Black holes
and the existence of extra dimensions

Rosemarie Aben, Milenna van Dijk, Nanne Louw

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Abstract

The addition of extra dimensions to our four dimensional universe, which is suggested in many theories, has drastic consequences for all physical behaviour in nature. By evaluating compactification and brane theories, the differences in gravitational force, volumes and mass in more dimensions become apparent. This can clearly be seen when considering black holes. Other characteristics, such as the lifetime, Schwarzschild radius and Hawking temperature, also depend on dimensionality. In the case that the fundamental Planck scale is of order TeV black holes can be produced in the next generation of particle colliders. The signatures of a black hole in detectors can be predicted by assessing its decay and enlighten us on the dimensions of the universe.

Contents

1 Foreword

In our daily experience we are very aware of the three spatial dimensions we live in. Even adding a fourth dimension, time, is not considered surprising. However, when scientists first came up with ground-breaking theories about higher dimensional space, it was put aside as science fiction or a fabrication of great fantasy. Currently, these theories, which result from the up-and-coming string theory, are beginning to be accepted as the next major step for theoretical physics [1].

The idea of extra dimensions originates in the search for a unified theory of all forces in nature. According to physicists, the electromagnetic force, the weak and the strong force and the gravitational force were combined as one at the time of the Big Bang. For these forces to merge, they would have to be the same strength in high energies. Gravity, however, is found to be much weaker than the other forces [2].

Currently, a unification is sought in string theory. According to this theory, in which general relativity and quantum mechanics are reconciled, the world is no longer described by interaction of particles but by the interaction of 1-dimensional objects, or *strings*. However, the only way in which string theory will be consistent is for it to add six or seven dimensions to our world [1].

In the first part of our paper we will review the consequences of these extra dimensions to the world as we know it. The structural properties of the dimensions will be assessed by studying compactification theories and brane theories and gravitational laws will be calculated for different numbers of dimensions. Furthermore, the Planck scale and its modifications in extra dimensions will be evaluated.

A very interesting consequence of the existence of extra dimensions is that the fundamental Planck scale might be lowered to TeV scale. This opens up the possibility that future particle colliders can produce black holes. By evaluating black holes and their properties in a more dimensional world we can examine whether black holes provide us with proof that there are indeed extra dimensions in our universe. The second part of our study is therefore devoted to black holes, their characteristics and the way they might be produced and decay in particle colliders. We have carried out all calculations and derivations in both sections ourselves, except for those where we refer to a certain reference.

2 Introduction into extra dimensions

2.1 Introduction

Before we begin our journey through the physics of higher dimensions, we should ask ourselves some questions. To begin with: what do we mean with a dimension?

You can define the number of spatial dimensions as the number of independent directions along which you can travel. When we look around us, we see one direction in front of us, one beside us and one above. We count three directions and the conclusion is rapidly drawn that we must live in a 3-dimensional world. You can also define the number of dimensions as the (precise) number of coordinates you need to completely pin down a point in space. Mathematicians walk yet another path. They use the notation (x, y, z) and add a new symbol for every new dimension. We can visualize the three dimensions we just counted by three coordinate axes. One extra dimension simply requires adding another axis, independent of the first three axis. However, when extra dimensions come along we better stop visualizing and continue with words and equations.

Everything around us suggests that we are living in three spatial dimensions. This state of mind remains for almost every person on earth. A small group who is introduced to special relativity discovers the combination of our three dimensions of space with one dimension of time, forming the space-time continuum. In this construct space and time are inseparable. This group of privileged people is offered time as the fourth dimension. Mathematically speaking this means that you need four coordinates to describe an event in space-time. There is an even smaller crowd of people who are considering even more spatial dimensions. This last group consists of well educated people, most of them physicists. These are normally not the kind of individuals who would believe in science fiction. What has gotten into them? Why would they seriously consider extra dimensions that cannot even be seen?

First we have to note that even though other dimensions cannot be seen, their existence is not ruled out. No physical theory states that there can only be three dimensions of space. Still, many believe there should be more than that. The best motivation comes from the search for the holy grail of physics: unification. Our heavyweight theory of today is the Standard Model. This theory describes three out of four fundamental forces: the strong, weak and electromagnetic forces. However, the Standard Model is not a complete theory. It does not tell us how the gravitational interaction takes place at subatomic level. Currently, the unification of all the fundamental forces into one theory is the main goal of physics: the fusion of Einstein's general relativity, which gives an accurate description of gravity, with the standard model. The only serious candidate that can make this dream come true is string theory. In this model of theoretical physics particles and forces are presented as tiny extended object, strings. It appears that string theory can only be consistent if there are many additional spatial dimensions, six or seven, depending on how you look at it.

2.2 Newton's law of Gravitation in more than four dimensions

As explained above, there are diverse reasons for the addition of extra dimensions to our universe. The next step to take will be to look at physics in a world with more spatial dimensions. Let us consider Newton's gravitational force law. This icon of classical physics tells us how the gravitational force, $F(r)$, depends on the distance, r, between two massive objects m and M:

$$F(r) = \frac{G_N m M}{r^2} \tag{1}$$

With G_N the gravitational proportionality constant.

Every grown-up multidimensional theory which includes gravity should reproduce this formula. This will be a check point along the way. How this *inverse square law* depends on distance is strongly linked with the number of spatial dimensions. This number tells us how gravity diffuses as it spreads in space. Before we adjust our formula to more than three spatial dimensions we should have some idea of how this spreading takes place.

