Computability of Diagrammatic Theories for Normative Positions

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Contributions of this work

- representing logical relations between normative positions using Aristotelian diagrams;
- drawing connections between various families of notions (e.g. different forms of power);
- building logical theories over diagrams which allow one to perform selected inferences on selected kinds of formulas (diagrammatic theories);
- providing an algorithm to decide whether a finite set of normative positions can be derived from another (i.e., a procedure to gain normative knowledge from a finite set of assumptions).

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    - symmetries facilitate perception and improve retention
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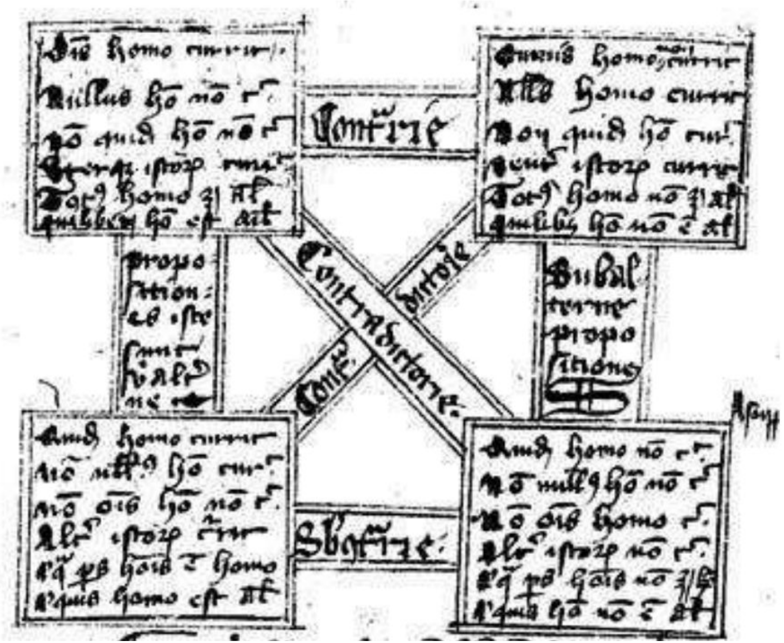
Two types of diagrams are generally referred to
when discussing about normative concepts:
squares of opposition and Hohfeldian squares.

Square of opposition (Aristotelian square)

- Logical construct described by Aristotle, centuries later represented in a diagrammatic form
- Related to syllogisms
- Abandoned with the advent of modern logic



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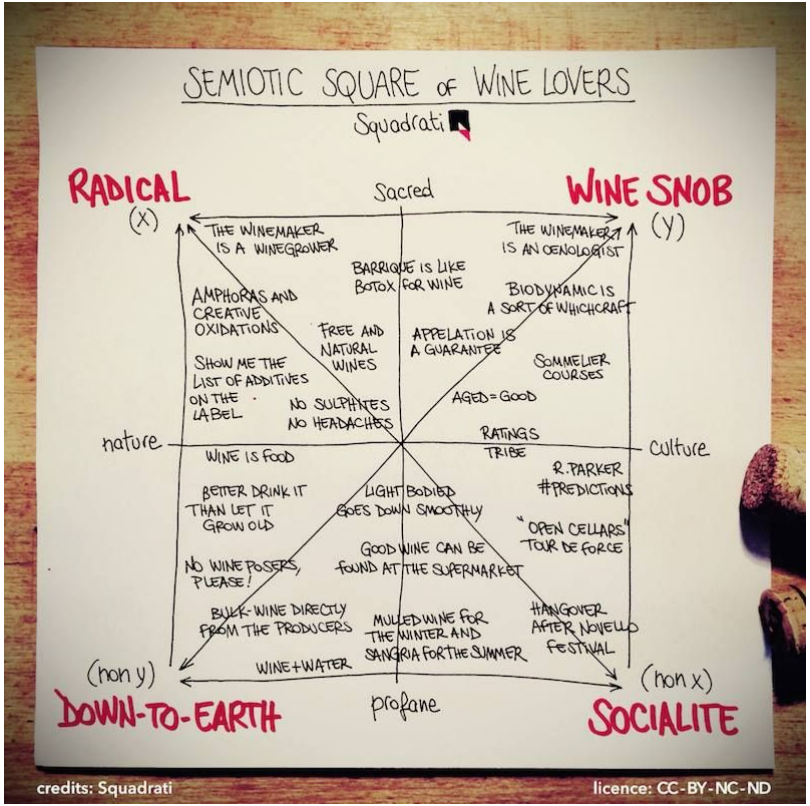

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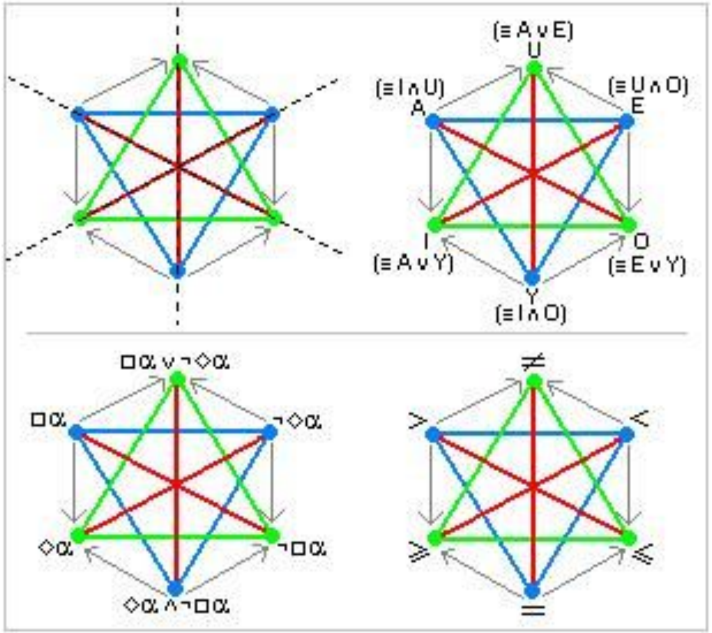
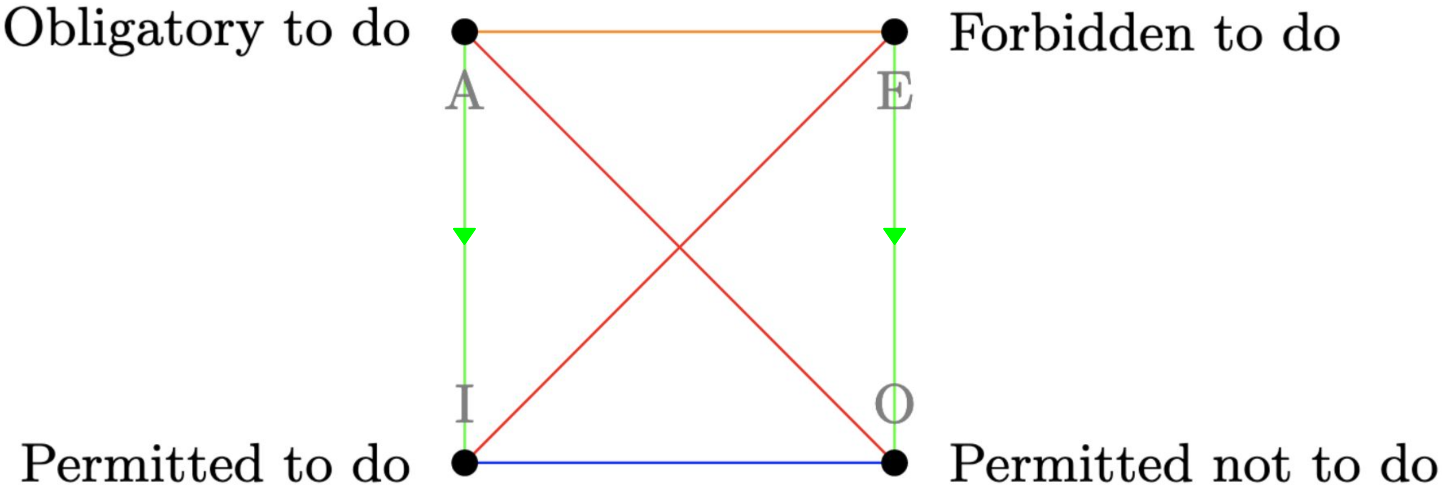
- Recent renewed interest on the formal and computational side



