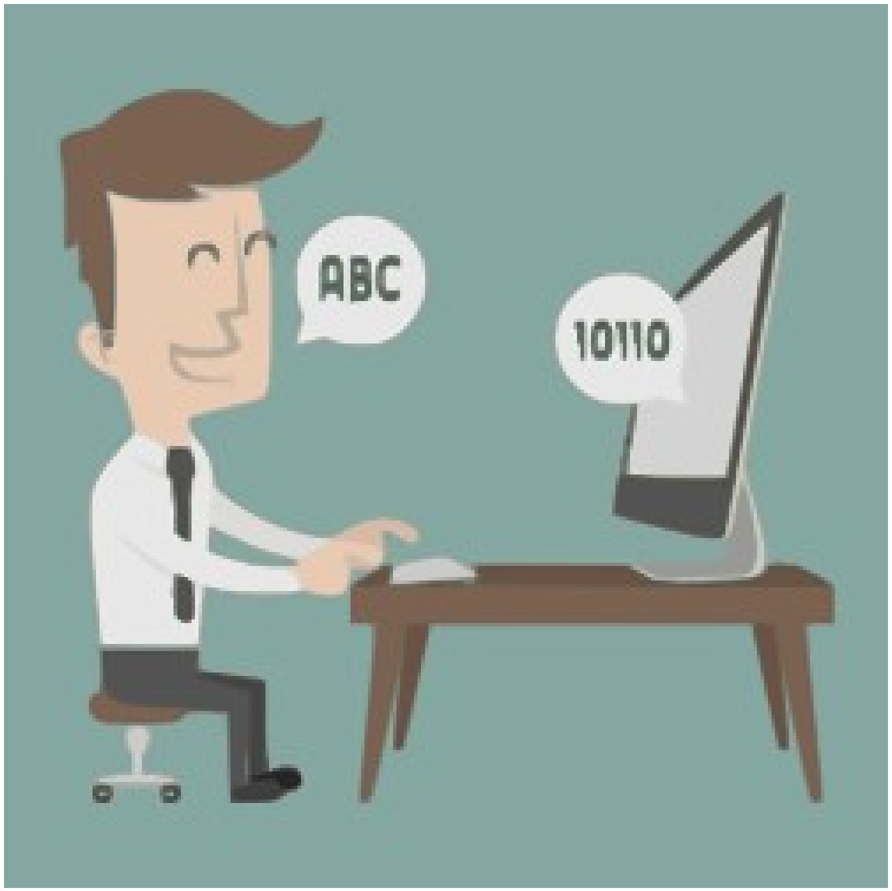
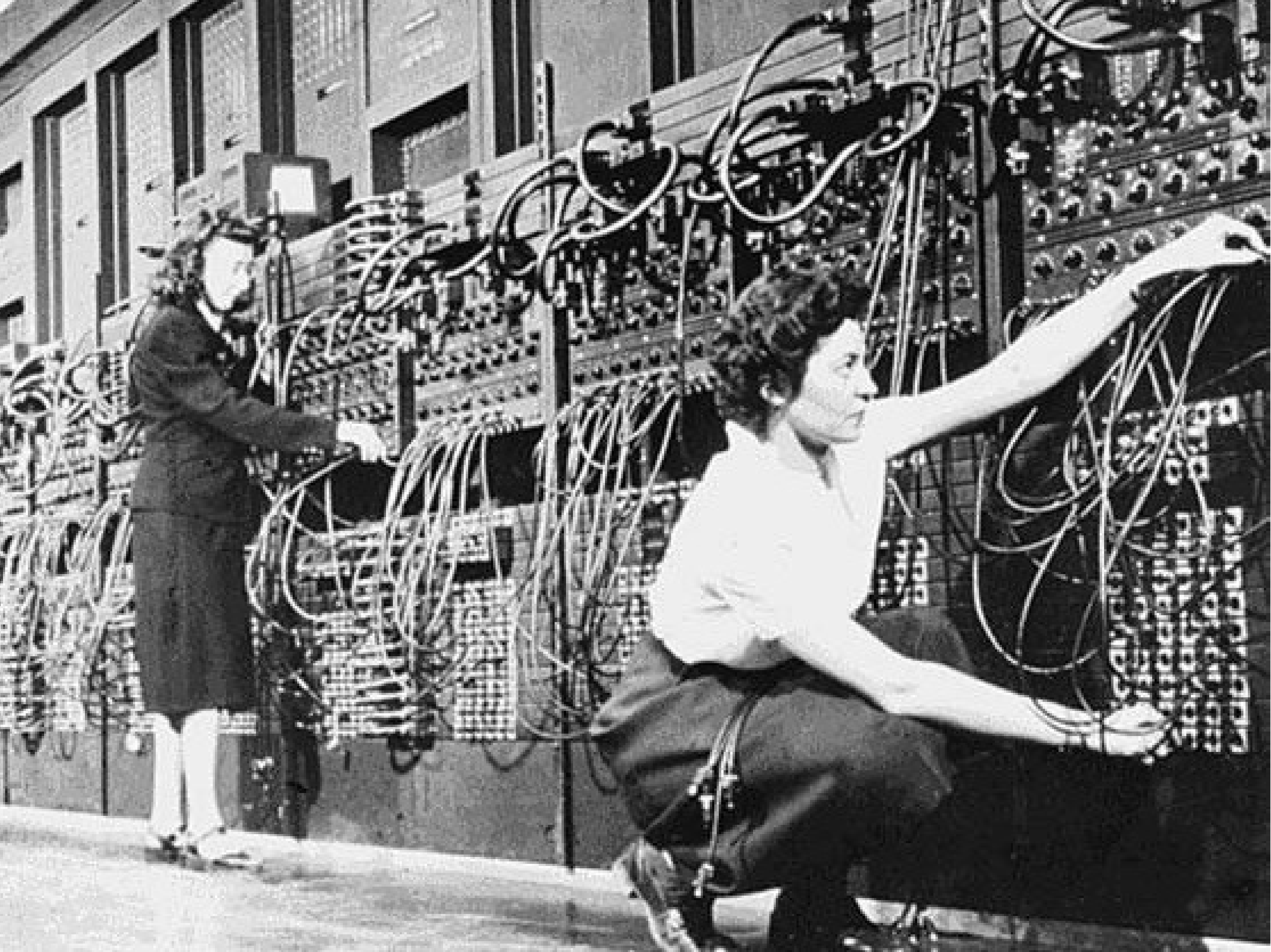
...basis for Good Old Fashioned AI (GOFAI)

Machines as symbol handlers



Starting from the Pascaline, computing machines respond to the need to displace tedious, repetitive (symbolic) work.

but **how to say to the machine what to do?**



Machine code/instructions

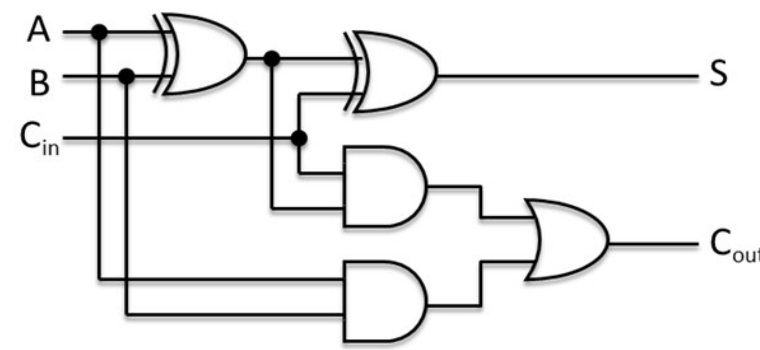
Related to the physical structure of the computer.

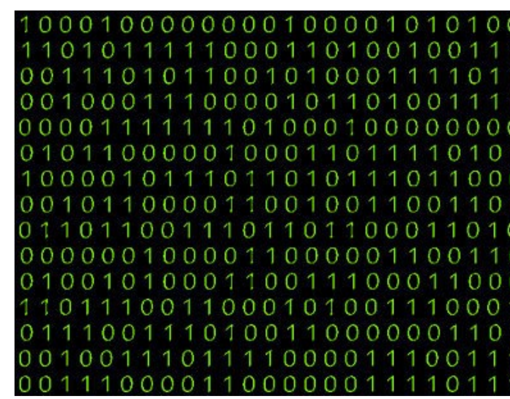
\+ poweful and fast

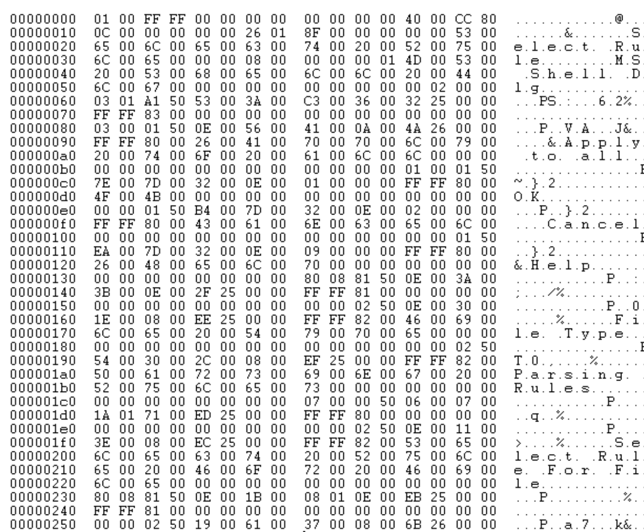
\- long programs

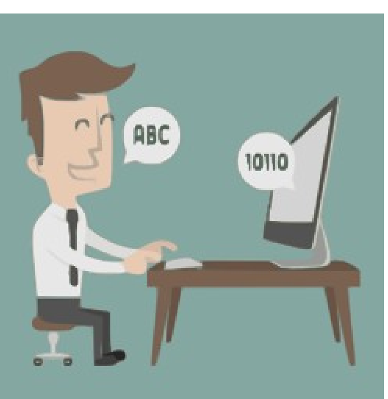
\- difficult to be written

\- difficult to be revised






```
00000000  01 00 FF FF 00 00 00 00  00 00 00 00 40 00 CC 80  ............@...
00000010  0C 00 00 00 00 00 26 01  8F 00 00 00 00 00 53 00  ......&.......S.
00000020  65 00 6C 00 65 00 63 00  74 00 20 00 52 00 75 00  e.l.e.c.t. .R.u.
00000030  6C 00 65 00 00 00 08 00  00 00 00 01 4D 00 53 00  l.e.........M.S.
00000040  20 00 53 00 68 00 65 00  6C 00 6C 00 20 00 44 00   .S.h.e.l.l. .D.
00000050  6C 00 67 00 00 00 00 00  00 00 00 00 00 02 00 00  l.g.............
00000060  03 01 A1 50 53 00 3A 00  C3 00 36 00 32 25 00 00  ...PS.:...6.2%..
00000070  FF FF 83 00 00 00 00 00  00 00 00 00 00 00 00 00  ................
00000080  03 00 01 50 0E 00 56 00  41 00 0A 00 4A 26 00 00  ...P..V.A...J&..
00000090  FF FF 80 00 26 00 41 00  70 00 70 00 6C 00 79 00  ....&.A.p.p.l.y.
000000a0  20 00 74 00 6F 00 20 00  61 00 6C 00 6C 00 00 00   .t.o. .a.l.l...
000000b0  00 00 00 00 00 00 00 00  00 00 00 00 01 00 01 50  ...............P
000000c0  7E 00 7D 00 32 00 0E 00  01 00 00 00 FF FF 80 00  ~.}.2...........
000000d0  4F 00 4B 00 00 00 00 00  00 00 00 00 00 00 00 00  O.K.............
000000e0  00 00 01 50 B4 00 7D 00  32 00 0E 00 02 00 00 00  ...P..}.2.......
000000f0  FF FF 80 00 43 00 61 00  6E 00 63 00 65 00 6C 00  ....C.a.n.c.e.l.
00000100  00 00 00 00 00 00 00 00  00 00 00 00 00 00 01 50  ...............P
00000110  EA 00 7D 00 32 00 0E 00  09 00 00 00 FF FF 80 00  ..}.2...........
00000120  26 00 48 00 65 00 6C 00  70 00 00 00 00 00 00 00  &.H.e.l.p.......
00000130  00 00 00 00 00 00 00 00  80 08 81 50 0E 00 3A 00  ...........P..:.
00000140  3B 00 0E 00 2F 25 00 00  FF FF 81 00 00 00 00 00  ;.../%..........
00000150  00 00 00 00 00 00 00 00  00 00 02 50 0E 00 30 00  ...........P..0.
00000160  1E 00 08 00 EE 25 00 00  FF FF 82 00 46 00 69 00  .....%......F.i.
00000170  6C 00 65 00 20 00 54 00  79 00 70 00 65 00 00 00  l.e. .T.y.p.e...
00000180  00 00 00 00 00 00 00 00  00 00 00 00 00 00 02 50  ...............P
00000190  54 00 30 00 2C 00 08 00  EF 25 00 00 FF FF 82 00  T.0.,....%......
000001a0  50 00 61 00 72 00 73 00  69 00 6E 00 67 00 20 00  P.a.r.s.i.n.g. .
000001b0  52 00 75 00 6C 00 65 00  73 00 00 00 00 00 00 00  R.u.l.e.s.......
000001c0  00 00 00 00 00 00 00 00  07 00 00 50 06 00 07 00  ...........P....
000001d0  1A 01 71 00 ED 25 00 00  FF FF 80 00 00 00 00 00  ..q..%..........
000001e0  00 00 00 00 00 00 00 00  00 00 02 50 0E 00 11 00  ...........P....
000001f0  3E 00 08 00 EC 25 00 00  FF FF 82 00 53 00 65 00  >....%......S.e.
00000200  6C 00 65 00 63 00 74 00  20 00 52 00 75 00 6C 00  l.e.c.t. .R.u.l.
00000210  65 00 20 00 46 00 6F 00  72 00 20 00 46 00 69 00  e. .F.o.r. .F.i.
00000220  6C 00 65 00 00 00 00 00  00 00 00 00 00 00 00 00  l.e.............
00000230  80 08 81 50 0E 00 1B 00  08 01 0E 00 EB 25 00 00  ...P.........%..
00000240  FF FF 81 00 00 00 00 00  00 00 00 00 00 00 00 00  ................
00000250  00 00 02 50 19 00 61 00  37 00 08 00 6B 26 00 00  ...P..a.7...k&..
00000260  FF FF 82 00 00 00 00 00                           ........
```

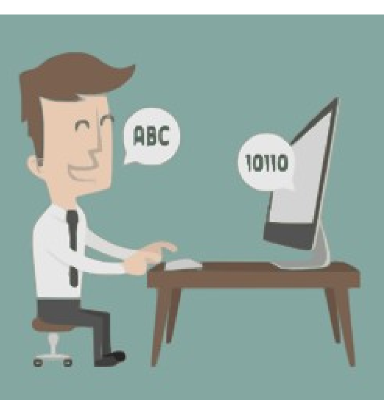


Natural (human) language

It is the language we use in all our communications, learned since our childhood.