As a descriptive explanation we picture the problem of watering a plant in a garden. We distinguish between giving the plant the water through a nozzle or through a sprinkler. Figure 1 depicts the differences between the two methods. When using the spout all the water lands on the plant. While using the sprinkler, merely a part of the water will end up on the plant. Futhermore, the distance between the sprinkler and the plant, matters as well. With the nozzle this is not the case. The fundamental distinction between the two watering methods is jumping to a higher dimension. The spout only gives water to a point (one dimension), other than the sprinkler, which distributes the water on to a surface (two dimensions). In general we can say that anything that is spread in more than one direction, will have a lower impact on objects that are further away. Similarly, gravity will spread more quickly with increasing distance.

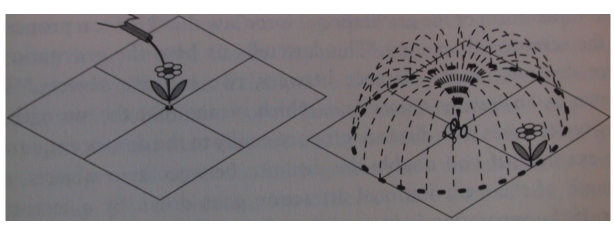

Figure 1: Depiction of the difference of watering plants with a nozzle (left) and with a sprinkler (right). When using a nozzle, all the water is poured onto one point, while the water through the sprinkler is spread over a great area. This shows us that in a more dimensional world, forces are diluted [3], pp 44 .

We will represent the strength of gravity by field lines (in analogy with the sprinkler: the water flux). The line-density indicates the strength of the gravitational force at a given point. Since gravity attracts all the surrounding mass isotropically, the field lines will go radially outwards. As you can see in figure two the same number of field lines intersect a sphere of any radius. The fixed number of gravitational field lines is spread over a sphere's surface, therefore it has to decrease with the squared radius.

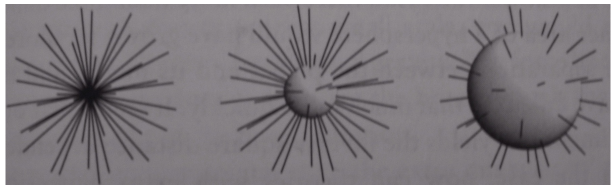

Figure 2: This figure shows that gravity (depicted by field lines) becomes weaker as a function of the distance to the gravitational source. This dependence goes as $1/r^2$. As you can see, this is caused by the fact that as the radius increases, the same amount gravitational field lines pierce through a larger area [3]). pp45.

The mathematical description of the above is given by Gauss' law, which, in a gravitational field, gives the relation between the gravitational flux flowing out of a closed surface and the mass enclosed by this surface. (see page 42 - 46 of [3]) We will use Gauss' law, which is stated below, to derive Newton's law of gravitation in more than four dimensions (we are taking space-time into account):

$$\int_{\text{surface}} \vec{g} \cdot d\vec{S} = -4\pi G_N M \tag{2}$$

where, $\vec{g}$, is the the acceleration due to gravity, caused by a point mass, M and the integral is over any surface which completely surrounds the mass.

We choose the surface around the point mass to be a sphere. The vector $d\vec{S}$ is a unit vector pointing radially out of the sphere, whereas the direction of $\vec{g}$ is the negative radial direction, so (2) now reads:

$$\int_{\text{surface}} -g dS = -4\pi G_N M \tag{3}$$

where g is the length of $\vec{g}$. Notice the minus sign in the integral. This results from the fact that $\vec{g}$ and $d\vec{S}$ are antiparallel.

This integral is easy to solve, because we can extract g (it has the same value everywhere on the surface) and the integral is just the surface area of a sphere.

$$-g \int_{\text{surface}} dS = -g \cdot 4\pi r^2 = -4\pi G_N M \tag{4}$$

To get (1) all there is left to do is to fill in Newton's second law $F = m \cdot a$ with g as a and we obtain the desired result.

We are interested in expanding this relation to more dimensions. Assuming that (2) holds for more than three dimensions we can easily rewrite (1) for d spatial dimensions in terms of the surface of a $(d-1)$-dimensional sphere (note: $(d-1)$-spheres live in d dimensions). Notice that if (2) holds in d spatial dimensions, then (4) must do so as well. For three dimensions the integral on the left was just the surface area of a sphere, however, now that we are in d dimensions it becomes the area of the surface of a $(d-1)$-sphere: $V_{d-1}(r)$ (We shall refer to this as their "volume"):

$$gV_{d-1}(r) = 4\pi G_N M \Rightarrow F = \frac{4\pi G_N mM}{V_{d-1}(r)} \tag{5}$$

So evaluating the volume of a $(d-1)$-dimensional sphere leads us directly to Newtonian gravity in d dimensions.

2.2.1 The "volume" of a $(d-1)$-dimensional sphere

We have been confronted with the surface or "volume" of a hypersphere several times. Let us look at this concept more precisely. A point in d-dimensional Euclidean space is represented by $(x_1, x_2, \ldots x_d)$. To evaluate the surface of a $(d-1)$-dimensional sphere we need its radius R, which is defined by the equation:

$$x_1^2 + x_2^2 + \ldots + x_d^2 = R^2 \tag{6}$$

For a 0-dimensional sphere (6) reads $x_1^2 = R^2$, so $x_1 = \pm R$. A 0-dimensional sphere is given by two points at $+R$ and $-R$, living in a 1-dimensional world (We will not evaluate the volume just yet, but we shall see later on that it is 2).