Deontic square of opposition



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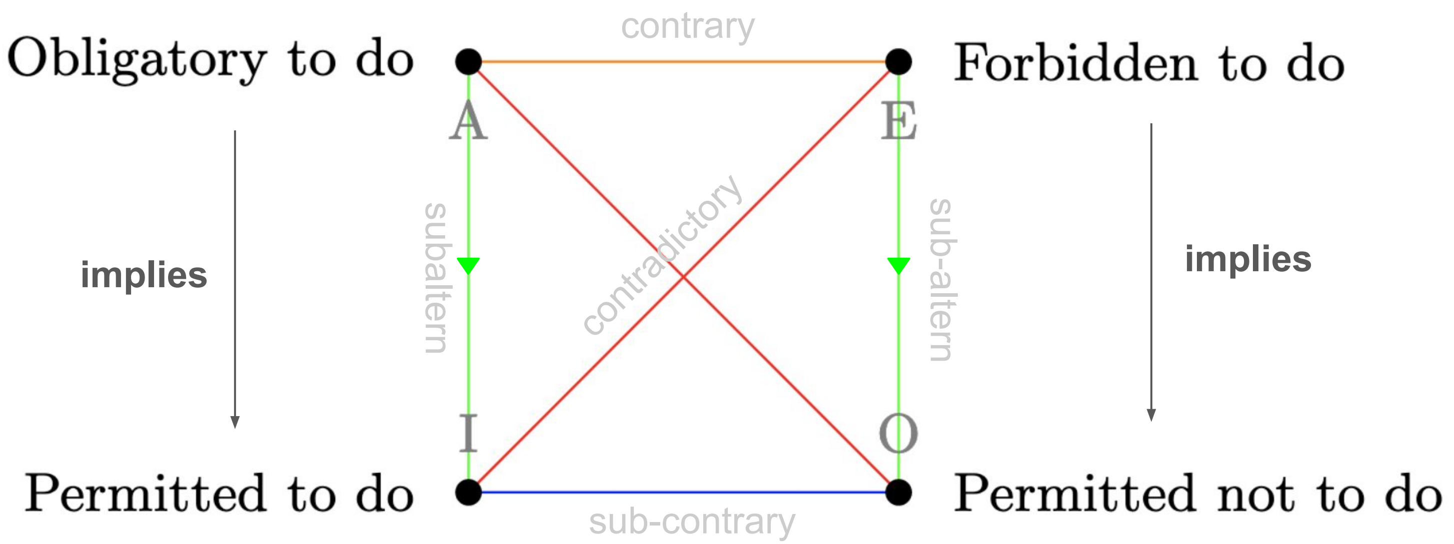

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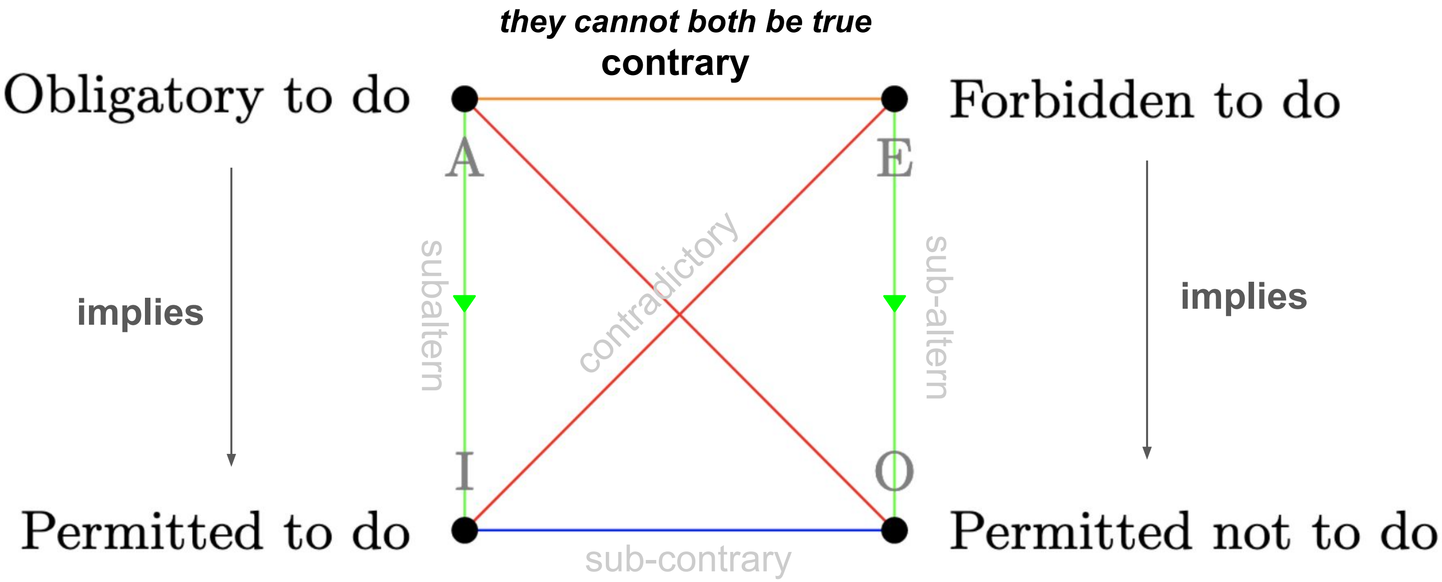

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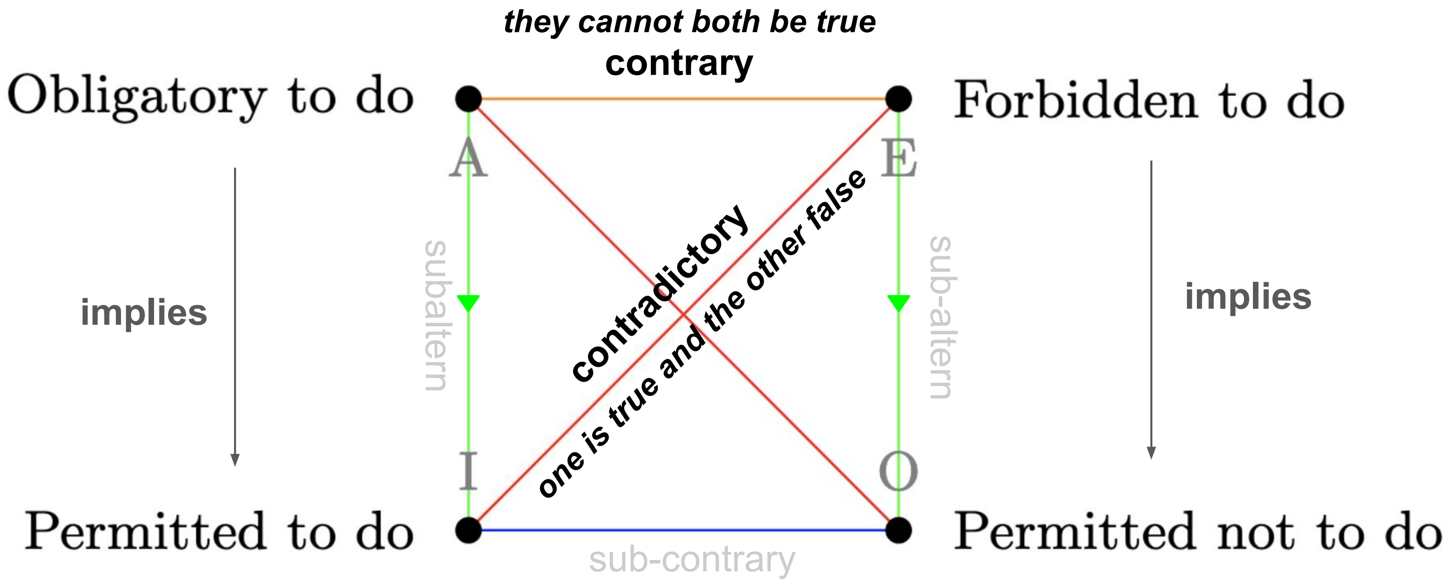