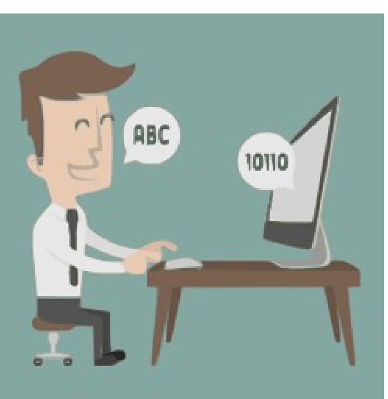
+ Expressively rich.

- Ambiguous, redundant.

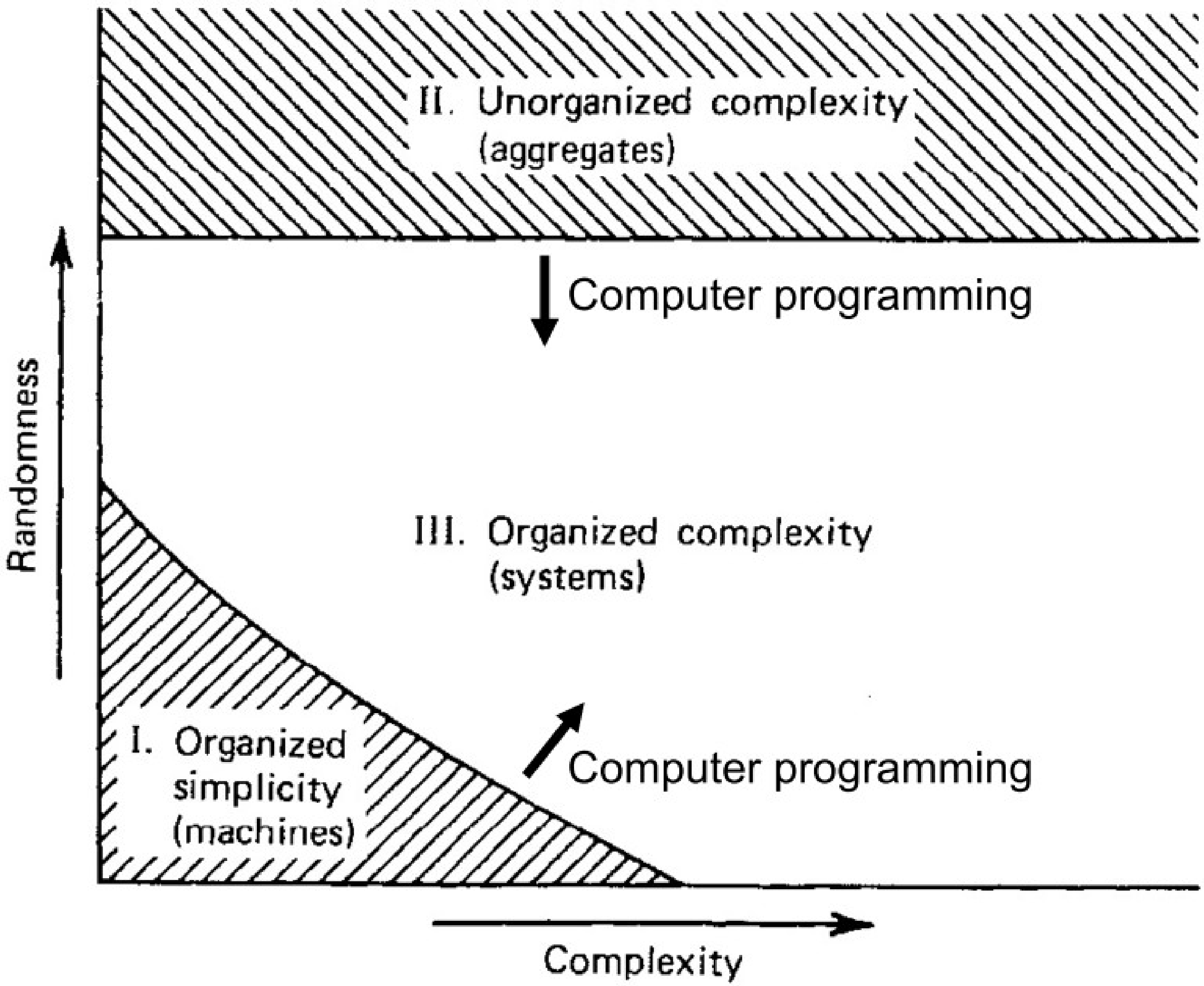


Programming languages

A programming language is a language which is *intermediary* between machine code and natural language.



Programming languages

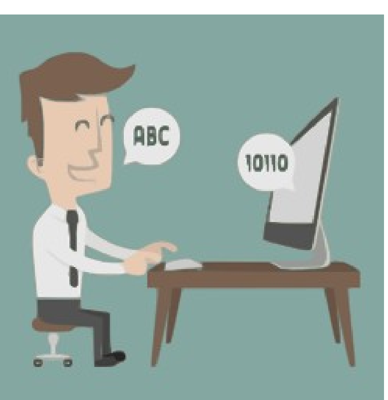


VanRoy 2009,
Weinberg 1977

Programming languages

A programming language is a language which is *intermediary* between machine language and natural language.

- But **what** we have to tell to the machine?



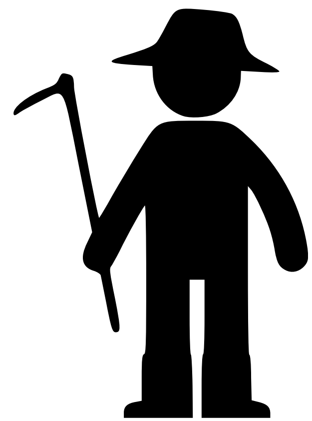
Programming as problem-solving

Problem solving terms

- A *well-defined* problem is usually defined in terms of

  - an **initial state** or *situation*
  - a **goal state**, i.e. a *desired outcome*,
  - certain **resources** (which put contraints on the possible paths towards the goal).

An ancient puzzle ~ 9th century



- Once upon a time a farmer went to market and purchased a **fox**, a **goose**, and a **bag of beans.** On his way home, the farmer came to the bank of a river. In crossing the river by boat, the farmer could carry only himself and a single one of his purchases - the fox, the goose, or the bag of the beans.

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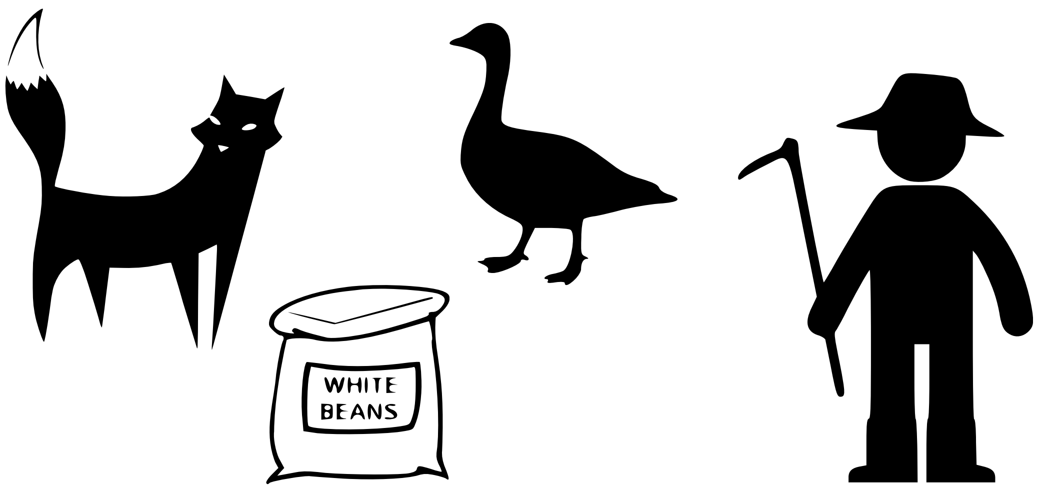

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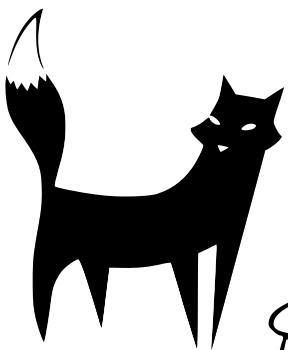

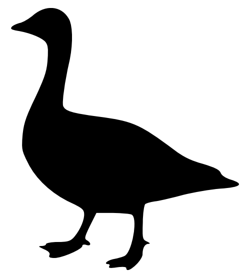

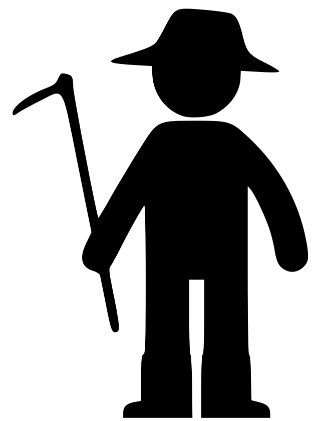

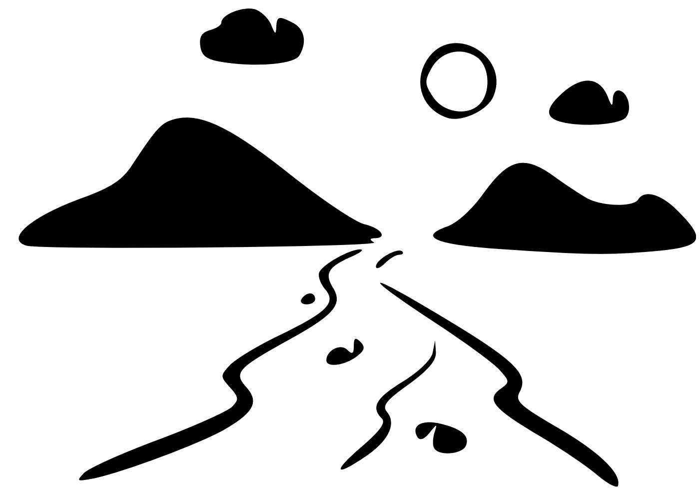

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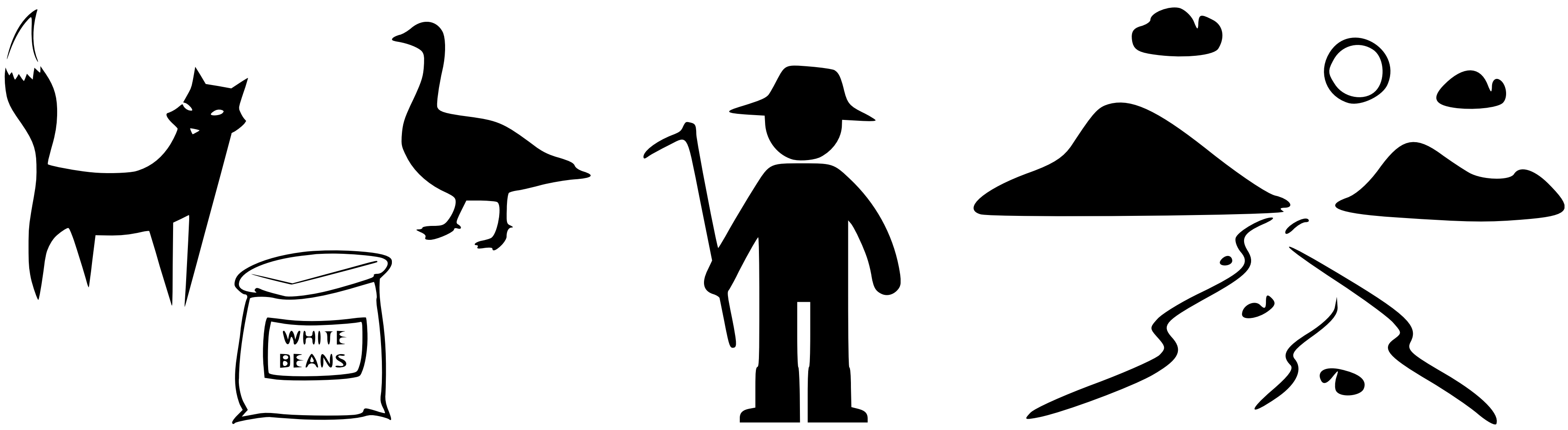
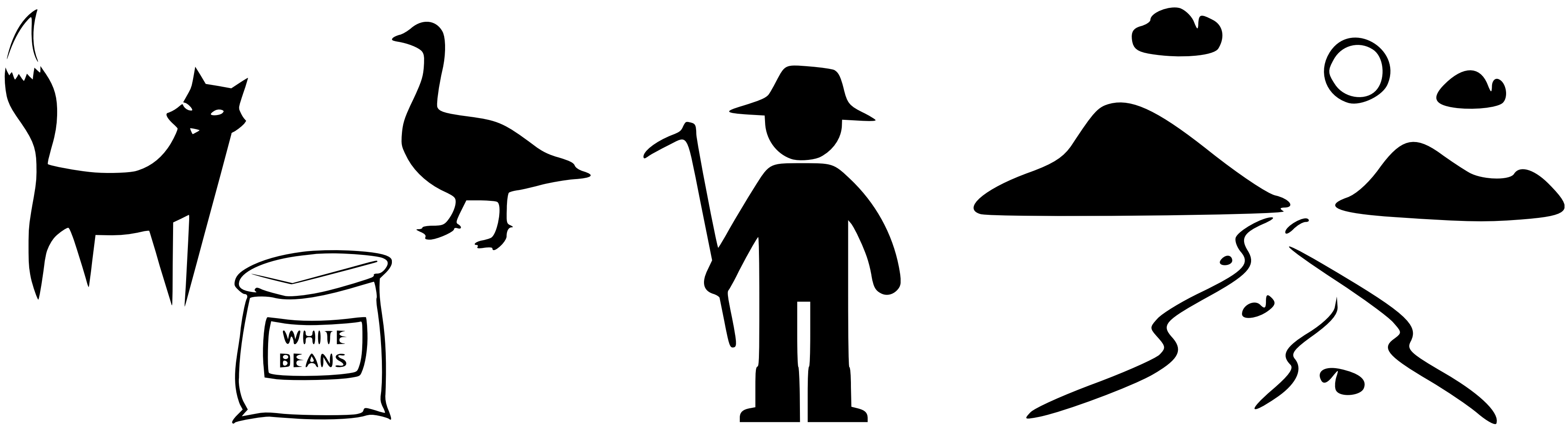


- If left together, the fox would eat the goose, or the goose would eat the beans.
- The farmer's challenge was to carry himself and his purchases to the far bank of the river, leaving each purchase intact. How did he do it?