A 1-dimensional sphere is given by the equation $x_1^2 + x_2^2 = R^2$. This is a circle in two dimensions and its volume can be obtained by integrating a infinitesimal bit of the circle's circumference $Rd\phi$ over the entire circle:

$$V_1 = \int_0^{2\pi} Rd\phi = 2\pi R \tag{7}$$

The 2-dimensional sphere is what we commonly hold for a sphere and its surface is given by $x_1^2 + x_2^2 + x_3^2 = R^2$. The volume of this sphere can also be obtained by integration, only one more variable is needed, because there is a dimension more in this problem. We must integrate the infinitesimal bit $R\sin(\theta)d\theta$ over half a circle (the other integral will take it around the whole sphere). We obtain:

$$V_2 = \int_0^{2\pi} Rd\phi \int_0^{\pi} R\sin(\theta)d\theta = 4\pi R^2 \tag{8}$$

From all this we can make the generalization that the volume of a $(d-1)$-sphere must depend on $r^{(d-1)}$: $V_{d-1} = cr^{d-1}$. Moreover from (5) we see that the gravitational force F must depend on $r^{-(d-1)}$.

But how about the volume of spheres with dimensions higher than two? We will now derive a formula for V_{d-1} so we can plug that into equation (5). To do this we use a trick. We will evaluate the integral I given by:

$$I = \int\limits_{\text{all space}} d^d r \; e^{-r^2}$$

in two ways; in Cartesian coordinates and in polar coordinates.
In Cartesian coordinates we know that $r^2 = x_1^2 + x_2^2 + \ldots + x_d^2$. Since the integral must be over all space, which is from $-\infty$ to $+\infty$ for every variable x_i with $1 \leq i \leq d$, the integral I becomes:

$$I = \int_{-\infty}^{+\infty} dx_1 dx_2 \ldots dx_d \; e^{-(x_1^2+x_2^2+\ldots+x_d^2)} \tag{9}$$

This is the known integral $\int_{-\infty}^{\infty} dx e^{-x^2} = \sqrt{\pi}$ in d dimensions. Hence, $I = \sqrt{\pi}^d = \pi^{\frac{d}{2}}$.

Now take a look at the problem in polar coordinates. To evaluate the integral over all space is the same as taking the volume of a $(d-1)$-sphere as a function of r and integrating that over all r (from 0 to ∞).

$$I = \int_0^{\infty} dr \; e^{-r^2} V_{d-1}(r) \tag{10}$$

We already know how V_{d-1} depends on r and d ($V_{d-1} = c \cdot r^{d-1}$, where c is a constant) so we can fill this in the equation above.

$$I = \int_0^{\infty} dr \; e^{-r^2} c r^{d-1} \tag{11}$$

We can solve this in terms of the Gamma function:

$$\Gamma(x) \equiv \int_0^{\infty} t^{x-1} e^{-t} dt$$

if we substitute $t = r^2$:

$$I = \frac{c}{2} \int_0^{\infty} dt \; e^{-t} t^{\frac{d}{2}-1} \tag{12}$$

(Here we use that if $t = r^2$, then $dr = \frac{dt}{2r}$ and $r = t^{\frac{1}{2}}$) The integral is just the Gamma function for $x = \frac{d}{2}$ and we already know from the Cartesian coordinates what the answer should be.

$$I = \frac{c}{2} \Gamma(d/2) = \pi^{\frac{d}{2}} \tag{13}$$

This can be solved for c, which gives us a result for V_{d-1}:

$$V_{d-1}(r) = \frac{2\pi^{\frac{d}{2}}}{\Gamma(\frac{d}{2})} r^{d-1} \tag{14}$$

We can check this equation for $d = 1, 2, 3$ (remember: $\Gamma(n) = (n-1)!$, when n is an integer, $\Gamma(1/2) = \sqrt{\pi}$ and $\Gamma(3/2) = \sqrt{\pi}/2$). We find the results we ran into before and now we see why the volume of a 0-dimensional sphere equals two.

We have almost achieved our goal. We can substitute our expression for V_{d-1} in equation (5) to end up with the expression for F in a d-dimensional space:

$$F = \frac{2\pi G_N m M \Gamma(\frac{d}{2})}{\pi^{\frac{d}{2}} r^{d-1}} \tag{15}$$

Now we have the gravitational force in more dimensions. With this we can evaluate the behaviour of the world in more dimensions.

2.2.2 An application: Planetary orbits in higher dimensions

With the expression for the gravitational force in more dimensions we can check if there can be stable planetary orbits in higher dimensions. In the case that the planetary orbits are stable in more dimensions it is possible that the size of the extra dimensions can be as large as our solar system, maybe even infinite. However, if planets cannot orbit stably in a more dimensional world we will have to conclude that the extra dimensions have to be smaller than astronomical scales. To do this analysis, we consider the total energy of the planet:

$$E = \frac{1}{2} m v^2 + V(r) \tag{16}$$

where the gravitational potential $V(r)$ is a function of r and changes when we vary the amount of spatial dimensions. We can evaluate the potential by integrating the gravitational force:

$$V(r) = -\int_r^{\infty} F(r) dr = k^* \frac{1}{(2-d) r^{d-2}} \tag{17}$$

where k^* is a constant which is different for different dimensions: $k^* = \frac{2\pi G_N m M \Gamma(\frac{d}{2})}{\pi^{\frac{d}{2}}}$ We will include the $(2-d)$ factor in k^* and call it k. Note that k is always negative, since there are at least three spatial dimensions in our universe.