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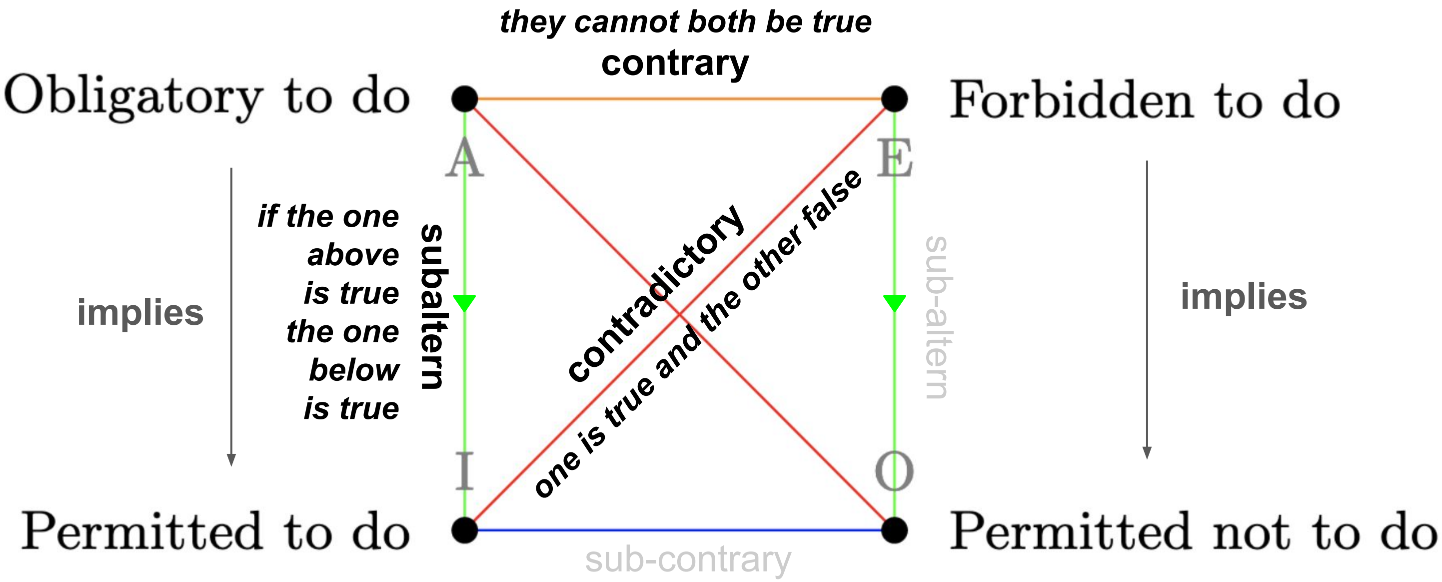

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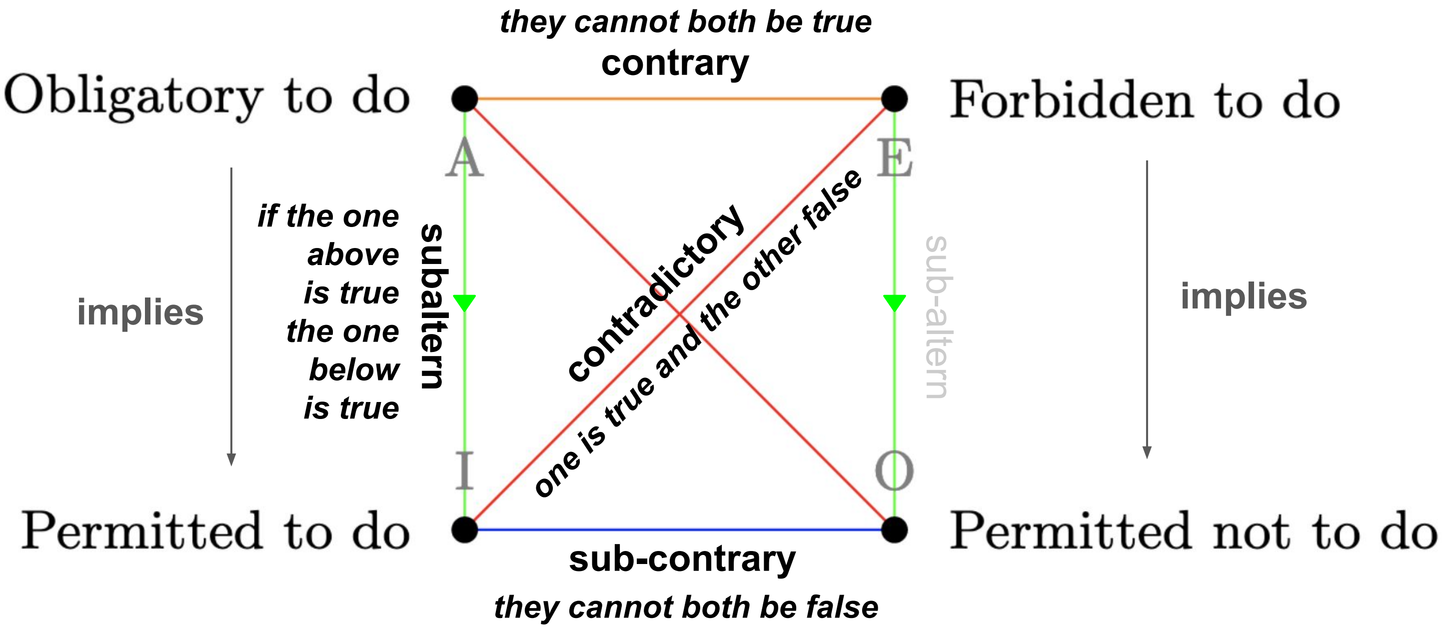

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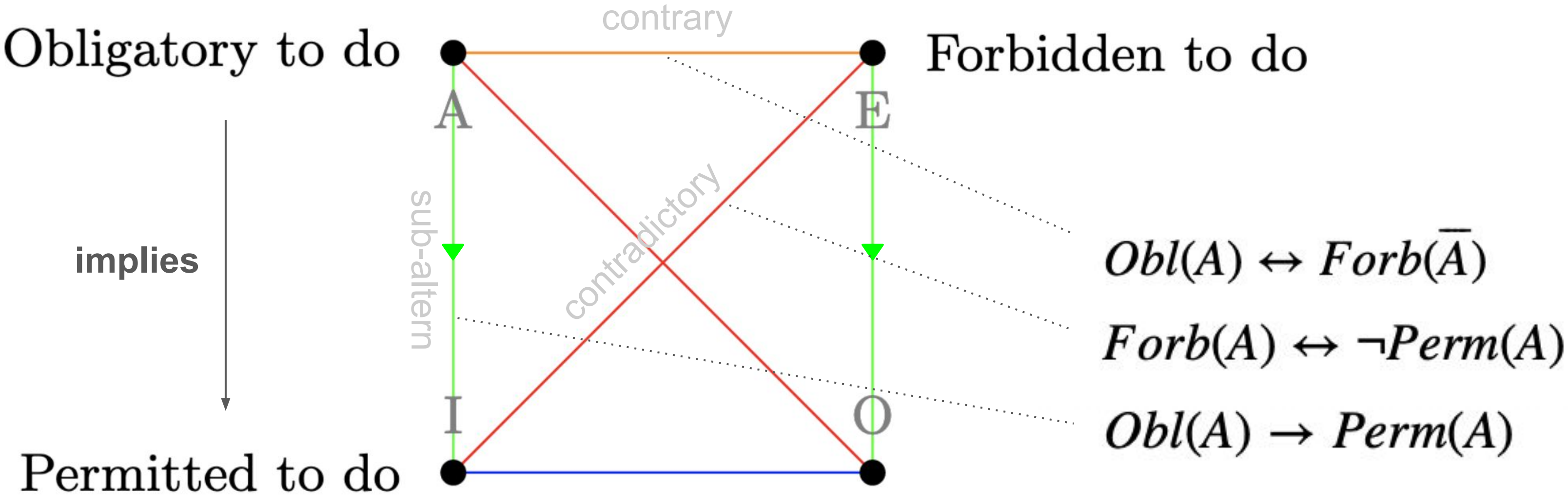
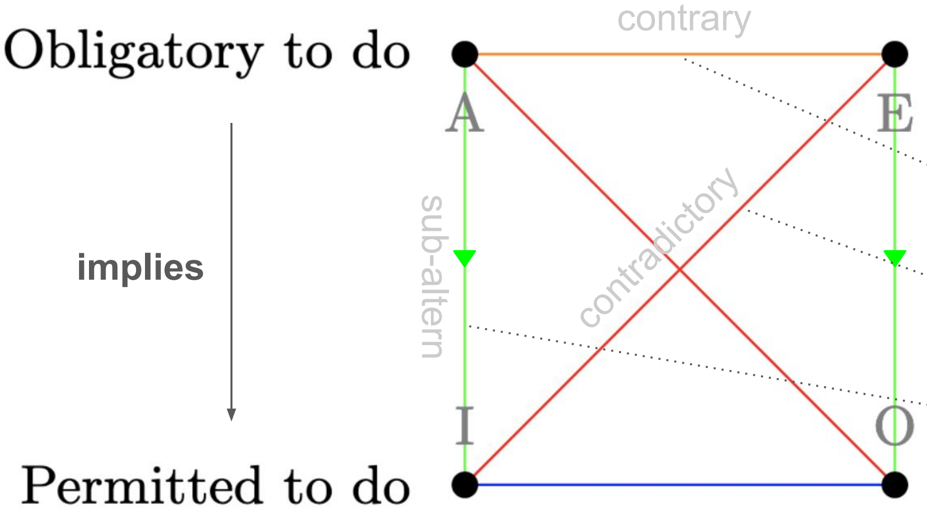