An ancient puzzle ~ 9th century



- What is the goal?
- What is the initial situation?
- Which are the resources/constraints?

From *problems* to *solvers*

Problem

↓ Analysis

Problem solving method

↓ Planning

Problem solving task

↓ Execution

Solution

Problem

↓ Analysis

Algorithm

↓ Programming

Program

↓ Execution

Outcome

From *problems* to *solvers*

Problem

↓ Analysis

Problem solving method

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Solution

Problem

↓ Analysis

Algorithm

↓ Programming

Program

↓ Execution

Outcome

The "real" problem is not programming
but finding the path toward the solution.

Imperative vs Declarative

Imperative vs Declarative

- Imperative:
  - programming focusing on the **sequence of operations** necessary to solve the problem (which in turn usually stays implicit)
- Declarative
  - programming focusing on describing the **problem** (while the sequence of operations to be performed is left implicit)

Imperative programming

Focus: *how to compute*

Based on instructions, correspondent to actions **commanded** to the machine.

- It assumes that the computer can maintain the changes (the *side-effects*) caused by the computation process.

Imperative programming

Focus: *how to compute*

- Most popular programming languages implement the imperative paradigm:
  - it most closely resembles the actual machine itself, so the programmer thinks in a much closer way to the machine;
  - because of such closeness, it was until recently the only one efficient enough for widespread use.

Imperative programming

- Advantages
  - efficient as close to the machine
  - popular
  - familiar
- Disadvantages
  - a program can be complex to understand, because the *referential transparency* does not hold (due to **side effects**)
  - *abstraction* is more limited
  - **order** is crucial, which is not suited in certain problems

Declarative programming

Focus: *what to compute (as desired outcome)*

- It is not concerned about how to do things, but what should be obtained.
  - Languages: domain specific (e.g. HTML), query (SQL), logic (Prolog).

Declarative/Logic programming

Focus: *what to compute (as desired outcome)*

- Various logical assertions about a situation are made, describing all known **facts** and **rules** about the modeled world. Then **queries** are made.
- The role of the computer is to *maintain data* and to perform *inferences.*

Algorithm = Logic + Control

"An algorithm can be regarded as consisting of

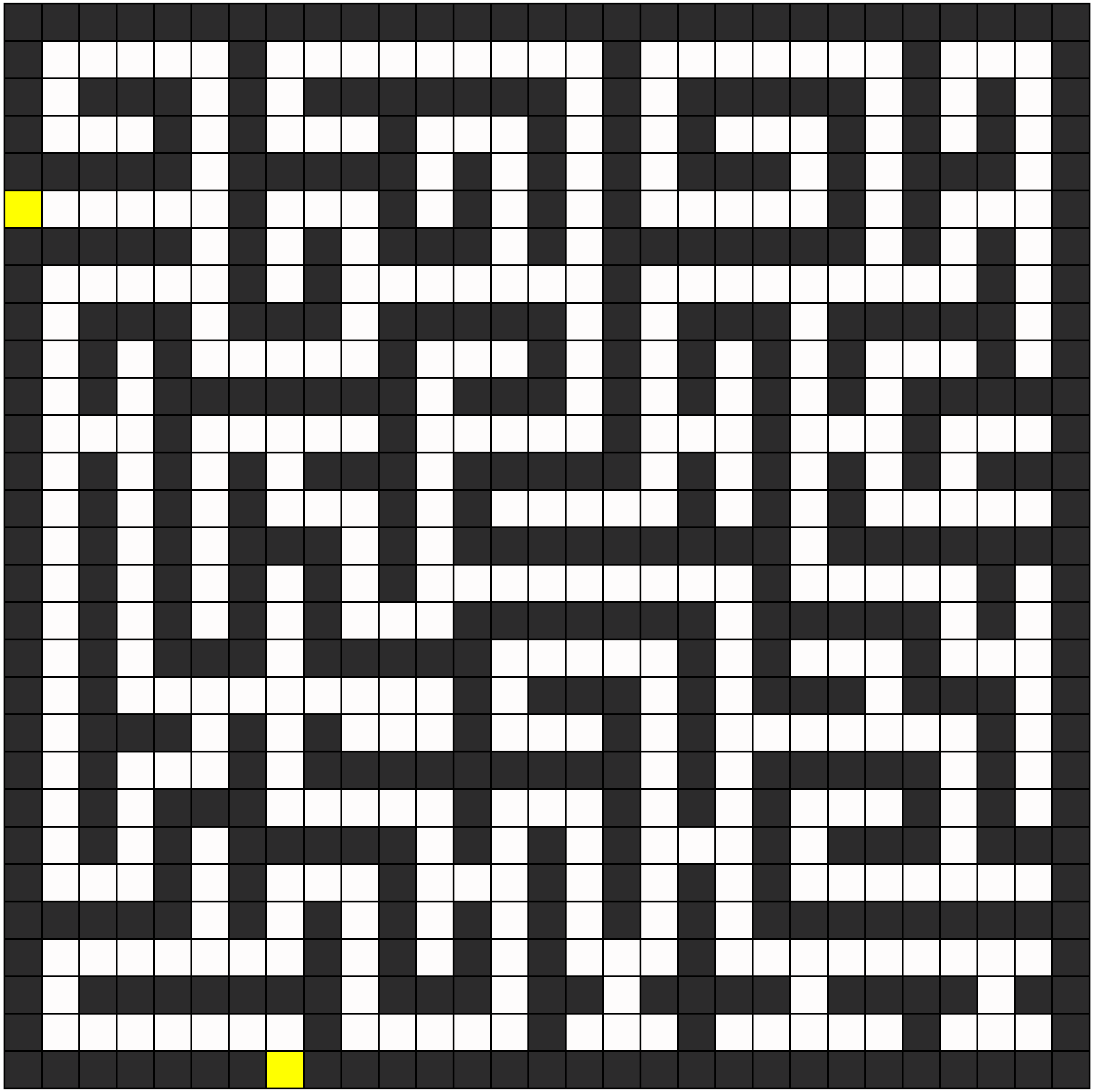
- a **logic component**, which specifies the *knowledge* to be used in solving problems, and
- a **control component**, which determines the *problem-solving strategies* by means of which that knowledge is used.

The logic component determines the meaning of the algorithm whereas the control component only affects its efficency."

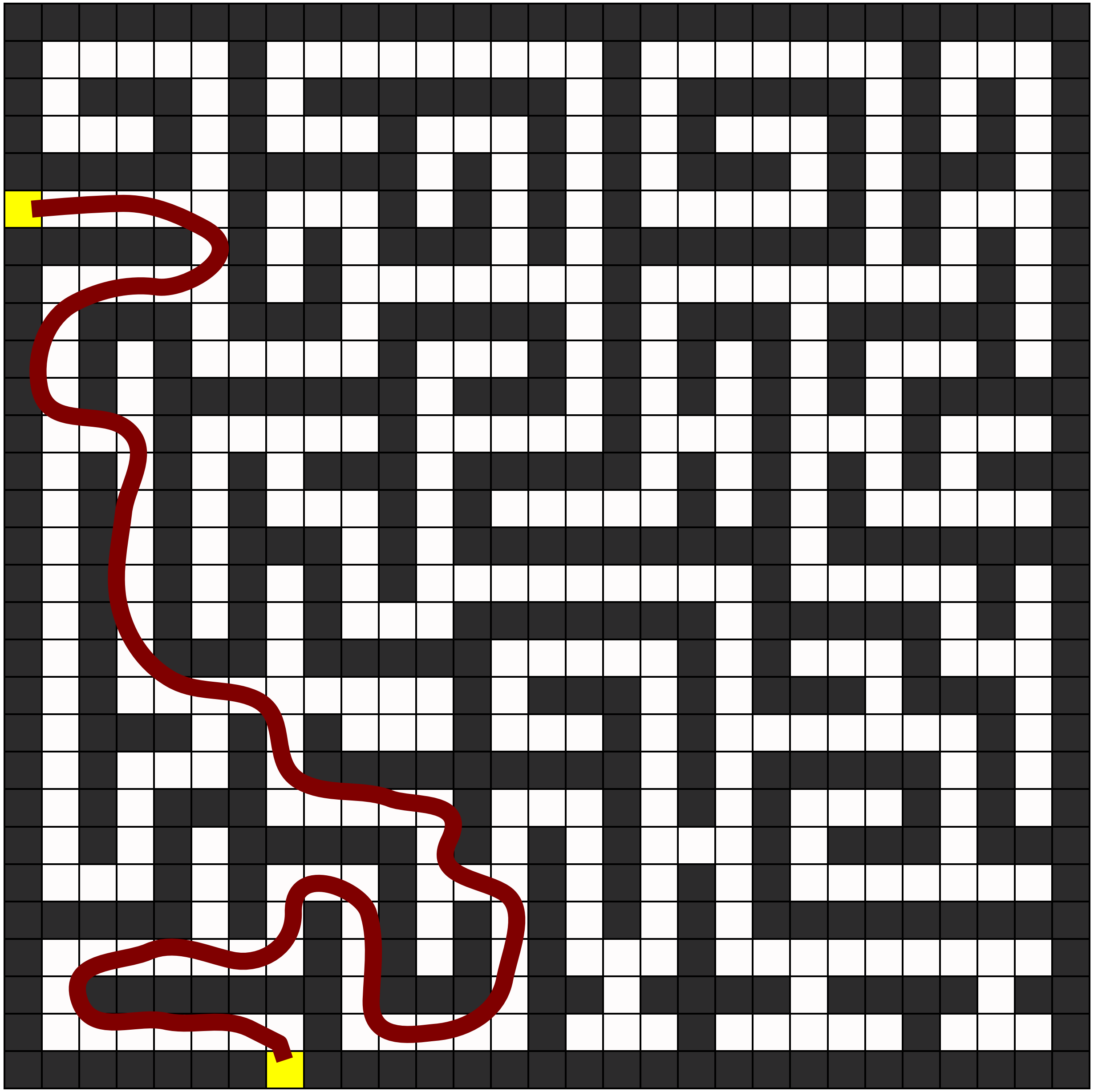
Robert Kowalsky, Algorithm = Logic + Control (1979)

Imperative vs Declarative

- Imperative:
  - **inside-to-outside** approach: all execution alternatives are explicitly specified and new alternatives must be explicitly added
- Declarative
  - **outside-to-inside** approach: constraints implicitly specify execution alternatives as all alternatives that satisfy the constraints; adding new constraints usually means discarding some execution alternatives