Since we are considering planetary orbits, we can be satisfied with a 2-dimensional description of the orbit. We can therefore write $\vec{r}$ and $\vec{v}$ in terms of 2-dimensional polar coordinates $(\hat{r}, \hat{\theta})$. $\vec{r}$ becomes $r\hat{r}$, while $\vec{v}$ (the time derivative of $\vec{r}$) becomes $\dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$.
The angular momentum of the system must be constant, because we are looking at an orbit.

$$\vec{l} = \vec{r} \times \vec{p} = r\hat{r} \times m(\dot{r}\hat{r} + r\dot{\theta}\hat{\theta}) \tag{18}$$

$$l = 0 + r^2\dot{\theta} \Rightarrow \dot{\theta} = \frac{l}{r^2} \tag{19}$$

If we fill in the potential and $\vec{p} = m(\dot{r}\hat{r} + r\dot{\theta}\hat{\theta})$ in (16) we get the expression:

$$E = \frac{1}{2} m(\dot{r}\hat{r} + r\dot{\theta}\hat{\theta})^2 + \frac{k}{r^{d-2}} \tag{20}$$

Working out the quadratic factor and filling in (19) for $\dot{\theta}$ we get:

$$E = \frac{1}{2}m\dot{r}^2 + \frac{ml^2}{2r^2} + \frac{k}{r^{d-2}} \tag{21}$$

We have now reduced the problem to a one-dimensional situation (there is only r-dependence) and we have split (21) into two parts; one dependent on $\dot{r}$ and associated with the kinetic energy of the system and one dependent on r and associated with the potential of the system. To see whether a planetary orbit is stable, consider the potential of the system in different dimensions.

$$V(r) = \frac{ml^2}{2r^2} + \frac{k}{r^{d-2}} \tag{22}$$

When plotting this function for different dimensions it shows that the potential only has a minimum value for $(d-1) < 3$. For higher dimensions than three there is either no extreme value (for $d = 4$) or a maximum, which means that planetary orbits are not stable in higher dimensions. They could exist, but the slightest knock would push the planets out of orbit. The radial position of the planets could *not* be stable.

This result should not be too surprising, since we have never seen a deviation to the inverse square law on ordinary distances. We *know* that Newton‘s law behaves as $\frac{1}{r^2}$ so this automatically leads to the conclusion that the number of spatial dimensions is three. Furthermore the assumption that the extra dimensions are of infinite size seems very unlikely. If they were, why would we be incapable of seeing them, or take a walk in them?

2.3 Compactification and Kaluza-Klein Reduction

To provide a reasonable explanation to the problem mentioned above, we can imagine the extra dimensions are compactified, meaning they are curled up so small that we cannot see them. In this section we will look at the first theory of compactification [2]. Another explanation could be localisation, using entities called branes. Branes can be described as surfaces on which numerous spatial dimension can be localised, see figure three. Their characteristics make them interesting for string theory for matter and forces can be confined to branes and branes can carry energy. The reason why we do not see the branes is because their radius is much smaller than the world we live in, see figure four.
In the 1920s Theodor Kaluza and Oskar Klein tried to combine electromagnetism and gravity in one covering geometrical scheme. To accomplish this, a curled-up fifth dimension was added to our universe [1].

Let us look a bit closer at the Kaluza-Klein compactification. We will consider relativistic particles to explain this compactification. First consider one relativistic particle in five dimensions (four spacial and time) with mass M. The 5-momentum for this particle is:

$$p \equiv (\frac{E}{c}, p_1, p_2, p_3, p_4) \tag{23}$$

From this 5-momentum we can derive an expression for the energy E in terms of its 4D spatial-momentum and M. Special relativity tells us:

$$p^{(5)} \cdot p^{(5)} = -\frac{E^2}{c^2} + p_1^2 + p_2^2 + p_3^2 + p_4^2 = m^2c^2 \tag{24}$$

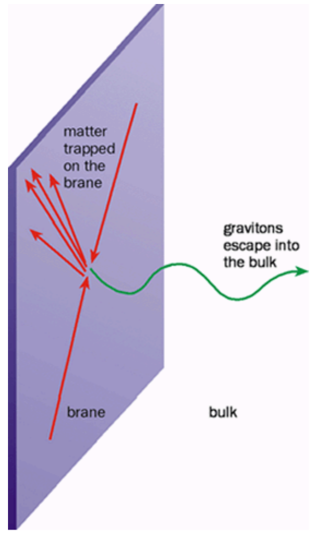

Figure 3: The brane can be seen as a surface with three spatial dimensions. Trapped on this surface are the electromagnetic, weak and strong forces and all the matter in the universe. Gravitons, however, are able to leave the surface and move throughout the bulk. The bulk is all the volume outside the brane.

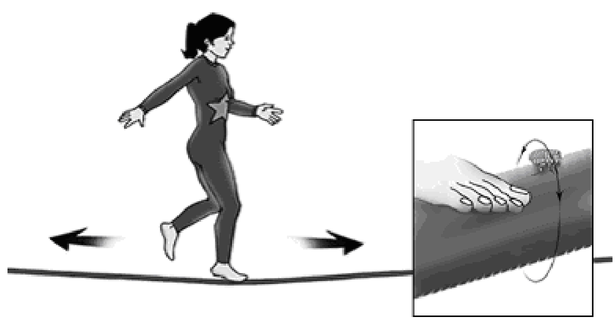

Figure 4: The 2-dimensional surface looks 1-dimensional when its radius is small. For the dancer the surface is small and she sees it as a one-dimensional rope. However, the ant is small compared to the rope and it sees it as a 2-dimensional surface. That is why for us we do not see the branes, their radius is much smaller than the world we live in.