typical deontic relations support the square

Deontic square of opposition



internal negation

$$Obl(A) \leftrightarrow Forb(\overline{A})$$

$$Forb(A) \leftrightarrow \neg Perm(A)$$

$$Obl(A) \rightarrow Perm(A)$$

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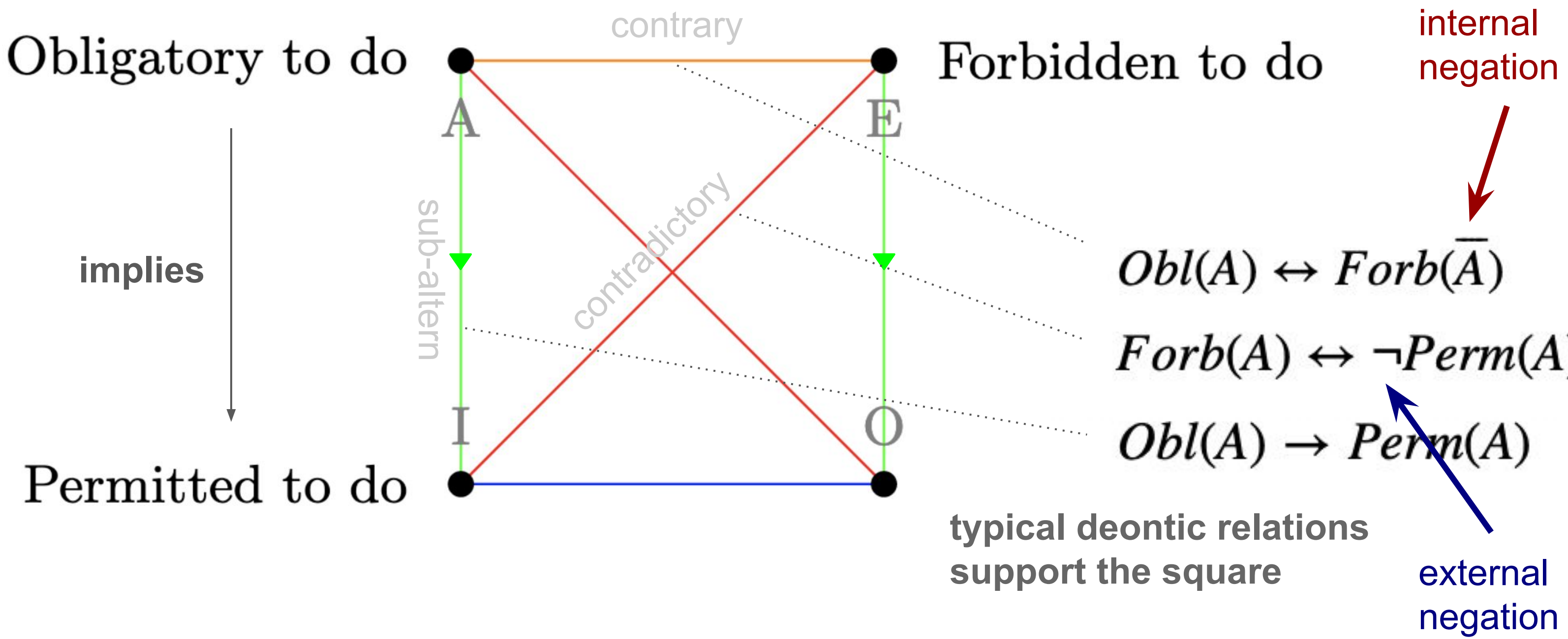

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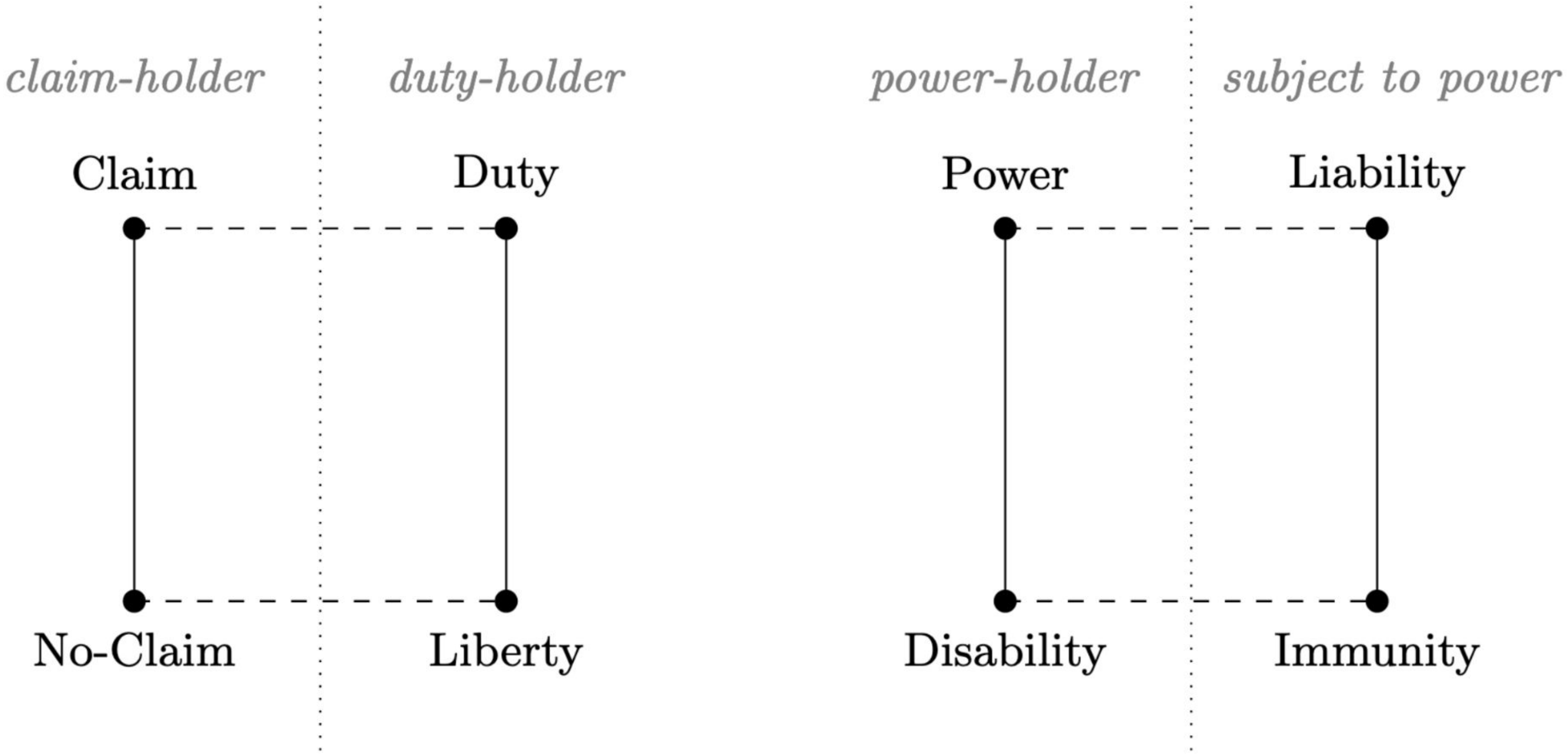
Hohfeldian squares



From Hohfeldian to Aristotelian squares

- Aristotelian squares *directly encode* logical relations between statements (contrariety, contradiction, subalternation, sub-contrariety). Therefore, they can be used as a starting point to build simple logical theories.
- By contrast, the logical interpretation of Hohfeldian squares is not straightforward – see, e.g., the discussion in Andrews (1983) or Markovich (2020).
- Here we opt for some interpretations of Hohfeldian squares presented in Sileno (2016), Sileno & Pascucci (2020) and Pascucci & Sileno (2021). These include alternative analyses of the notion of power in terms of the notion of ability.