Imperative:
you command
the directions



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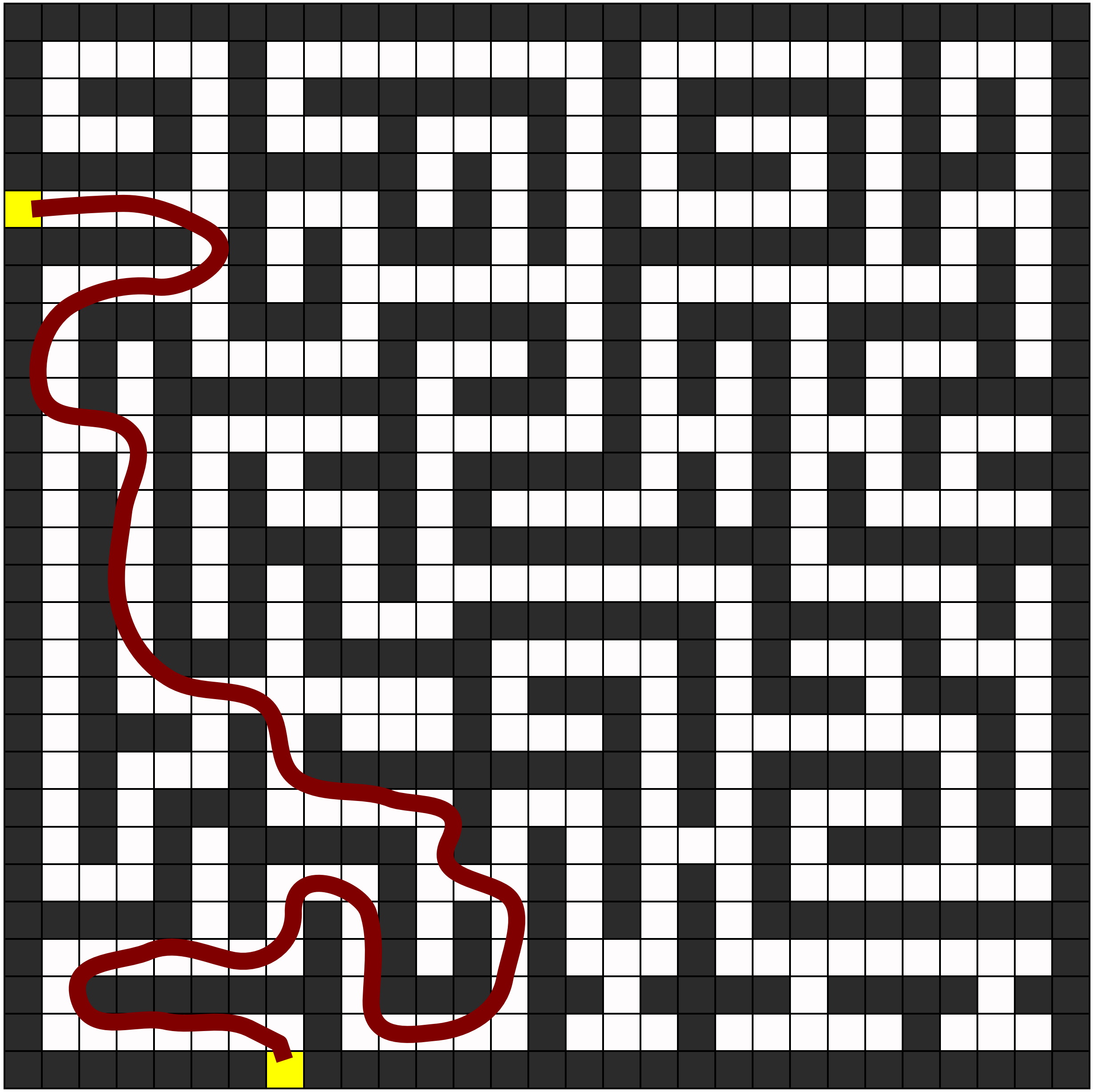
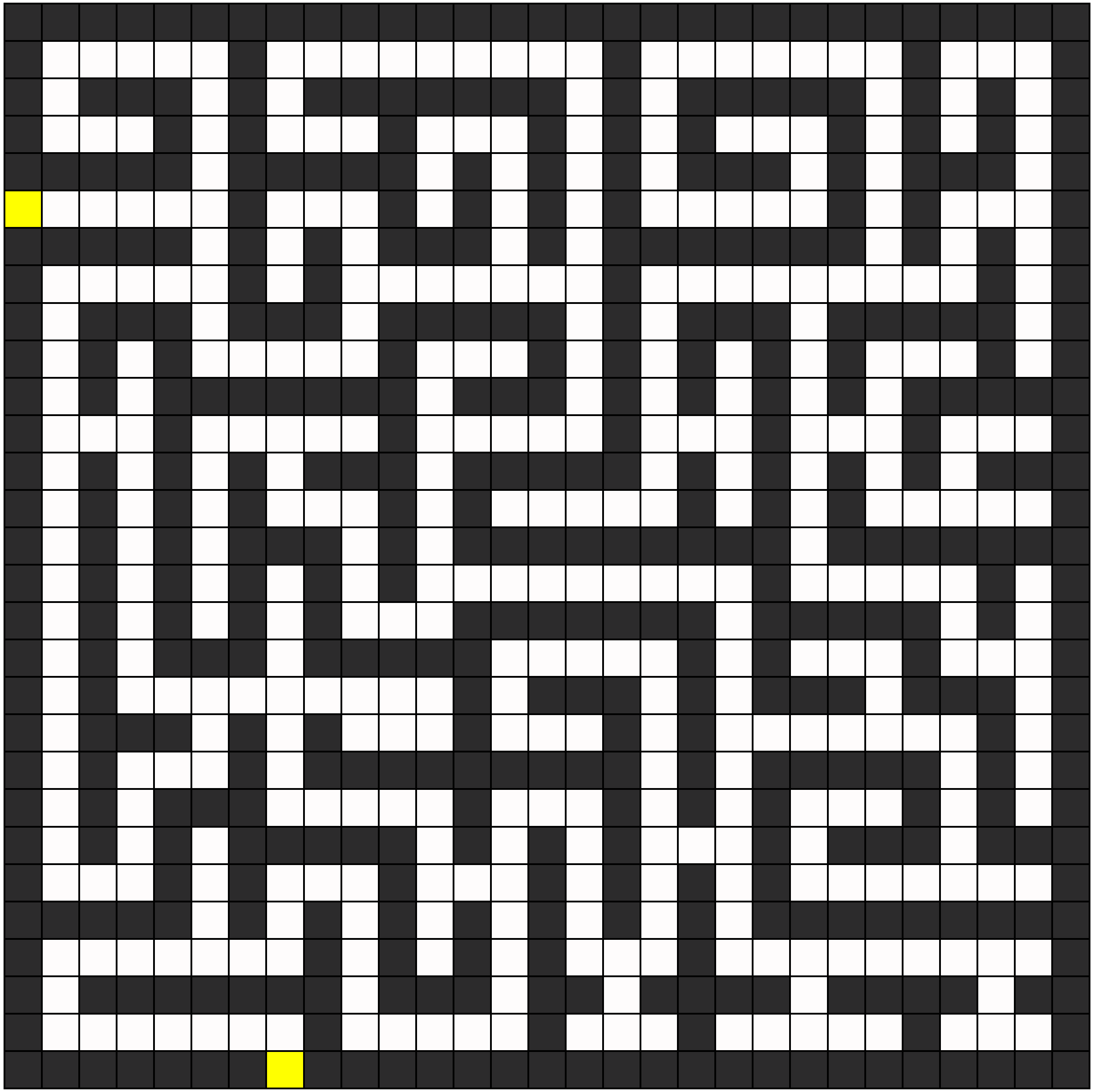
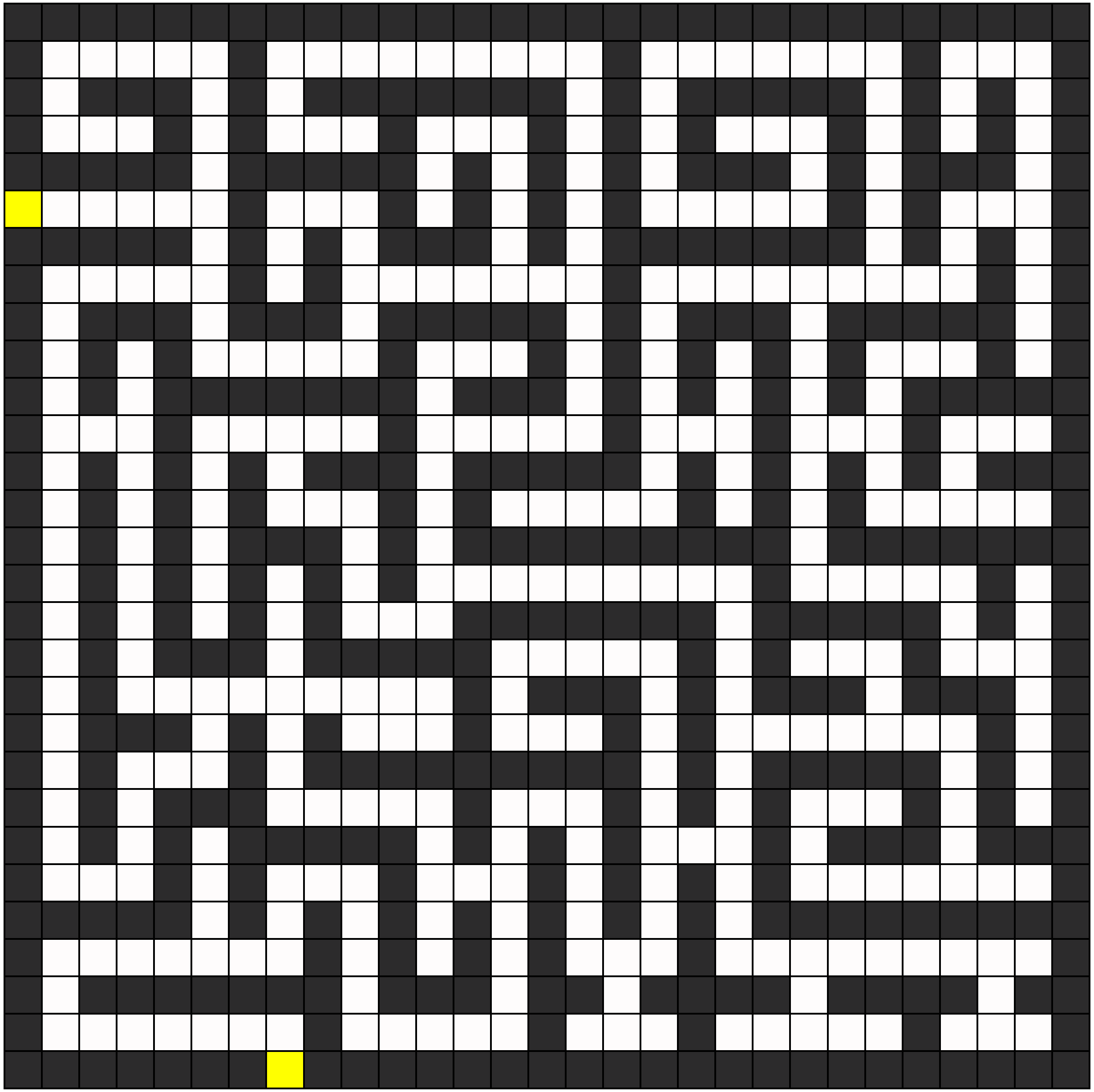


Imperative: you command the directions

- What if the labyinth changes?



Declarative: you give just the labyrinth. The computer finds the way.



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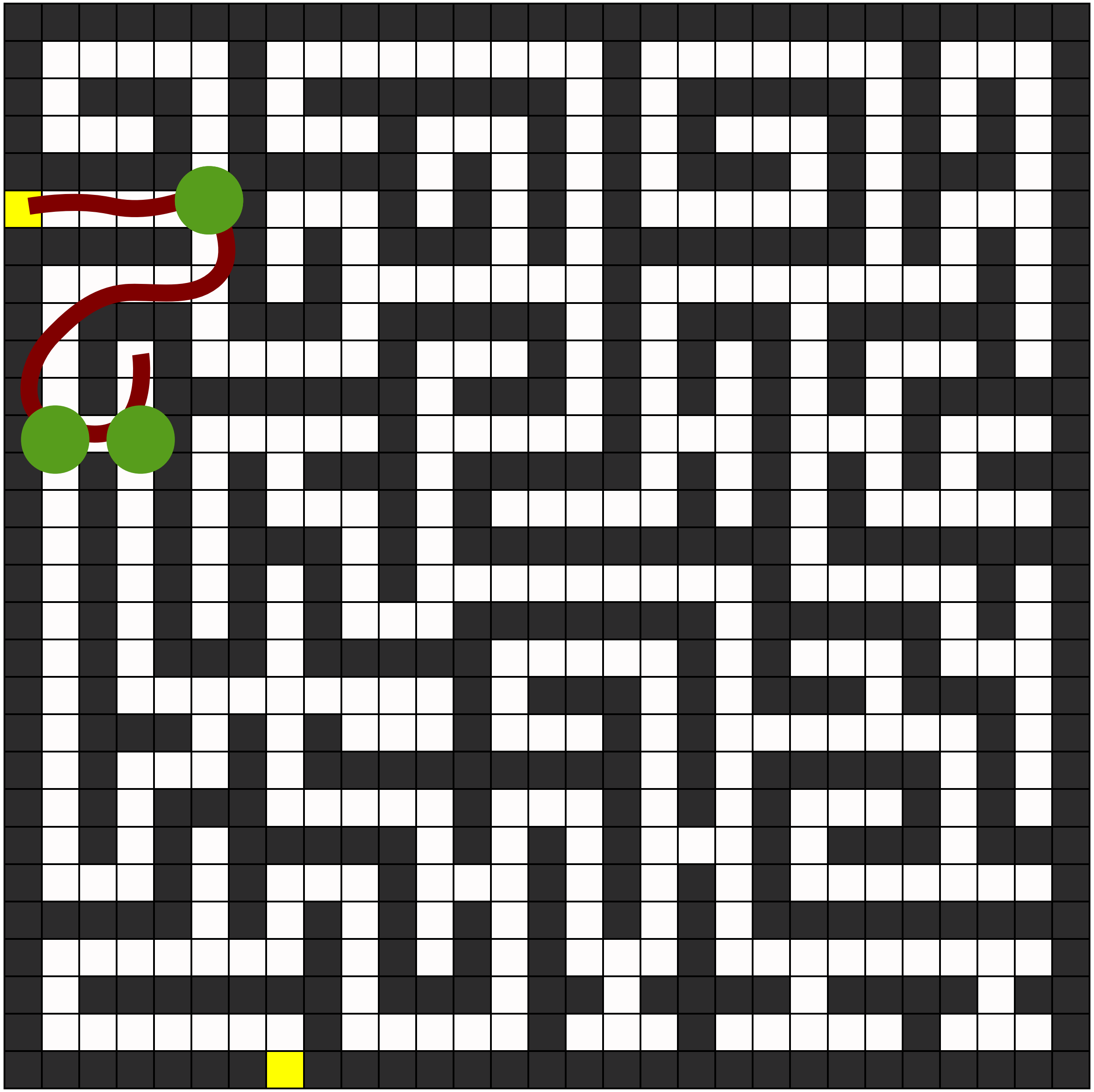
- For instance, via *trial, error* and **backtracking**



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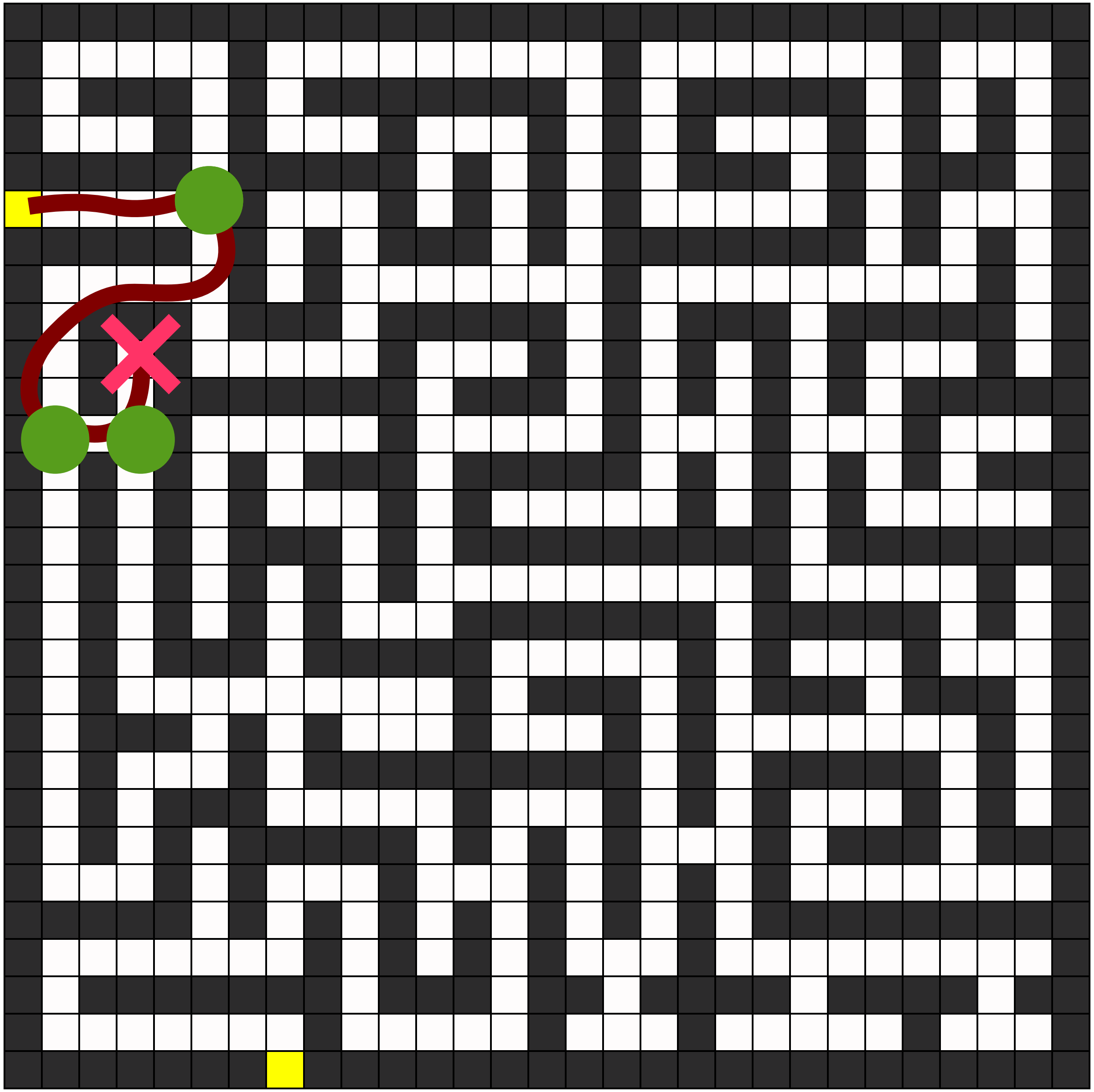