The next step will be to write the previous in correct terms from which we acquire:

$$E^2 = m^2c^4 + \tilde{p}^2c^2 + p_4^2c^2 \tag{25}$$

with $\tilde{p}$ the spatial momentum in 4D.
For futher analysis it is necessary to take the compactification into account. Take the fourth spatial dimension, x_4, compactified on a circle of radius R. Now we can express its energy and 3D spacial momentum in terms of p_4 and M.
When we compare this equation with the equation for the energy in 4D: $\frac{E^2}{c^2} = m^2c^2 + \tilde{p}^2$ we see there is an extra term, p_4, in the equation for five dimensions. This means that, since

E, c, and p are the same for both 4D and 5D, the mass in 5D has to be smaller than the mass in 4D to compensate for the extra factor p_4^2. Assuming we are confined to four of the five dimensions, the mass seems to be larger to us than it is in 5D.

Now we observe two particles in 5D instead of just one. x_4 is still compactified on a circle of radius R. Take the 5-momenta of the two particles to be p and p'. Then the gravitational force between the particles, when they are at rest in 4D (so p equals zero) and are separated by $r \gg R$, can be approximated by the semi-relativistic expression:

$$F(r) \simeq \frac{G_N}{c^2}\frac{p \cdot p'}{r^2} \tag{26}$$

Let us consider this force in 5D, with the momentum of the particles in the x_4 direction. We would like to show that it will look like a combination of a gravitational force and an electrostatic Coulomb force. We may assume that the mass in 5D equals zero. Furthermore we consider the momenta to be quantized. This results from the fact that the extra dimensions are compactified. The dimensions are compactified on circles with perimeter $2\pi R$. The wavelengths have to fit a whole number of times (N) on this circle: $N\lambda = 2\pi R$. This means that the wavelengths are quantized. The momentum is proportional to the wavelength: $p = \frac{\hbar}{\lambda}$. As a result we see that: $p = \frac{N\hbar}{R}$, the momentum is quantized.

First consider the inner product of the two momenta:

$$p \cdot p' = -\frac{EE'}{c^2} + \frac{nn'\hbar^2}{R^2} \tag{27}$$

We already know that: $\frac{E^2}{c^2} = m^2c^2 + \tilde{p}^2 + p_4^2$ and $p_4 = \frac{n\hbar}{R}$. When you compare these equations, we acquire an equation for the energy:

$$E = \frac{|n|\hbar c}{R} \tag{28}$$

We can write $|n|$ in another way. We know the inner products of the 5-momenta and 4-momenta, with the knowledge that the inner product of the 5-momenta is zero, you can compare the two and get:

$$|n| = \frac{McR}{\hbar} \tag{29}$$

Finally we can fill (27), (28) and (29) in into (26) to get an expression for $F(r)$ in four dimensions:

$$F(r) = -\frac{G_N M'M}{r^2} + \frac{G_N nn'\hbar^2}{c^2R^2r^2} \tag{30}$$

The two assumed particles both have charges contributing to the Coulomb force. The second part in (30) should be the Coulomb force. Therefore we have to compare it to the known equation for Coulomb force, which is: $F_c = \frac{qq'}{4\pi\epsilon_0 r^2}$. Clearly, the charges only depend on n and n', because the other terms are constants. Therefore: $q \propto n$ and $q' \propto n'$.

As we all know the gravitational force is attractive. The gravitational force in (30) has a minus sign, which means that, minus stands for attraction. Now let us look at the second term, the Coulomb force. The attraction or repulsion depends on n and n'. Suppose the particles attract each other. This means that one n has a minus sign and so the Coulomb force will have a minus in front and therefore will be attractive. On the other hand, when the particles repel each other both have one or none minus sign, which means that there will be no minus signs in front of the Coulomb force, and so it will be repulsive.

(26) shows the dependence on R, so we can calculate R in such a way that a particle in five dimensions with one unit of momentum along x_4 appears to have the elementary charge of an electron. After we fill in the right values for the constants we get a value for R of $1.8 \cdot 10^{-34}$ m.

The search for unification, which began around 1860, when Maxwell came up with a brilliant theory of electromagnetism, inspired Theodor Kaluza and Oskar Klein to come up with the compactification when they tried to combine electromagnetism and gravity in one covering geometrical scheme. To accomplish this, they needed the curled-up fifth dimension. However, this theory cannot explain why gravity is so much weaker than the electromagnetic force, nor can it be combined with quantum mechanics, which was rapidly developing in their day [1].

2.4 Gravitational potential in n compactified dimensions

The result that Newton's gravity and Coulomb's electromagnetic force could be combined in one expression is still promising for further unification. This provides a good reason to consider the gravitational potential in more than one compactified dimension.

In section 2.2.1 en 2.2.2 we have derived Newton's gravitational potential in d spatial dimensions (17):

$$V(r) = k\frac{1}{r^{d-2}} \tag{31}$$

where $k = \frac{2\pi G_N mM\Gamma(\frac{d}{2})}{(2-d)\pi^{\frac{d}{2}}}$.
It is important to take in account the different variables for the dimensions. We have defined d as the number of spatial dimensions: in our world $d = 3$. Furthermore we have n as the number of extra spatial dimensions. The relation between d and n is: $d + 1 = 4 + n$.
We can express the potential in $4 + n$ space-time dimensions, using G_{n+4}: Newton's gravitational constant in $(n + 4)$ space-time dimensions:

$$V(r) = -\frac{G_{n+4}mM}{r^{n+1}} \tag{32}$$

We now turn to the question what the gravitational potential would look like if the additional n dimensions were compactified on to a circle with radius R. We can visualise the compactification by considering a mass M on a cylinder. The length of the cylinder represents the three unfolded spatial dimensions and the radius the n compactified dimensions. We

can solve the problem using the method of images (just like the electrodynamic problem of a point charge on a cylinder).
If we first imagine n to be one, we can unfold the extra dimension, so to get an infinite extra dimension, with the mass M repeated every $2\pi R$. (32) now becomes an infinite sum over all the masses. The distance to the mass becomes $\sqrt{r^2+(b2\pi R)^2}$ where b is an integer going from $-\infty$ to ∞. The potential becomes:

$$V(r) = -\sum_{b=-\infty}^{\infty} \frac{G_{n+4}mM}{[r^2+(b2\pi R)^2]^{\frac{1}{2}}} \tag{33}$$