First-order Hohfeldian concepts

Ternary relations among two normative parties and an action type

CLAIM, NO-CLAIM, DUTY, LIBERTY

- Each of these can be taken as primitive and used to define the others
- Each choice of a primitive notion may give rise to an Aristotelian square.

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For instance, with claim:

$$\boxed{\begin{aligned} \mathsf{NoClaim}(x, y, A) \\ \mathsf{Duty}(y, x, A) \\ \mathsf{Liberty}(y, x, A) \end{aligned}} \begin{aligned} &\equiv \neg\mathsf{Claim}(x, y, A) \\ &\equiv \boxed{\mathsf{Claim}(x, y, A)} \\ &\equiv \neg\mathsf{Claim}(x, y, \overline{A}) \end{aligned}$$

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external negation

subalternation relation

$$\mathsf{Claim}(x, y, A) \to \neg\mathsf{Claim}(x, y, \overline{A})$$

If y has a duty-of-A towards x,
then y has no duty-of-not-A towards x

Second-order Hohfeldian concepts

Relations among two normative parties and an action type

POWER, LIABILITY, DISABILITY and IMMUNITY

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$$\begin{aligned} \mathsf{Disability}(x, y, A) &\equiv \neg\mathsf{Power}(x, y, A) \\ \mathsf{Liability}(y, x, A) &\equiv \mathsf{Power}(x, y, A) \\ \mathsf{Immunity}(y, x, A) &\equiv \neg\mathsf{Power}(x, y, A) \end{aligned}$$

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external negation

internal negation?

But then, how to give rise to an Aristotelian square from power?

Second-order Hohfeldian concepts

We will then increase the granularity, relying on a definition of power based on the concept of **ability** -- for possible semantics see e.g. Sileno et al. (2019) [logic programming and event-calculus], or Sileno and Pascucci (2020) [modal logic]:

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"Canonic" form of power: the ability or competence to create a claim/duty

$$\mathsf{Power}(x, y, A) \equiv \exists \beta : \mathit{Ability}(x, \beta, \mathsf{Claim}(x, y, A))$$

agent

action-type
stimulus

configuration
manifestation

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...we can now individuate distinct forms of power and build the corresponding Aristotelian squares.

Outcome-centered power

The notion of power at its core is centered around the outcome produced.

We can distinguish between the **power to issue** a duty (canonic power) and the **power to release** from a duty.

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internal negation

sub-alternation relation

$$\mathsf{Power}(p, q, A) \rightarrow \neg\overline{\mathsf{Power}}(p, q, A)$$

If x is able to create y's duty-of-A,
x is not able to release y's duty-of-A.

Change-centered power

First analysed in a rigorous way by O’Reilly (1995).

The notion of power at its core concerns the ability of a normative party p to **affect** another normative party q **with respect to a certain relation** $\boldsymbol{R}$. We redefined it using *ability*...

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Focusing on "canonic" power, R is about a duty

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— *x is able to create a duty upon y*

— *x able to create a prohibition upon y*

— *x able to create (partial) liberties upon y*

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$\mathsf{Power}^{+}(x,y,A) \equiv \exists\beta : \mathsf{Power}_{\mathsf{OReilly}}(x,y,\beta,A)$ $\longleftarrow$ *The agent can do something changing R*

$\mathsf{Power}^{-}(x,y,A) \equiv \exists\beta : \neg\mathsf{Power}_{\mathsf{OReilly}}(x,y,\beta,A)$ $\longleftarrow$ *The agent can do something without changing R*

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If x is not able to not affect a y's duty-of-A, then x is able to affect it.

internal negation

$$\mathsf{Power}^{+}(x, y, A) \equiv \exists\beta : \mathsf{Power}_{\mathsf{OReilly}}(x, y, \beta, A)$$

$$\mathsf{Power}^{-}(x, y, A) \equiv \exists\beta : \neg\mathsf{Power}_{\mathsf{OReilly}}(x, y, \beta, A)$$

sub-alternation relation

$$\neg\mathsf{Power}^{-}(x, y, A) \rightarrow \mathsf{Power}^{+}(x, y, A)$$

Force-centered power

First observed in Sileno et al. (2014): the notion of power can be put in analogy to physical notions as *attraction* and *repulsion* towards a certain relation.

- **positive-force power**: to attract [*create a duty to perform*] a certain action type A
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same stimulus

opposite manifestations

The Dutch Declaration of Independence: Act of Abjuration (1581)

"Know all men by these presents [..] we have unanimously and deliberately declared [..] that the King of Spain has forfeited, *ipso jure*, all hereditary right to the sovereignty of those countries, and [they] are determined from henceforward not to acknowledge his sovereignty or jurisdiction [..], nor ***suffer others to do it.***"

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[we will punish who follows the orders of the King of Spain → the King has a *negative-force* power]

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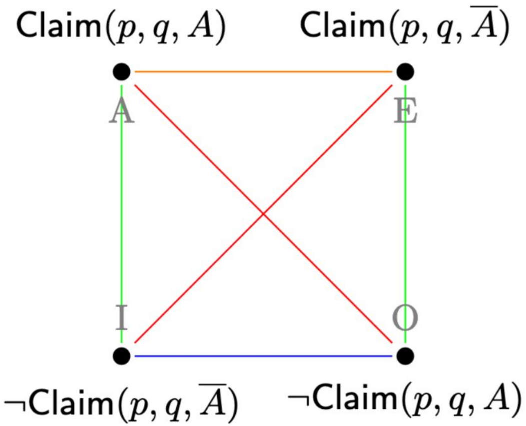
internal
negation

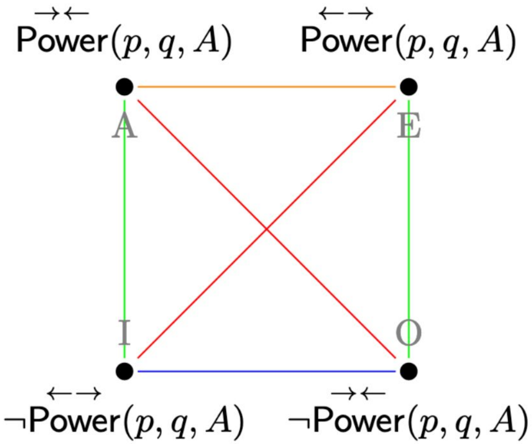
sub-alternation relation

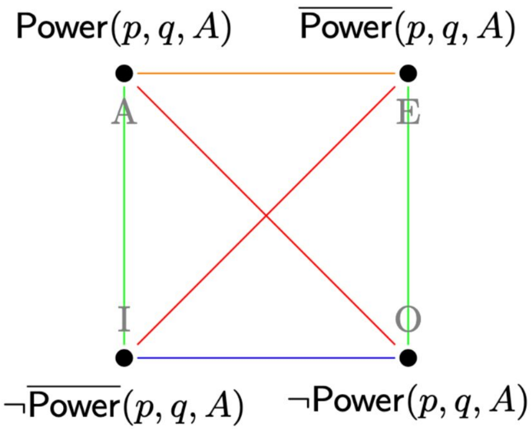
$$\overset{\rightarrow\leftarrow}{\mathsf{Power}}(x, y, A) \rightarrow \neg\overset{\leftarrow\rightarrow}{\mathsf{Power}}(x, y, A)$$

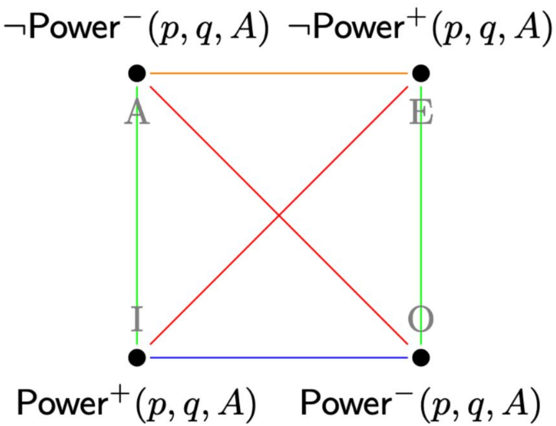
If x is able to create y's duty-of-A by commanding A,
x is not able to create y's duty-of-not-A by the same act.