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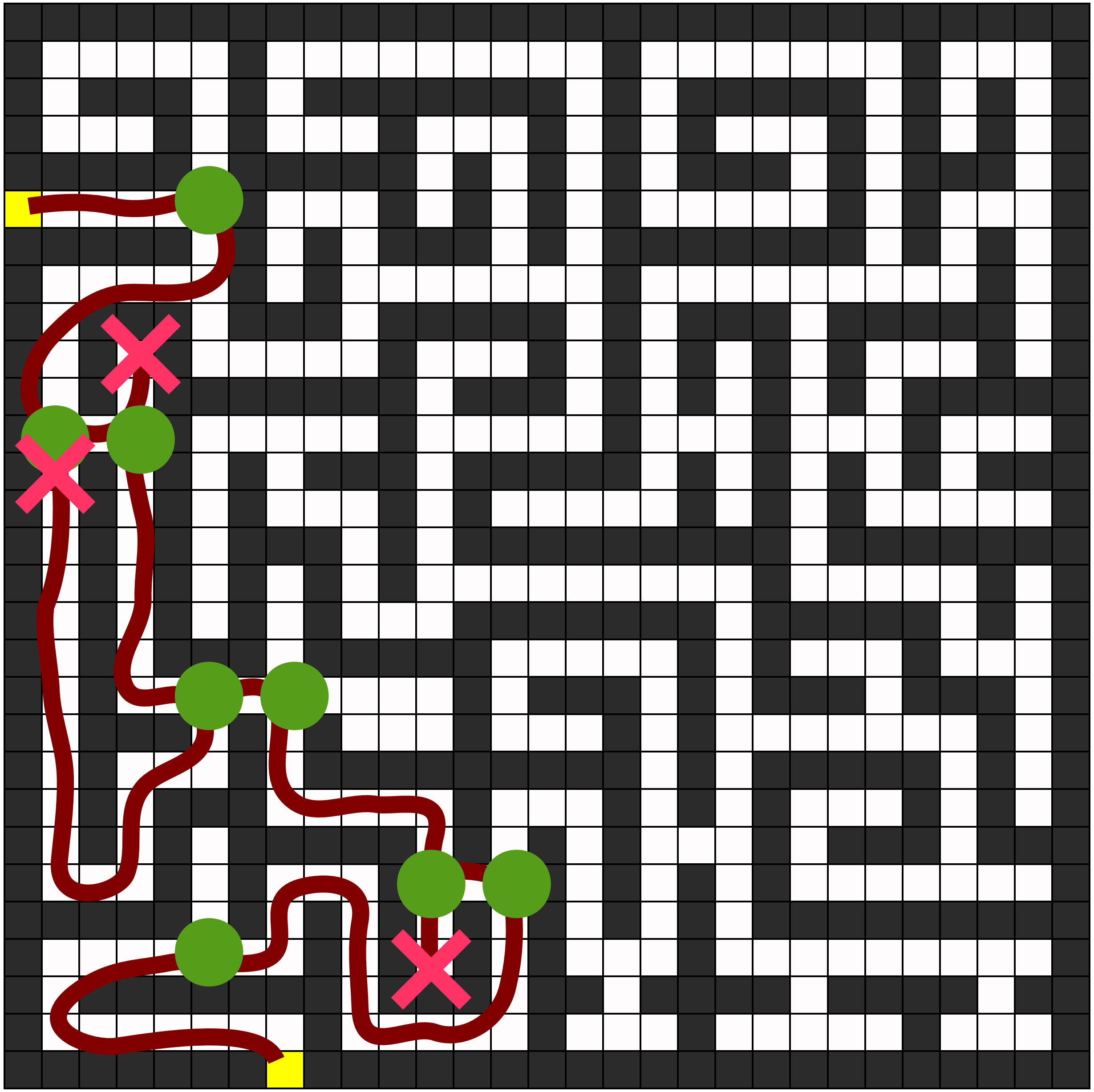

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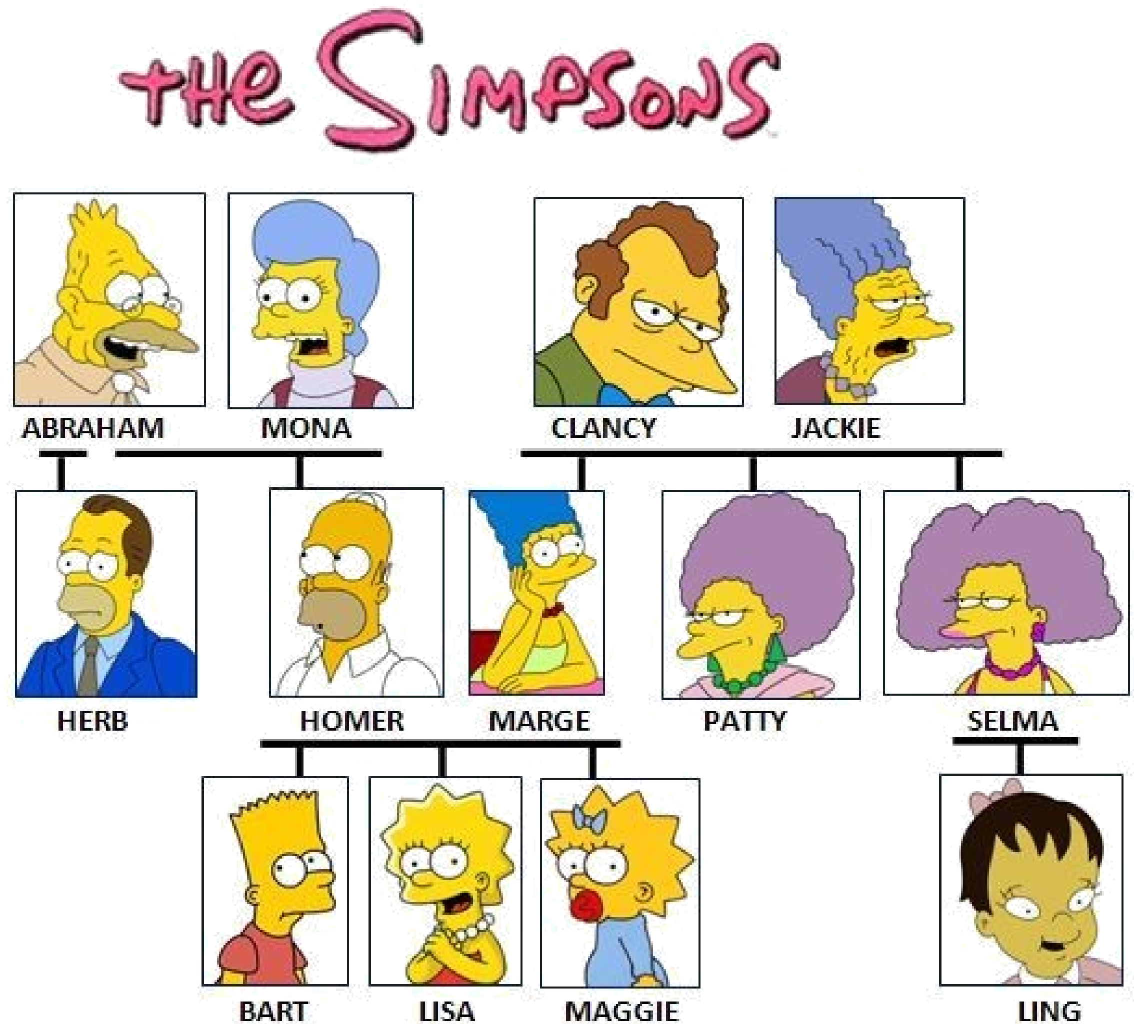
First steps in Prolog

Histrory of Prolog

1965	Resolution algorithm by J. A. Robinson followed by SLD resolution by R. Kowalski
1972	***PROgrammation en LOGique*** created by A. Colmerauer and P. Roussel in Luminy, Marseille
1980	Prolog acknowledged as a major A.I. language
	...
now	various versions, used in ***Constraint Programming***, or basis/reference for alternative techniques, as *DataLog*, *Anwser Set Programming* (ASP), etc.

First program

```
parent(marge, lisa).
parent(marge, bart).
parent(marge, maggie).
parent(homer, lisa).
parent(homer, bart).
parent(homer, maggie).
parent(abraham, homer).
parent(abraham, herb).
parent(mona, homer).
parent(jackie, marge).
parent(clancy, marge).
parent(jackie, patty).
parent(clancy, patty).
parent(jackie, selma).
parent(clancy, selma).
parent(selma, ling).
```



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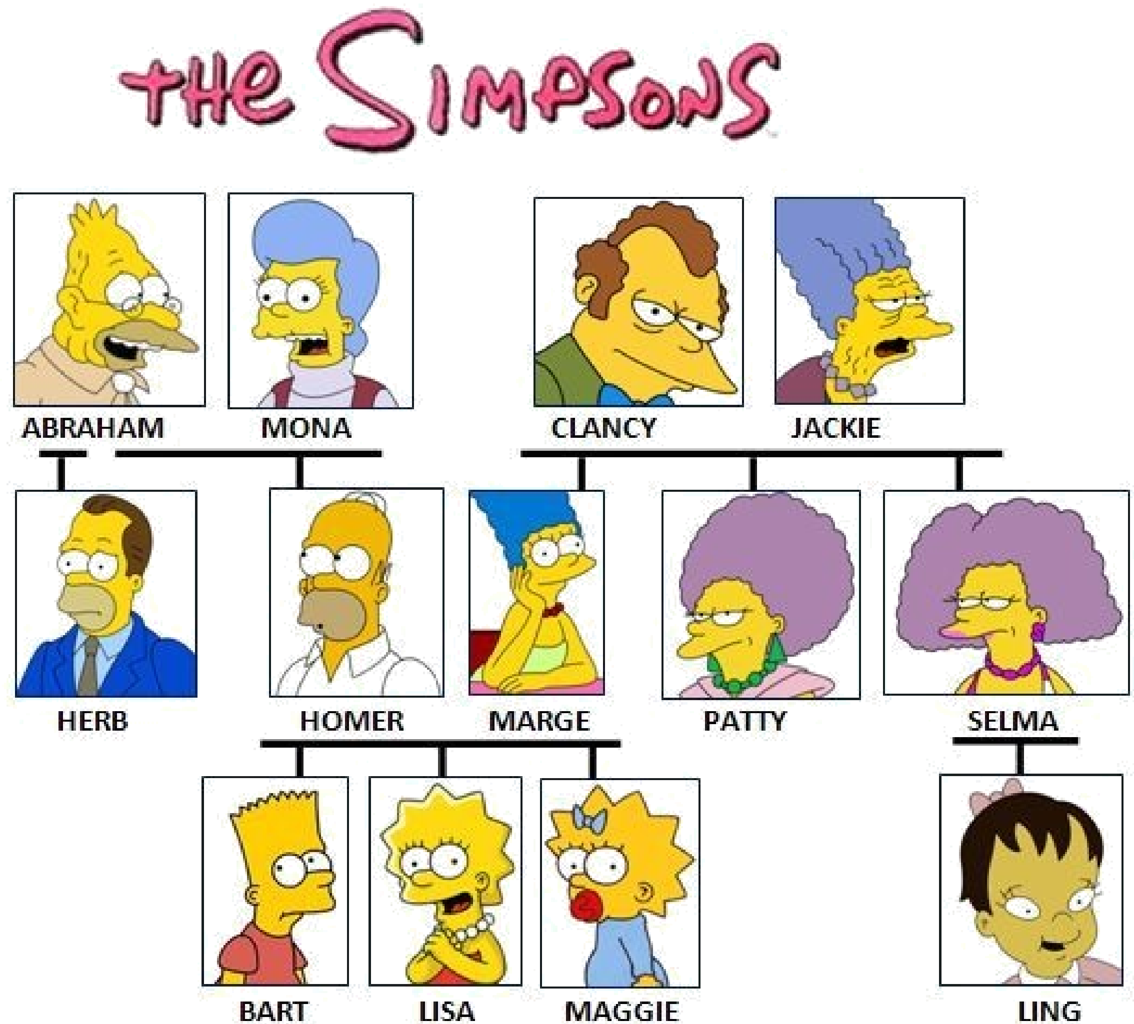
child(X,Y) :-
    parent(Y,X).
```



Prolog *clauses*

- **Fact**

```
female(marge).
```

Prolog *clauses*

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```
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```

- **Rule**

```
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```

Prolog *clauses*

- **Fact**

```
female(marge).
```

- **Rule**

```
child(X,Y) :- parent(Y,X).
```

Exercise: write

```
mother(X,Y)     grandparent(X,Y)
ancestor(X,Y)   cousin(X,Y)
```

Prolog syntax and bonuses

Prolog syntax: Terms

- **Constants**

  Identifiers strings of letters, digits, or underscore "_" that start with lower case letters.

  ```
  marge lisa x25 x_25 alpha_beta
  ```

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- **Constants**

  Identifiers strings of letters, digits, or underscore "_" that start with lower case letters.

  ```
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  ```

  Numbers

  ```
  1.001 2 3.03
  ```

Prolog syntax: Terms

- **Constants**

  Identifiers strings of letters, digits, or underscore "_" that start with lower case letters.