By generalising this to n compactified dimensions, we get an expression in terms of n infinite sums:

$$V(r) = -\sum_{b_1=-\infty}^{\infty} \cdots \sum_{b_n=-\infty}^{\infty} \frac{G_{n+4}mM}{[r^2+(b_1 2\pi R)^2+\cdots+(b_n 2\pi R)^2]^{\frac{n+1}{2}}} \tag{34}$$

In the limit of $r \gg R$ these sums can be replaced by integrals. This is because the fractions $2\pi R$ are so small in comparison to r, that it can be approximated as infinitesimal. Now (34) can be written as:

$$V(r) = -\int_{b_1=-\infty}^{\infty} \cdots \int_{b_n=-\infty}^{\infty} \frac{G_{n+4}mM}{[r^2+(b_1 2\pi R)^2+\cdots+(b_n 2\pi R)^2]^{\frac{n+1}{2}}} db_1 \dots db_n \tag{35}$$

To clean up this expression we divide the denominator by r^2 and substitute $x_i = \frac{b_i 2\pi R}{r}$ to get:

$$V(r) = -\frac{G_{n+4}mM}{r(2\pi R)^n} \int_{x_1=-\infty}^{\infty} \cdots \int_{x_n=-\infty}^{\infty} \frac{1}{(1+x_1^2+\cdots+x_n^2)^{\frac{n+1}{2}}} dx_1 \dots dx_n \tag{36}$$

Now we can change to polar coordinates using the volume of a $(n-1)$ dimensional sphere ($V_{n-1}(\rho)$) (I will use ρ for the radial variable, because we already have a different r in the expression). The integral can be written as follows:

$$V(r) = -\frac{G_{n+4}mM}{r(2\pi R)^n} \int_0^{\infty} V_{n-1}(\rho) \frac{1}{(1+\rho^2)^{\frac{n+1}{2}}} d\rho \tag{37}$$

If we now substitute u for ρ^2, we can solve this in terms of the Beta function [16];

$$B(p+1,q+1) = \int_0^{\infty} \frac{u^p du}{(1+u)^{p+q+2}} = \frac{\Gamma(p+1)\Gamma(q+1)}{\Gamma(p+q+2)} \tag{38}$$

Working out the integral and writing the Beta function in terms of Gamma-functions yields the following result [16]:

$$V(r) = -\frac{V_{(n-1)}G_{n+4}mM}{2\Sigma_n}\frac{1}{r} \tag{39}$$

Where $V_{(n-1)}$ is the volume of a n-dimensional unit sphere given by (14) and Σ_n is the size of the extra dimensions (in this case $\Sigma_n = (2\pi R)^n$). The equation looks just like the gravitational potential we know in 4D. This is what we would expect, since we are working in the limit of $r \gg R$. To get the regular expression in 4D, we have to realise that:

$$G_{n+4} = \frac{2G_N\Sigma_n}{V_{(n-1)}} \tag{40}$$

So if we assume the extra n dimensions to be compactified on a circle, we obtain the usual inverse square law for distances much greater than the size of the extra dimensions. However, if we look at distances close to R, the approximation of a sum by an integral no longer holds. Threfore we would expect the gravitational force to deviate from the inverse square law and pick up an extra correction term:

$$V(r) \sim \frac{1}{r}(1 + \alpha e^{-\frac{r}{\lambda}} + \ldots) \tag{41}$$

Physicists trying to find extra dimensions by looking at the gravitational force are looking for this extra term.

2.4.1 The exact gravitational potential for $n = 1$

We can check that the expression for the potential satisfies our expectations by taking $n = 1$. With this we can work out (34). If we calculate the limit of $r \gg R$ we want to find the $\frac{1}{r}$ potential, while taking the other limit ($R \gg r$) we would expect the potential derived in section 2.2.2, for we are looking at such small distances that the extra dimensions appear very large. For $n = 1$ (34) becomes:

$$V(r) = -\sum_{b=-\infty}^{\infty} \frac{G_{4+1}mM}{r^2 + (b2\pi R)^2} \tag{42}$$

Now we divide the denominator by $2\pi R$ and use the identity:

$$\sum_{m=-\infty}^{\infty} \frac{1}{m^2 + a^2} = \frac{\pi}{a}\coth(\pi a) \tag{43}$$

to get:

$$V(r) = -\frac{G_{4+1}mM}{2rR}\coth\left(\frac{r}{2R}\right) \tag{44}$$

We will first check the limit $R \gg r$, so we need the limit $\lim_{x\downarrow 0}\coth x = \frac{2+x^2}{2x}$. Here we can forget about the quadratic term, because in this case it would be extremely small.