A collection of squares of opposition





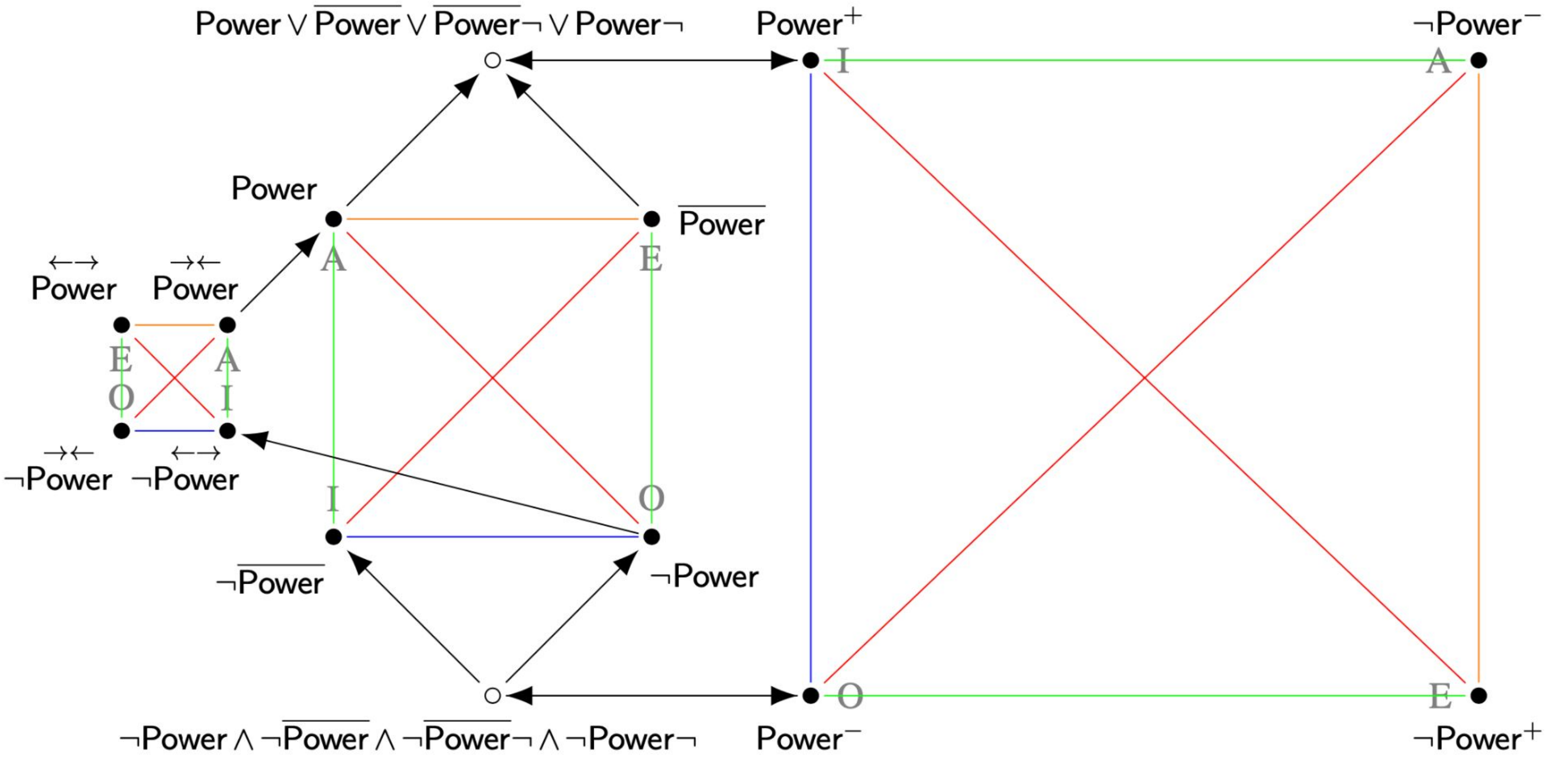




A map of potestative relations

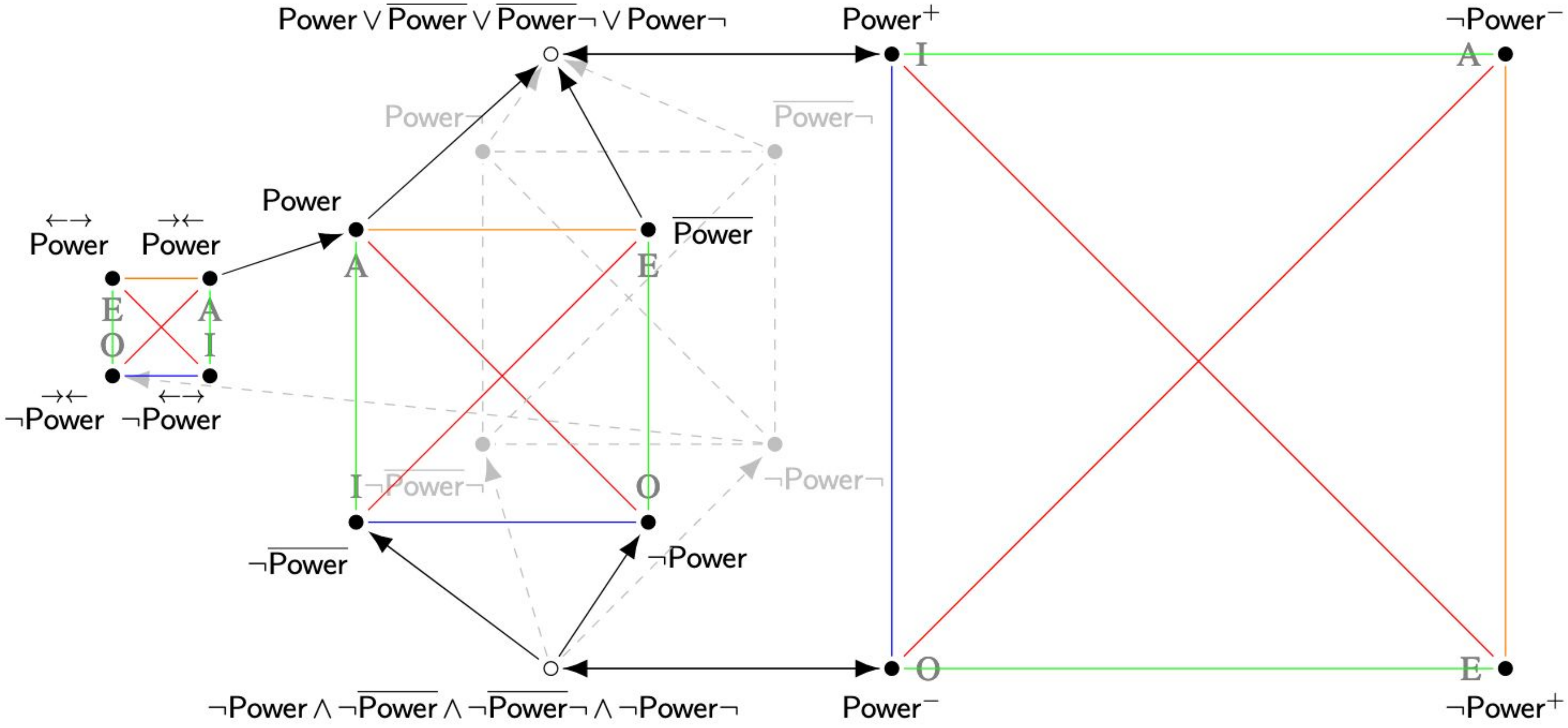
- Aristotelian diagrams can be expanded and combined among them by adding further relations of contrariety, sub-contrariety, contradiction and subalternation.
- For instance, putting together the three squares for power, and expanding the outcome-centered one to an *hexagon*, we get a complex diagram showing connections between the various senses of power.

A map of potestative relations



Further connections can be drawn, enabling one to form 3D maps...