  ```
  marge lisa x25 x_25 alpha_beta
  ```

  Numbers

  ```
  1.001 2 3.03
  ```

  Strings enclosed in single quotes

  ```
  'Mary' '.01' 'string'
  ```

  Note that, because of the enclosing, strings can start with upper case letter, or can be a number now treated as a string.

Prolog syntax: Terms

- **Constants**

  Identifiers strings of letters, digits, or underscore "_" that start with lower case letters.

  ```
  marge lisa x25 x_25 alpha_beta
  ```

  Numbers

  ```
  1.001 2 3.03
  ```

- **Variables** strings of letters, digits or underscore that start with an upper case letter or the underscore

  ```
  _x, Anna, Successor_State,
  ```

  Note: undescore by itself ("_") is a special **anonymous** variable.

Prolog syntax: Terms

- **Structures**

  <identifier>(Term1, ..., Termk)

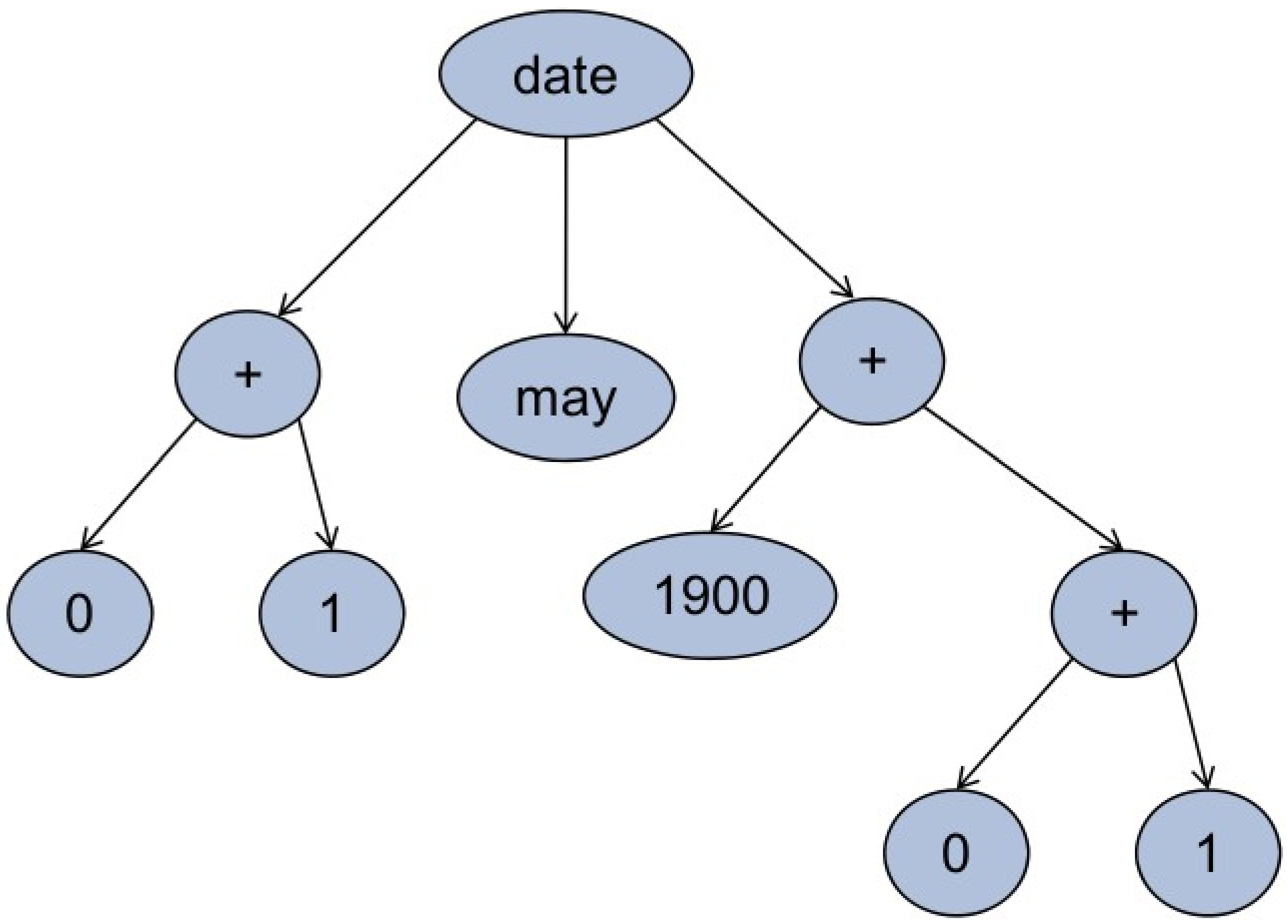
  recursive definition: each term can itself be a structure

```
date(1, may, 1983)
point(X, Y, Z)
date(+(0,1), may, +(1900,-(183,100)))
```

Prolog syntax: Terms

Structures can be thought as trees!

```
date(+(0,1), may, +(1900,-(183,100)))
```



Prolog syntax: Predicates

- **Predicates** are syntaxically the same as structures.

  <identifier>(Term1, ..., Termk)

```
parent(marge, bart)
female(marge)
```

Prolog syntax: Predicates

- **Predicates** are syntaxically the same as structures.

  <identifier>(Term1, ..., Termk)

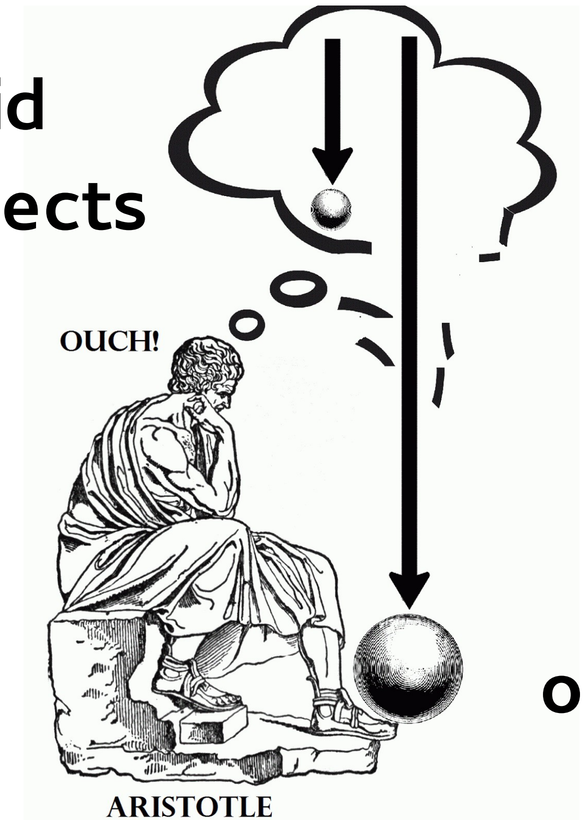
```
parent(marge, bart)
female(marge)
```

.. but they are not terms!

→ predicate identifiers ≠ structure identifiers

Prolog syntax: Predicates

things said about objects



objects

→ predicate identifiers ≠ structure identifiers

Prolog Syntax

- A prolog **program** is a sequence of *facts* and *rules*.
- **Facts** are *predicates* terminated by a period "**.**"

  `<identifier>(Term1, ..., Termk).`

Prolog Syntax

- A prolog **program** is a sequence of *facts* and *rules*.

- **Facts** are *predicates* terminated by a period "**.**"

  `<identifier>(Term1, ..., Termk).`

  Facts encode *assertions*, things that are declared certain!

```
female(marge).
parent(homer, bart).
```

Prolog Syntax

- A prolog **program** is a sequence of *facts* and *rules*.

- **Rules** are in the form:

  ```
  predicateH :- predicate1, .., predicatek.
  ```

  the left of "**:-**" is named **head**, the right **body**.

Prolog Syntax

- A prolog **program** is a sequence of *facts* and *rules*.

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  `predicateH :- predicate1, .., predicatek.`

- Rules encode ways of deriving or computing a new fact.

```
animal(X) :- elephant(X).
```

We can show that X is an animal if we can show that it is an elephant.

Prolog Syntax

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  `predicateH :- predicate1, .., predicatek.`

- Rules encode ways of deriving or computing a new fact.

  ```
  animal(X) :- elephant(X).
  ```

  We can show that X is an animal if we can show that it is an elephant.

  ```
  father(X,Y) :- parent(X,Y), male(X).
  ```

  We can show that X is a father of Y if we can show that X is a parent of Y and that X is male.

Prolog's main operation

- A **query** is a sequence of predicates

```
predicate1, predicate2, ..., predicatek
```

  - using the facts and rules in the Prolog program, the solver tries to *prove* that this sequence of predicates is true.
  - in proving the sequence it (should) perform the computation you want.

Prolog's main operation

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  - in proving the sequence it (should) perform the computation you want.

delicate point**: procedural computation** might be mixed with **declarative computation**

Prolog's resolution strategy

Horn clauses

- Facts and rules falls under the definition of **Horn clauses**, the structures on which SLD resolution was proven:

  General form : `F :- F1, F2,…, Fn.`

- **To prove `F`,** one must successively **prove `F1`, …, `Fn`.**

Horn clauses

- Facts and rules falls under the definition of **Horn clauses,** the structures on which SLD resolution was proven:

  General form : `F :- F1, F2,…, Fn.`

- **To prove `F`,** one must successively **prove `F1`, …, `Fn`.**

- `F` is the clause's ***head***.
- `F1`, `F2`, …, `Fn` constitute the clause's ***tail*** or ***body***.
- A fact is a clause with an empty tail.

Prolog's strategy

- To answer the question, Prolog builds a ***search tree*** :
  - set of possibilities (clauses matching the question) represented as a tree
  - each choice is a node of the search tree
  - trial and error sequential search, following the order of declaration

Prolog's strategy, example