$$V(r) = -\frac{G_{4+1}mM}{r^2} \tag{45}$$

which is exactly what we had expected!
Using the limit $\lim_{x\to\infty}\coth x = 1$ we obtain the potential in the limit $r \gg R$:

$$V(r) = -\frac{G_{4+1}mM}{2Rr} \tag{46}$$

which is exactly (39) for $n = 1$.
As a result we see that for $n = 1$ the potential satisfies our physical expectations. Furthermore the exponential power in (41) can be obtained by the first order correction to equation (47).

$$V(r) = -\frac{G_{4+1}mM}{2Rr} \tag{47}$$

We can calculate the correction term on (47) by keeping the first correction term of the approximation we made to get from (44) to (47). To simplify matters, we write

$$V(r) \sim \frac{1}{r}\coth\left(\frac{r}{2R}\right) \tag{48}$$

When we write out $\coth$ and multiply by $e^{\frac{-r}{2R}}$ we obtain

$$V(r) \sim \frac{1}{r}\left(\frac{1+e^{\frac{-r}{R}}}{1-e^{\frac{-r}{R}}}\right) \tag{49}$$

In the limit where $r \gg R$, the term $e^{\frac{-r}{R}}$ goes to zero. This is where we made the approximation

$$V(r) \sim \frac{1}{r}(1+e^{\frac{-r}{R}})(1+e^{\frac{-r}{R}}+\ldots) \tag{50}$$

Working away the brackets and omitting the last term leaves us with

$$V(r) \sim \frac{1}{r}(1+2e^{\frac{-r}{R}}) \tag{51}$$

2.4.2 The Planck scale

To find the scale at which gravity becomes a strong force, the fundamental units G_N, $\hbar$ and c can be combined into a new quantity with units of mass and another with units of length. This can be done by dimensional analysis. These quantities are called the *Planck mass* (M_p) and the *Planck length* (l_p):

$$M_p \equiv \sqrt{\frac{\hbar c}{G_N}} = 2,2 \cdot 10^{-8}kg \sim 1,2 \cdot 10^{28}eV \tag{52}$$

and

$$l_p \equiv \sqrt{\frac{\hbar G_N}{c^3}} = 1,6 \cdot 10^{-35}m \tag{53}$$

We see from (52) that Newton's gravitational constant is inversely proportional to the Planck mass (squared) and thus also related in this way to a concept called Planck energy (remember that we can convert between energy and mass just by putting in a c^2).

Before we continue our analysis it may be wise to look deeper into some concepts from high energy physics. In this branch of physics, people always talk about physical processes taking place at a certain energy scale. What is meant is related to the following.

In quantum mechanics there is a concept called the uncertainty principle. This principle states that the product of the uncertainties in the measurements of position and momentum must exceed Planck's constant. In other words, when you know everything about the position of a particle, you know almost nothing about its momentum. Particle physicists work with processes occurring at very small (read: relatively precise known) distances. Consequently these processes must contain very high momenta. In special relativity energy and momentum are related. When momenta are high the energies are high too. In conclusion we can say that you can only explore short distances by making use of high energies. Moreover, we can state that working at different distance scales requires working at different energies. We can convert an energy scale into a corresponding length scale with the formula $E = \frac{hc}{\lambda}$. This length scale illustrates the range of the associated force.

There is another way to come to the same conclusion. Picture the following. Only particles whose wave functions vary over small scales will be affected by short distance physical processes. However according to the de Broglie relation, particles whose wave function involve short wavelengths also have high momenta. Therefore de Broglie would also have us conclude that you need high momenta, and hence high energies, to be sensitive to the physics of short distances (see page 143 of [3]) .

In nature there are at least two fundamental energy scales. On the one hand we have the electro-weak energy scale ($\sim 10^3$ GeV). This scale appears in the Standard Model. Current experiments in particle accelerators are operating around this energy. When the Large Hadron Collider (LHC) is launched in 2007 at CERN we will be able to do experiments above the weak scale energy. On the other hand there is the Planck scale ($\sim 10^{18}$ GeV) which is much higher and related to gravity. We already saw that Newton's gravitational constant is inversely proportional to Planck energy. Gravity is weak because the Planck scale energy is large. Moreover, the Planck scale energy is the amount of energy that particles would need to have, for gravity to be a strong force. As told before we can convert an energy scale into a corresponding length scale which tells us about the range of the force in question. The enormous gap between the two energy scales bothers a lot of physicists. The existence of this gap is formulated as the hierarchy problem.

The supporters of the Grand Unified Theory would like to combine all physics in one theory. However, you can expect particles that experience similar forces, to be somewhat similar. The enormous desert in between the two energy scales does not help them much. For example, this gap results in a huge dissimilarity in the mass of the particles (we can convert between energy and mass with Einstein's formula). Therefore many physicists are determined to solve this mystery.