A map of potestative relations



Building diagrammatic theories

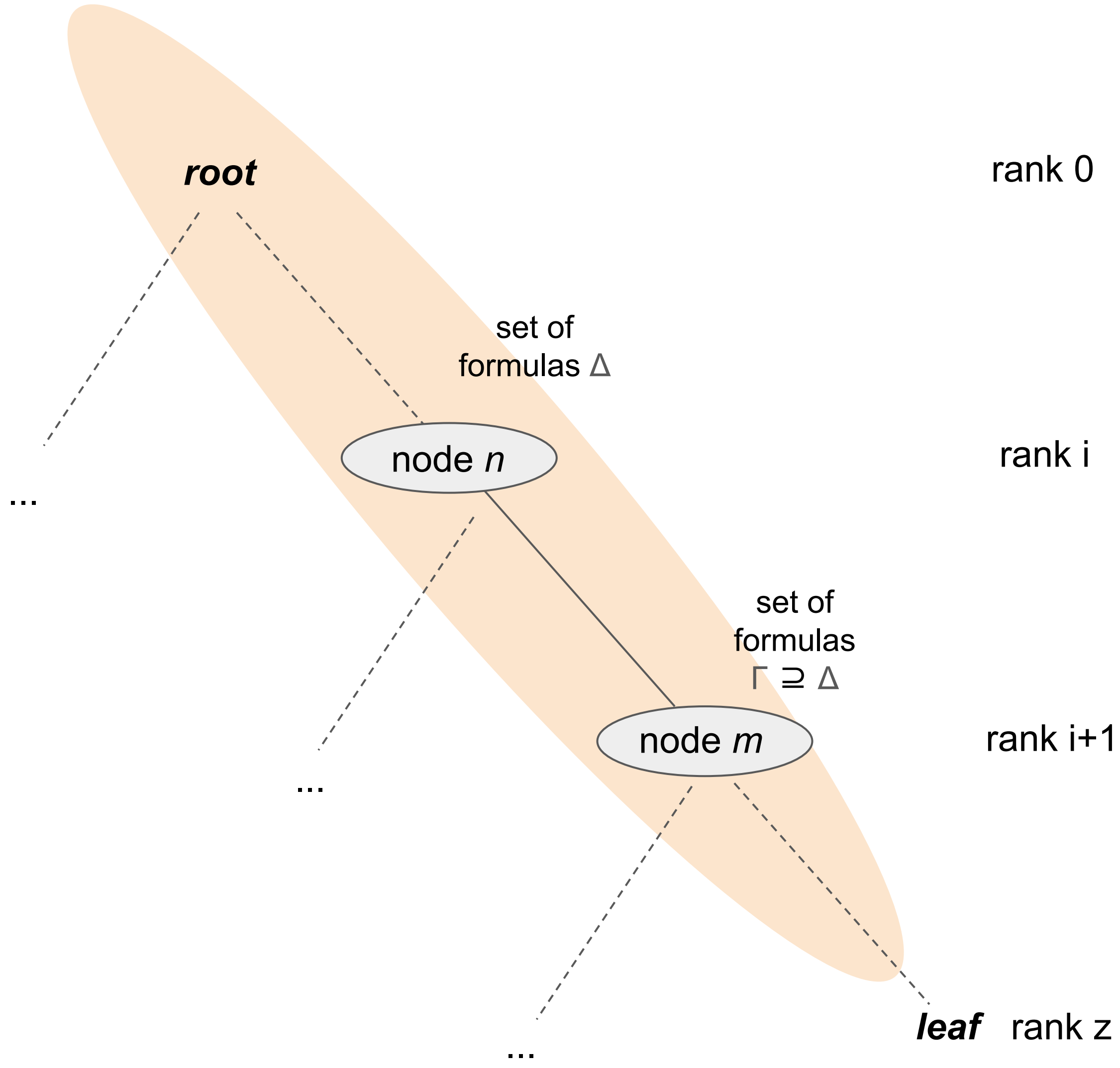
- We can define logical theories based on an Aristotelian diagram, and will name these *diagrammatic theories*;
- a diagrammatic theory **DT** over a diagram D encodes (at least) all logical relations among formulas used as labels in D;
- a diagrammatic theory will be presented as a set of inference trees, which capture selected instances of the consequence relation in a logical system.

Inference tree

Basic idea

Given a set of assumptions Δ, an inference tree T indicates which selected inferences can be performed from Δ so as to obtain a larger set Γ.

One locates Δ at some node *n* of a tree T and inspects the subsequent nodes.

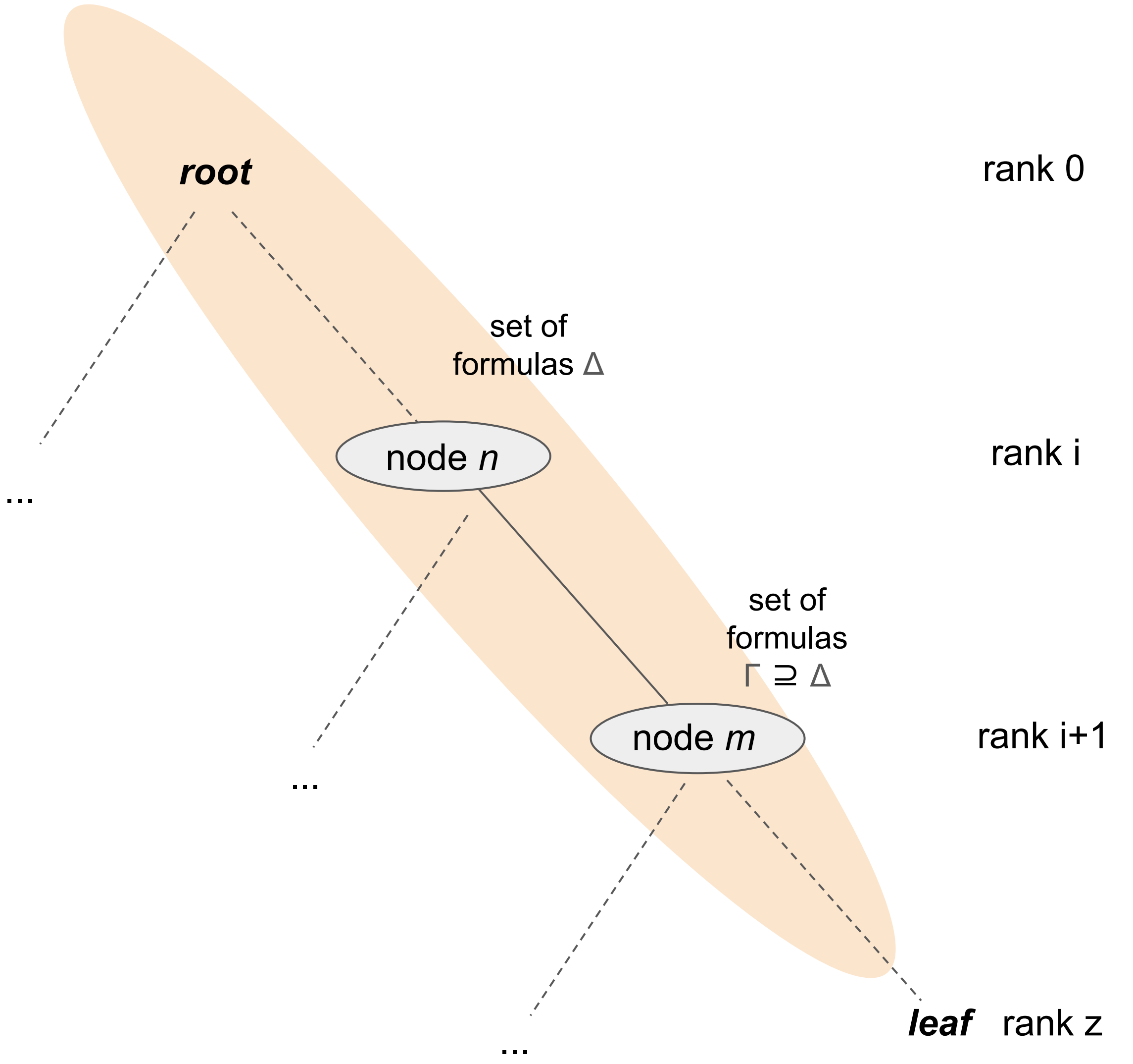


Inference tree

Set-inference

Σ can be inferred from Δ in a branch *b* of a tree T iff Σ ⊆ Γ for some Γ that occurs below Δ in *b*.

When this is the case for some branch *b* of a tree T, we say that T allows one to infer Σ from Δ.