```
p(1) :- a(1).
p(1) :- b(1).
a(1) :- c(1).
c(1) :- d(1).
c(1) :- d(2).
b(1) :- e(1).
e(1).
d(3).
```

- Query: `?- p(1).`

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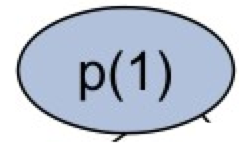

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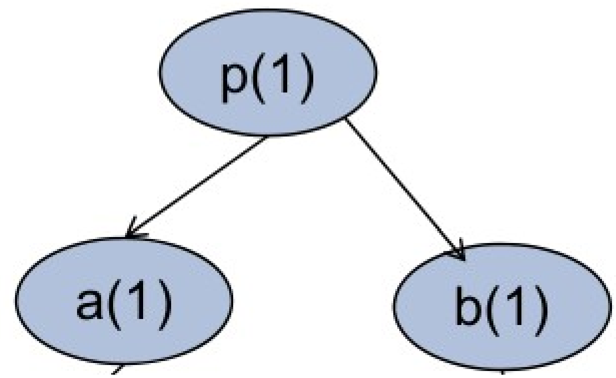

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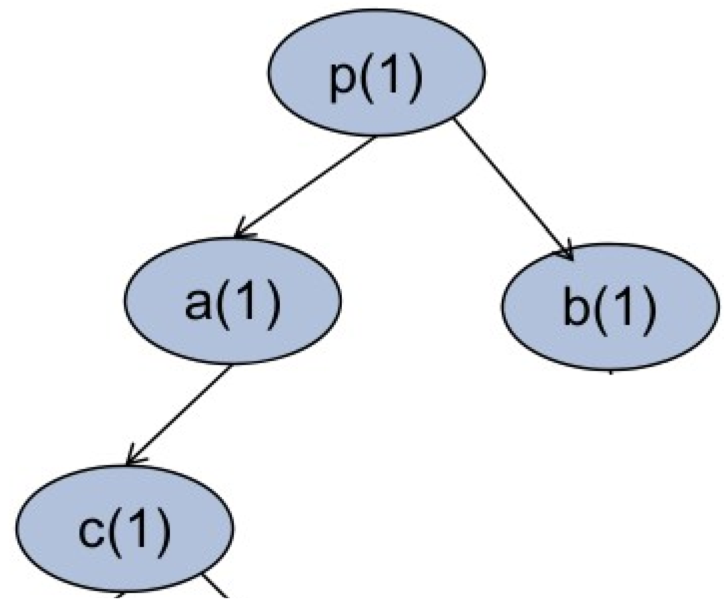

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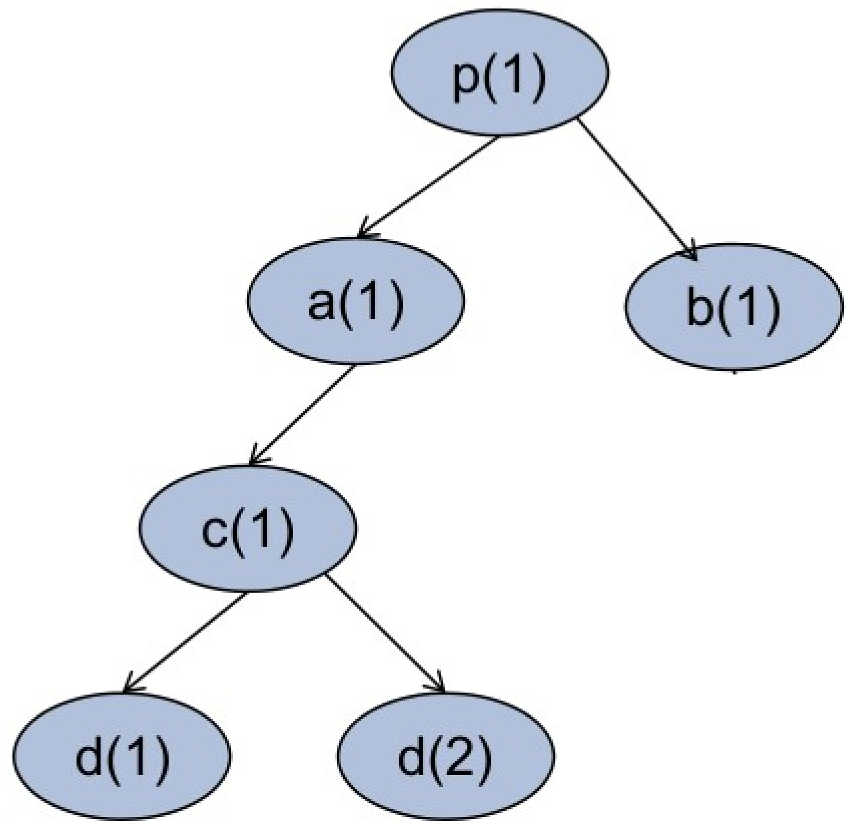

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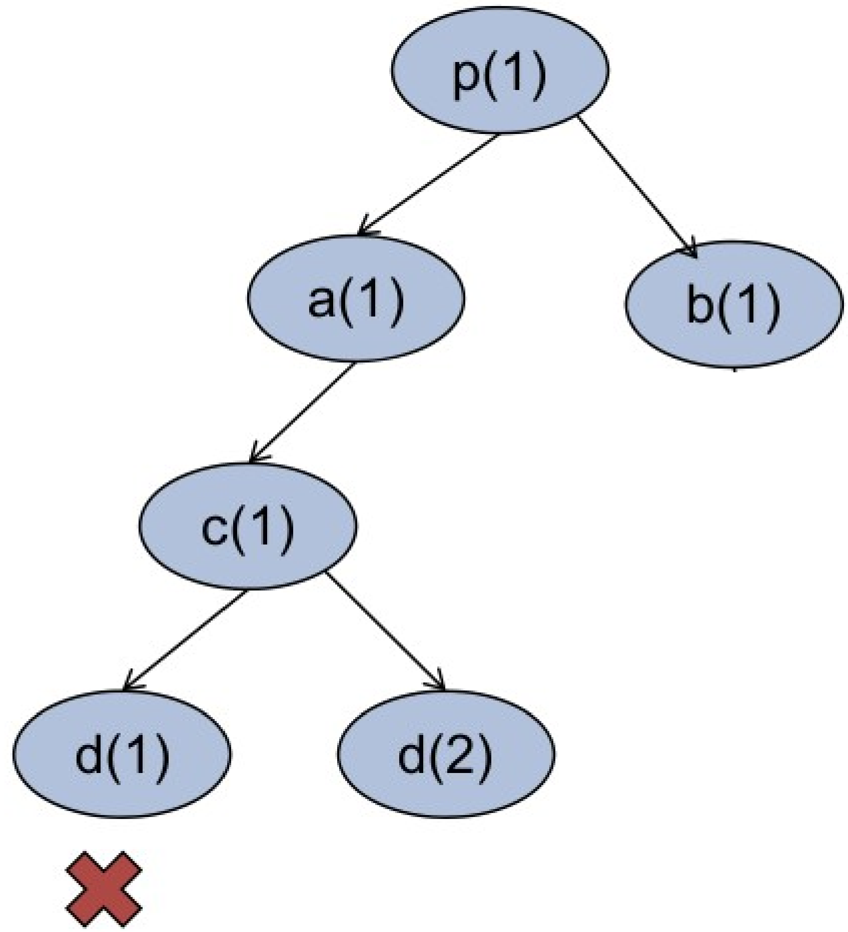

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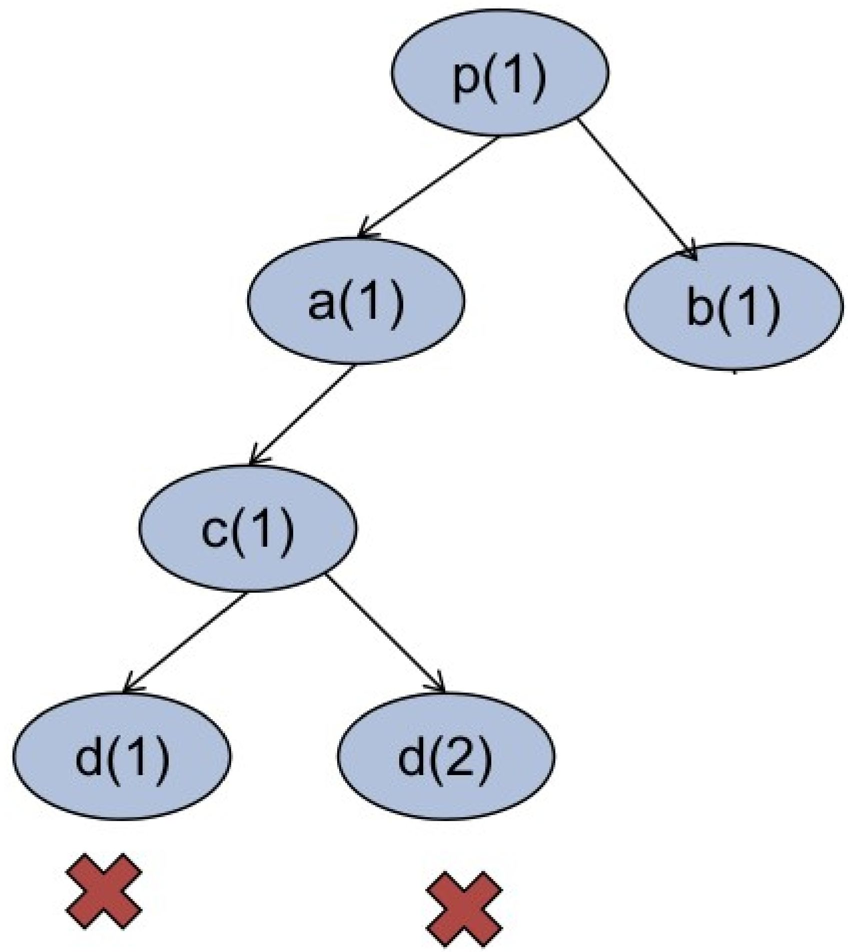

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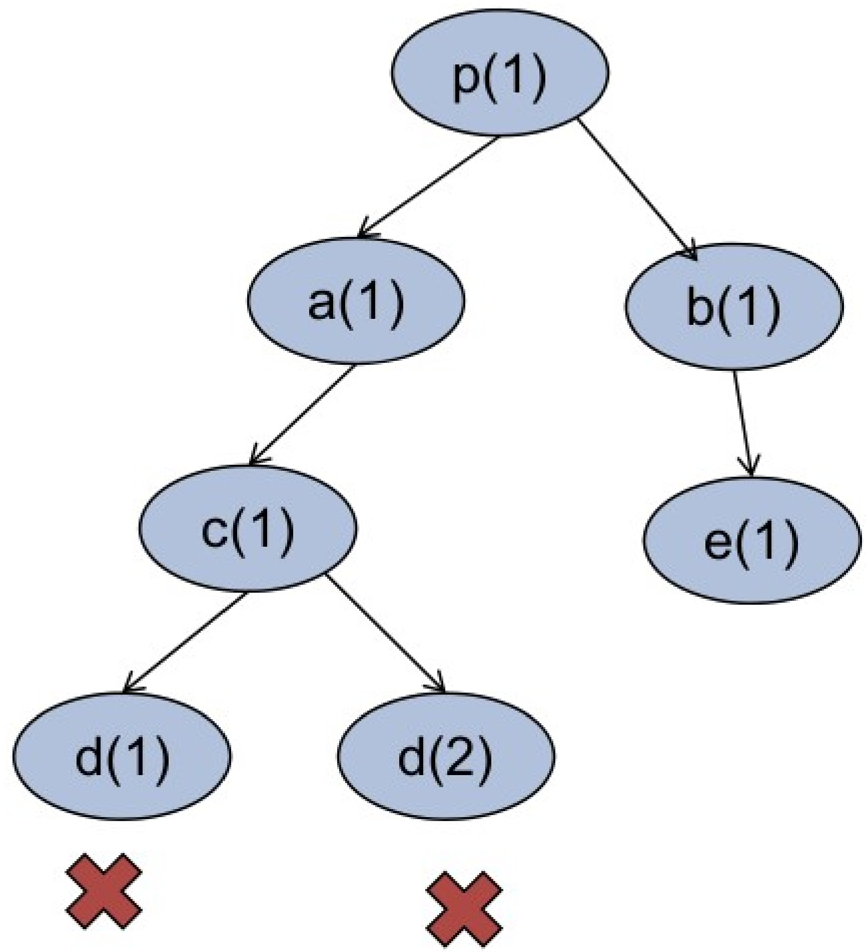

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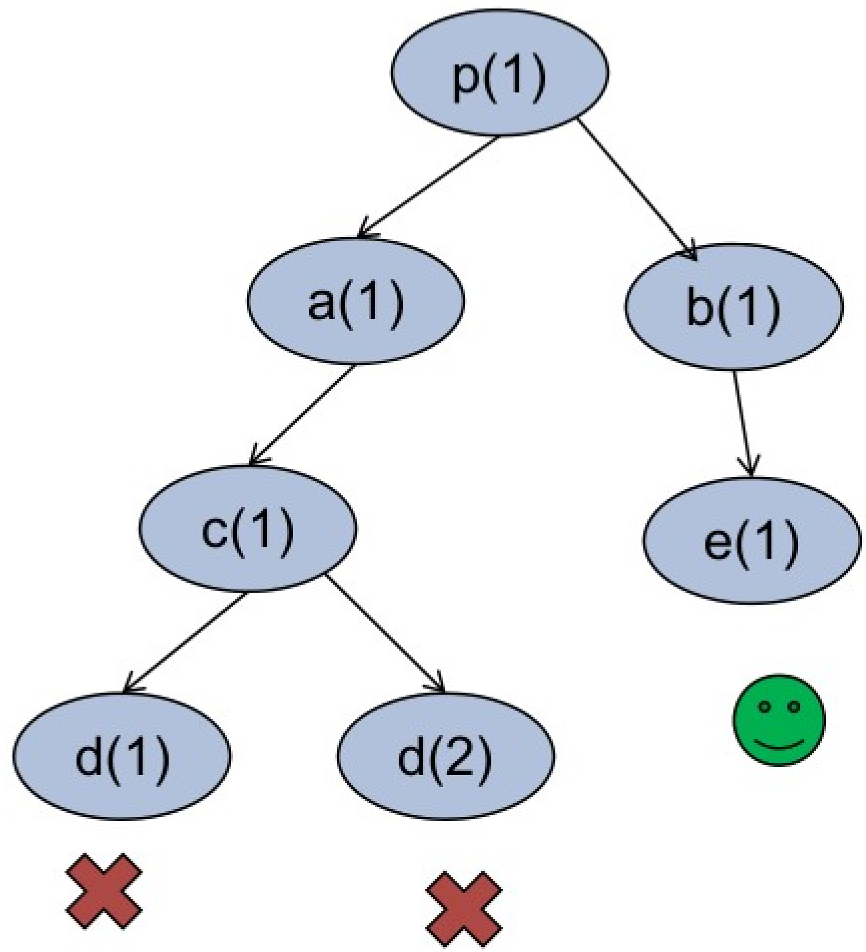

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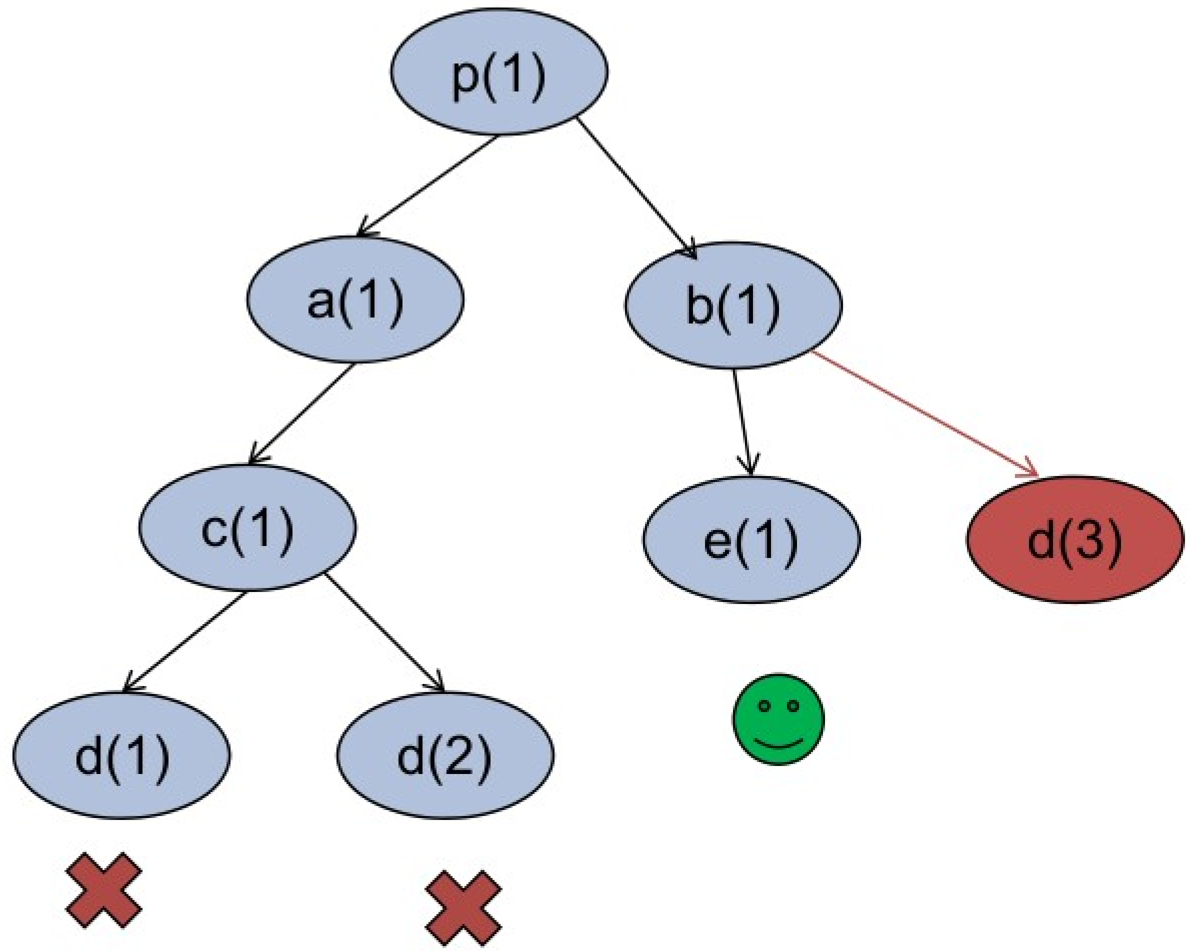
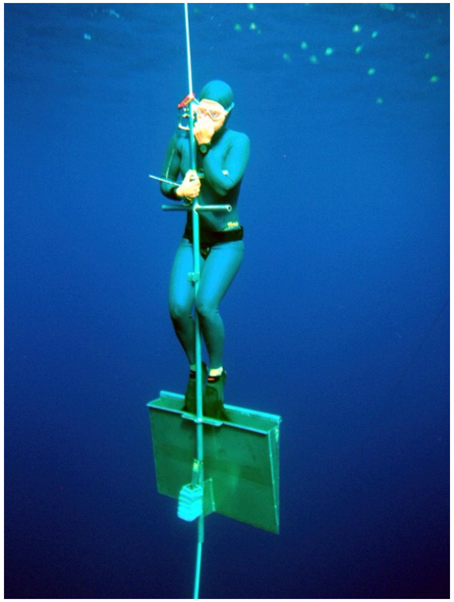


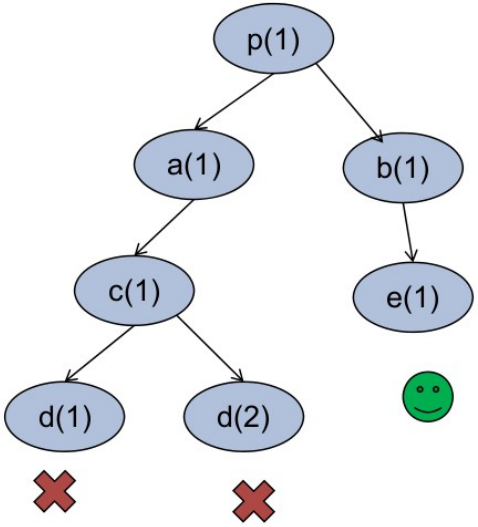
- Query: `?- p(1).`

with backtracking we can get more answers by using ";"

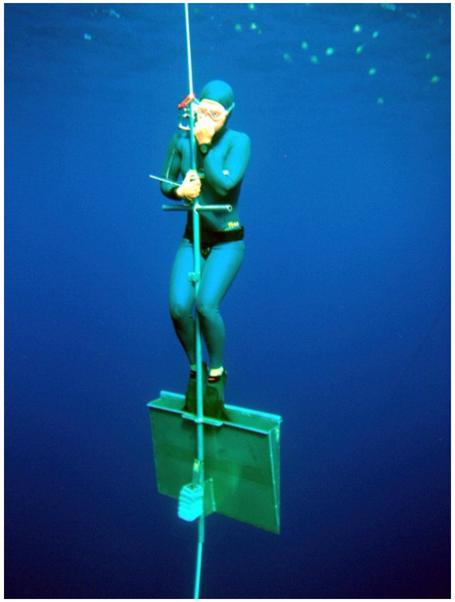
Prolog's strategy



- The solver follows a ***depth-first strategy***
  - **success node**: solution found! the solver displays it and stops.
  - **failure node**: the solver backtracks up in the tree, until it finds a choice point with unexplored branches.



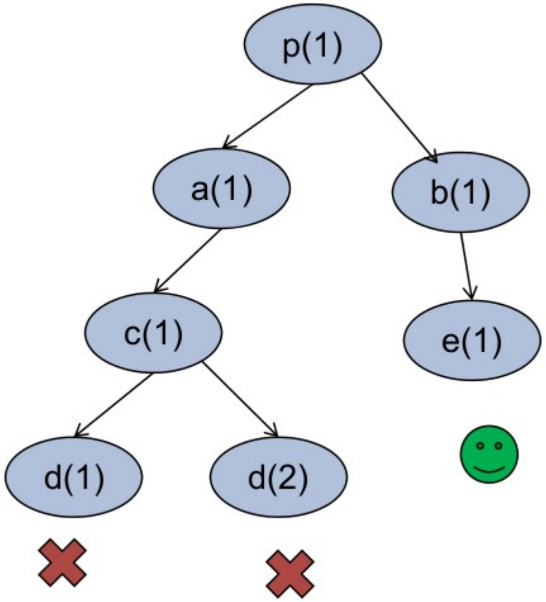
Prolog's strategy



- If backtracking encounters no choice point left, Prolog stops. No further solution.
- NOTE: Some branches are infinite! So search may not stop...

Take care of:

- The order of goals in clause tails
- The order of clauses



Resolution with variables

Using variables...

- Variables allow us to:
  - compute more than yes/no answers

```
?- parent(marge, X).
```

Using variables...

- Variables allow us to:
  - compute more than yes/no answers

```
?- parent(marge, X).
```

  - compress the program.

```
parent(marge, lisa) :- child(lisa, marge).
parent(marge, bart) :- child(bart, marge).
…
parent(X, Y) :- child(Y, X).
```

Using variables…

- Variables allow us to:
  - compute more than yes/no answers

    ```
    ?- parent(marge, X).
    ```

  - compress the program.

    ```
    parent(marge, lisa) :- child(lisa, marge).
    parent(marge, bart) :- child(bart, marge).
    …
    parent(X, Y) :- child(Y, X).
    ```

  - **reversibility** (when declarativity is maintaned)

    ```
    ?- parent(Y, lisa).
    ```

Variables matching via Unification

```
?- brother(bart, Who).

brother(X,Y) :-
   male(X),
   parent(X,Z),
   parent(Y,Z),
   X \== Y.
```

Unification with

```
brother(X,Y)
X=bart, Y=Who
```

```
male(bart)
parent(bart,Z)
parent(bart,Z)
bart \== Who
```

Unification examples

- Unification predicate: “=”

```
?- a(B,C) = a(2,3).
YES {B=2, C=3}

?- a(X,Y,L) = a(Y,2,carole).
YES {X=2, Y=2, L=carole}

?- a(X,X,Y) = a(Y,u,v).
NO
```

Unification examples

- Unification predicate: "="

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?- a(B,C) = a(2,3).
YES {B=2, C=3}

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YES {X=Y, Y=2, L=carole}

?- a(X,X,Y) = a(Y,u,v).
NO
```

Exercise

Unify `p(X,b(Z,a),X)` with `p(Y,Y,b(V,a))`

Unification algorithm

A **free variable** can be seen as a pointer to NIL. When not free, it is said **bound variable**. ***Dereferencing*** a variable means reaching the value to which it is bound.

t1, t2 are two terms

dereference is a function that dereferences bound variables and returns the input otherwise

```
procedure unify(t1,t2)

t3 := dereference(t1); t4 := dereference(t2)
if t3 is a variable then
    t3 points to t4; return success
else if t4 is a variable then
    t4 points to t3; return success
else if t3 is an atom and t4 is an atom then
    if t3 = t4 then return success
    else return fail
else let t3 = f(t31, .., t3n) and t4 = g(t41, .., t4m)
    if f = g and n = m then
        for i := 1 to n do
            if unify(t3i, t4i) fails then return fail
        return success
    else
        return fail
```

- **X** and **marge** where X is bound to the value marge will match.
- **X** and **Y** where X is bound to marge and Y is bound to marge will match,
- **X** and **marge** where X is bound to lisa will not match.

Lists

Representing lists

- **Lists** are a crucial data structure in Prolog. They are usually written as:

  `[a, b, c, d]`

  This corresponds to the structured term:

  `[a|[b|[c|[d|[]]]]]`

  where `[]` is a special constant the empty list.

Representing lists

- Each list is of the form [<head> | <rest_of_list>]

  <head> an element of the list (not necessarily a list).
  <rest_of_list> is a list (a sub-list).

`[a, b, c, d] = [a | [b, c, d] ]`

a: **head**; [b, c, d]: **tail**

`[a, b, c, d] = [a, b | [c, d]]`

`[_|_]` has at least one element.

Cut

Backtracking control using CUT

- Sometimes it is useful to control the backtracking, and this can be done using the "!", the cut operator.
  - once it is executed, it **disallows** backtracking

Backtracking control using CUT

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  - once it is executed, it **disallows** backtracking

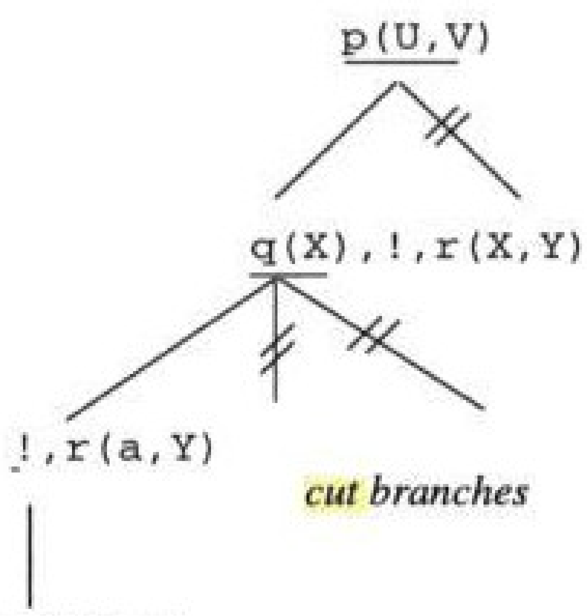
```
p :- b1, b2, !, a1, a2, a3.
p :- r1, r2 .
p :- r3 .
```

Before reaching cut, there might be backtracking on b1 and b2 or trying other rules for p if one of b1 or b2 cannot be satisfied. After reaching !, no more backtracking. The second and third rule will not be searched.

Backtracking control using CUT

- Sometimes it is useful to control the backtracking, and this can be done using the "!", the cut operator.
  - once it is executed, it **disallows** backtracking

```
p(X, Y) :-
   q(X),
   !,
   r(X,Y).
p(X, Y) :-
   s(X).
q(X) :-
   !, r(a,Y).
```



…undermining declarativity

- All times in which we play with the control we are undermining the declarative properties of the language.
- Therefore, uses of cut (and as we will see *negation as failure*) or any other properties having side effects remove properties as *reversibility.*

Prolog and logic

Prolog and Logic

- A Prolog clause is a *generalised disjunction*

  ```
  a :- b, c.
  ```

  In logic:

  $b \land c \Rightarrow a$

  As material implication $p \Rightarrow q$ equivalent to $\neg p \lor q$

  $\neg b \lor \neg c \lor a$

- Prolog inherits the correctness and completeness proof methods from First Order Logic, and, for the restriction to Horn Clauses, enjoyes **decidable algorithms**... More about this the next weeks.

Prolog and Logic - 2

- A Prolog clause

  ```
  a :- b.
  ```

  In logic *would* be:

  $b \Rightarrow a$

  which is equivalent to

  $\neg a \Rightarrow \neg b$

Prolog and Logic - 2

- A Prolog clause

  `a :- b.`

  In logic *would* be:

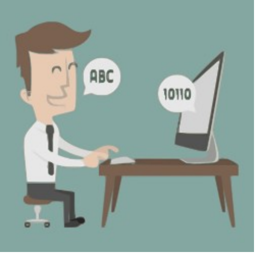
  $b \Rightarrow a$

  which is equivalent to

  $\neg a \Rightarrow \neg b$

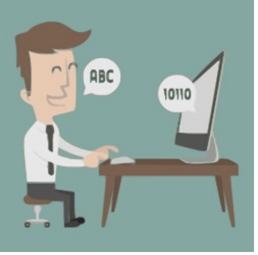
  but the clause does not specify that!!

...difference due to the different meaning of ***negation!***



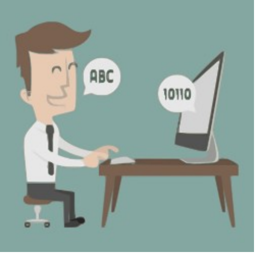
NAF – *Negation as Failure*

- In propositional logic, propositions can be **true** and **false**.



NAF – *Negation as Failure*

- In propositional logic, propositions can be **true** and **false**. But propositions could be also **unknown**.
  - Example: obviously, it may rain, or not rain. But if I'm indoor, and there is no window, I do not know!



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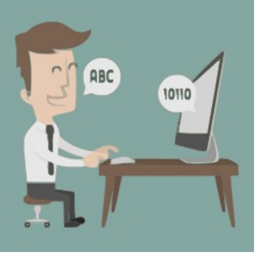
- When the inference engine is not able to assess if the proposition is true, the result will be false. This is called *Negation as Failure* (NAF).



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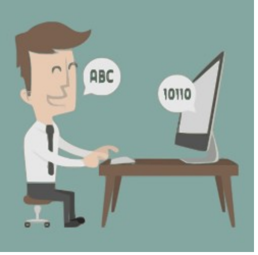
- In general, there may be two possible negations:

  - strong negation (↔ "classical" negation)

  - NAF negation (↔ "*undecidable*")



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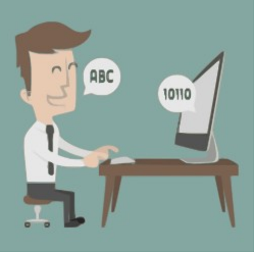

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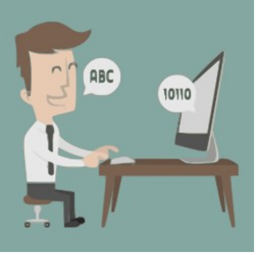

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then *it is the case*
   that ufos do *not* exist.

That's the case of Prolog!

Negation as failure

- Negation as failure can be used to implement *defaults*.

```
fancy(belle_d_argent).
fancy(maximus).

expensive(belle_d_argent).
affordable(Restaurant) :-
  not(expensive(Restaurant)).

?- fancy(X), affordable(X).
     X=maximus
```

Negation as failure

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?- fancy(X), affordable(X).
    X=maximus

?- affordable(X), fancy(X).
    FALSE
```

Negation as failure with CUT

- This predicate behaves just as **`not p(X)`**:

```
q(X) :-
  p(X),
  !,
  fail.
q(X).
```

Conclusions

Guidelines

- Go on **http://aicourse.r2.enst.fr:4242/SCIA**
  - Read and work with the material.
  - Responses to questions will be recorded up to a limit date. (generally 2 weeks from the lecture) and graded.
  - After sending a response you receive a possible correction.
- Try to be in scheduling with the course !
- **Collaborative learning is welcome, copying is prohibited !**
- Question and comments are welcome, just write me : gsileno@enst.fr