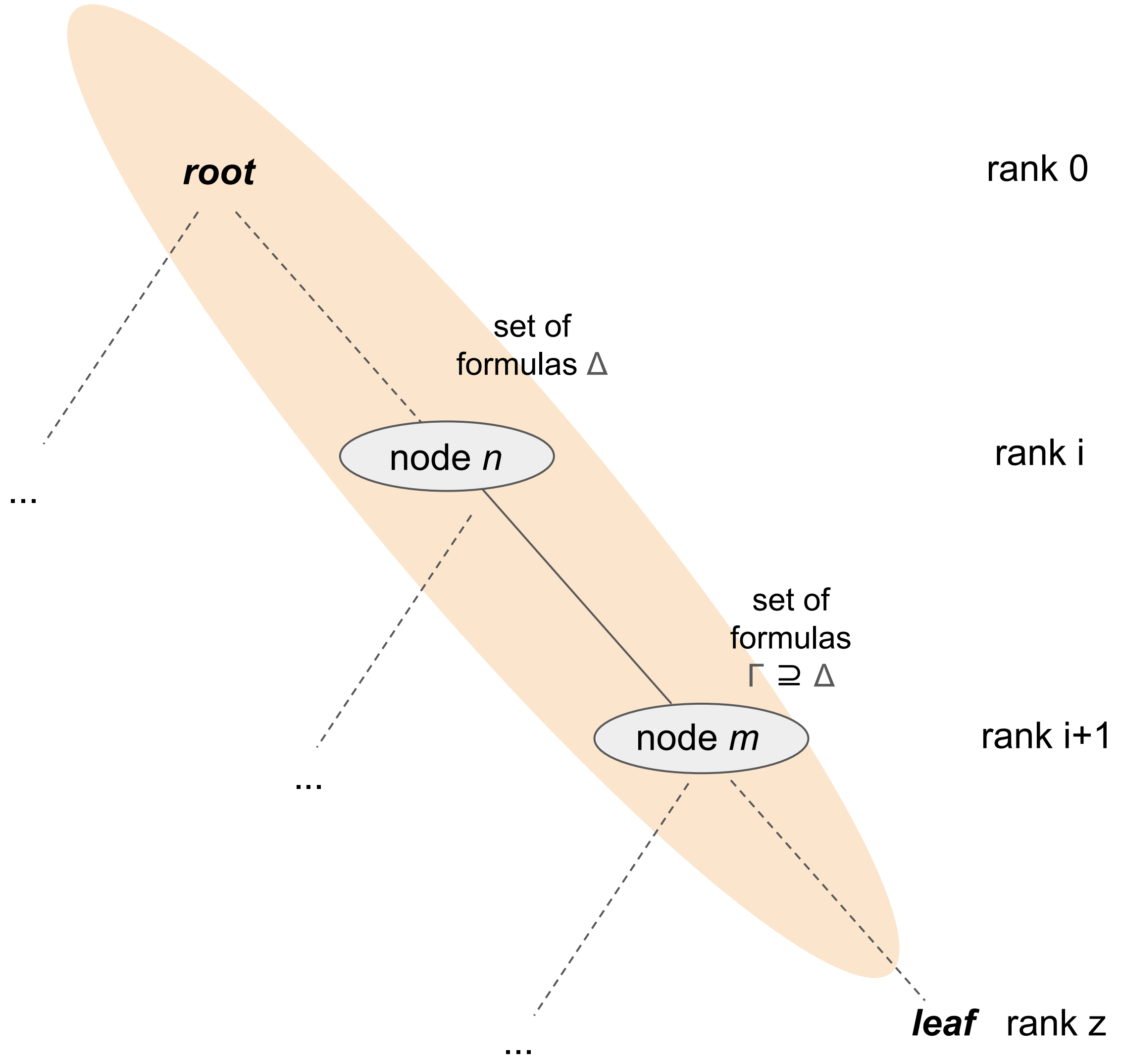


Inference tree

Set-derivation

Σ can be derived from Δ in a tree T iff for every branch *b* of T, Σ can be inferred from Δ in *b*.

When this is the case for some tree T in a diagrammatic theory **DT**, we say that **DT** allows one to derive Σ from Δ.



Decidability: algorithm

- We designed an algorithm to decide whether, for any finite set of formulas Γ and Δ and any diagrammatic theory **DT**, **DT** allows one to derive Γ from Δ.
- The algorithm consists of two steps:
  1. compare the two sets Γ and Δ in order to determine whether one is a subset of the other or not.
  2. consider the set Γ - Δ and perform procedures called traversals **with respect to the trees of DT**.

Decidability: tree traversal

The traversal of a tree T with reference to a formula φ and a set Δ can be described as follows (we assume that Δ occupies the root of T):

- Following the order of ranks, for any set of formulas Γ with rank i in T, we compare φ with all formulas in Γ and keep track of whether φ occurs in Γ or not.
- The procedure terminates when either (positive outcome) there is a rank j s.t. all sets of formulas with rank j include φ or (negative outcome) all sets of formulas with all ranks available in T have been checked.

Decidability: theory traversal

- The traversal of a diagrammatic theory **DT** with reference to a formula φ and a set of formulas Δ is the traversal of all trees T in **DT** with reference to φ and Δ. The outcome is positive iff it is positive for *some* T in **DT**.

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Complexity of the whole algorithm

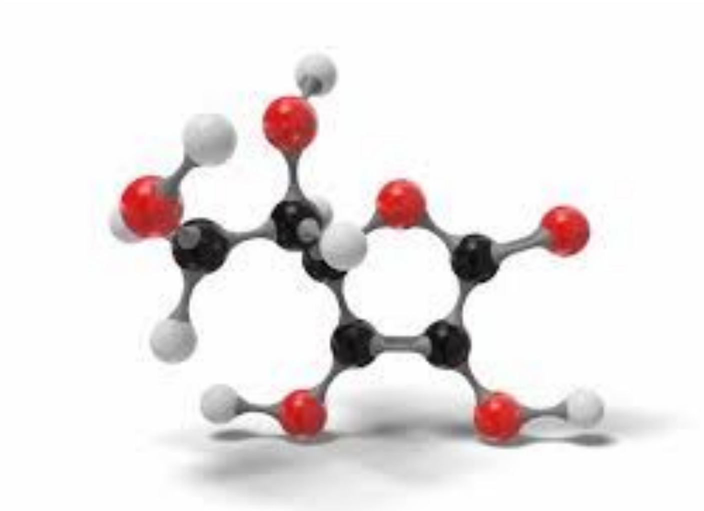
- The designed algorithm takes polynomial time with respect to *max*(|Γ, Δ|).

Conclusion

- We formalized and systematized previous contributions representing **normative positions** in Aristotelian diagrams. We showed how one can build simple logical theories based on Aristotelian diagrams via **inference trees**.
- We provided an algorithm for finite-sets-derivability-checking tailored on diagrammatic theories (hence, capturing only relevant instances of the consequence relation associated with a logical system).
- One of the main features of our approach is that we do not need the full deductive power of a logical system, since we only deal with formulas and inferences of a selected kind. In future work we will compare our approach with more general deductive approaches.

Work in progress…

- Intuitively, diagrams have also a strong potential for designing visualization interfaces. For instance, to “navigate” contracts as we do with molecules in chemistry. This remains to be further evaluated.



References

M. Andrews (1983). Hohfeld's cube. *Akron Law Review* 16: 471-485.

R. Markovich (2020). Understanding Hohfeld and formalizing legal rights: the Hohfeldian conceptions and their conditional consequences. *Studia Logica* 108: 129-158.

M. Pascucci and G. Sileno (2021). The search for symmetry in Hohfeldian modalities. In: *Proceedings of DIAGRAMS 2021*, pp. 87-102.

D.T. O'Reilly (1995). Using the square of opposition to illustrate the deontic and alethic relations. *University of Toronto Law Journal* 45: 279-310.

G. Sileno (2016). Aligning Law and Action. Phd Thesis, University of Amsterdam.

G. Sileno, A. Boer and T. van Engers (2014). On the interactional meaning of fundamental legal concepts. In: *Proceedings of JURIX 2014*, pp. 39-48.

G. Sileno and M. Pascucci (2020). Disentangling deontic positions and abilities: a modal analysis. In: *Proceedings of CILC 2020*, pp. 36-50.

Computability of Diagrammatic Theories for Normative Positions

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