However, there is a way out of the hierarchy problem. If we assume extra compactified dimensions, we could obtain a new expression for the Planck length and the Planck mass in $(n + 4)$ dimensions. As we shall see the Planck scale can become much weaker, even down to the TeV scale, which can be tested in near future collision experiments. We see

that extra dimensions provide a solution to the hierarchy problem.
Now we know that Newton's constant has different units in more dimensions. In 4 dimensions (that is: three spatial and one time dimension) it has units of $m^3 s^{-2} kg^{-1}$. In $(n+4)$ dimensions G_{4+n} has units of $m^{n+2} s^{-2} kg^{-1}$. You can easily check that the Planck length then becomes:

$$l_p = \left(\frac{\hbar G_{4+n}}{c^3}\right)^{\frac{1}{n+2}} \tag{54}$$

Next we can find an expression for the Planck mass in 4D (52) in terms of the Plank mass in $(4+n)$-dimensions M_{4+n}, given by:

$$M_{4+n} = \left(\frac{\hbar^{(n+1)}}{c^{(n-1)} G_{4+n}}\right)^{\frac{1}{n+2}} \tag{55}$$

Filling in G_{n+4} from (40) in (55) and using (52) works out to (we will denote M_p as M_4 for the 4 dimensional Planck mass to avoid confusion):

$$M_4^2 = \left(M_{4+n}\right)^{n+2} \frac{2\Sigma_n}{V_{n-1}} \frac{c^n}{\hbar^n} \tag{56}$$

With this result we can work out the size of the extra dimensions for gravity to become strong at the electro-weak scale as a function of the number of extra dimensions n.
This allows us to make the first step in calculating the actual size of the extra dimensions. As in the theory developed by Arkhani, Dimopoulos and Dvali (often referred to as the ADD-model) [2] we assume $M_{4+n} = m_{EW} \approx 1$TeV. Furthermore, from measurements we know $M_4 = 10^{16}$TeV. By rewriting (55), using the above numbers and exctracting R^n from Σ_n, we obtain:

$$R = \frac{10^{\frac{32}{n}}}{2\sqrt[n]{\pi^{\frac{n-1}{2}} \Gamma(\frac{n+1}{2})}} TeV^{-1} \tag{57}$$

Notice that this has dimensions TeV^{-1}. This results from the convention to set $\hbar = c = 1$. To express R in meters we use $E = \frac{2\pi\hbar c}{\lambda}$, this gives us:

$$R = \frac{1.98}{2\sqrt[n]{\pi^{\frac{n-1}{2}} \Gamma(\frac{n+1}{2})}} 10^{\frac{32}{n}-19} m \tag{58}$$

For $n < 7$, which are the relevant extra dimensions for our purpose, the factor in front of $10^{\frac{32}{n}-19}$ gives a factor ~ 0.5. We only want to get an indication of the size of the possible extra dimensions, so we can leave this factor outand simply write:

$$R \sim 10^{\frac{32}{n}-19} m \tag{59}$$

We can use this expression to calculate the size of the extra dimensions:

n	1	2	3	4	5	6
$R(m)$	10^{13}	10^{-3}	10^{-9}	10^{-11}	10^{-13}	10^{-14}

In this table we can see that if $n = 1$ the size of the extra dimension is comparable to distances in our solar system. However, as we have assessed in section 2.2.2, planetary orbits are not influenced by the quantum gravity effects we would expect if $r \sim R$. Therefore the possibility that $n = 1$ is excluded.

We have now calculated how properties as gravity and potential change as the number of dimensions changes. We have also assessed the structure extra dimensions might have. Furthermore we have found a relation between M_p and the number of extra dimensions n. By increasing n the Planck scale can be lowered to 1 TeV, providing a solution for the hierarchy problem. This alteration in gravity in more dimensions has numerous possible consequences, one of which is the production of black holes in future colliders. In the next section we will evaluate this production and look at some fascinating properties of black holes.

3 Black holes and extra dimensions

3.1 Properties of black holes

A black hole is a region of space that has such a concentration of mass or matter that nothing inside its event horizon can ever escape, due to enormous force of gravity. Not even light has a speed fast enough to escape the gravitational field [16]. A black hole consists of an event horizon and a gravitational singularity. A singularity is a point in space-time at which the laws of nature lose validity. The singularity has an infinitely small volume and an infinitely large mass. Here time and space cease to exist [15].
A black hole is defined by merely three characteristics: mass, charge, and angular momentum. Other information, for example about the mass that has fallen into the black hole is lost behind the event horizon. This phenomenon is called the no-hair theorem [15]. All other properties can be derived from these three characteristics.

As is shown in the previous chapter, things change drastically when we look at phenomena on the level of the Planck scale. In this case quantum gravity becomes important and black holes of this scale are considered totally different from astrophysical black holes. In this chapter we will assess a number of important qualities of black holes. First we will do this for black holes in four dimensions. Then we will investigate black holes in extra dimensions. However, most of the properties in four dimensions hold for black holes in extra dimensions, sometimes varying only by a constant.

3.1.1 Escape velocity

For any given gravitational field and a given position, the escape velocity is the minimum speed an object needs to have in order to move away indefinitely from the source of the field. The object is assumed to be influenced by no forces except for the gravitational field. The energy required to take an object from the surface of a planet to infinity is given by the integral of the gravitational force:

$$E = -\int_R^\infty -\frac{G_N M m}{r^2} dr = \frac{G_N M m}{r} \tag{60}$$

To calculate the escape velocity, the energy here calculated has to be equal to the kinetic energy:

$$\frac{1}{2} m v_{esc}^2 = \frac{G_N M m}{r} \tag{61}$$

This gives an escape velocity:

$$v_{esc} = \sqrt{\frac{2 G_N M}{r}} \tag{62}$$

3.1.2 Event horizon

The surface of a black hole is the so-called event horizon, an imaginary surface surrounding the mass of the black hole. When an object has crossed the event horizon it cannot escape, for it would have to travel with a speed greater than the speed of light. Objects that cross the event horizon will experience spaghettification [16], which means the object will be distorted, because the parts of the object closer to the singularity experience a stronger attraction than those parts further away. Consequently the object will be stretched radially with respect to the black hole. Furthermore, all parts of the object are pulled in the direction of the singularity, which results in the compression of matter in directions perpendicular to this axis.

The radius of the event horizon can be calculated with the use of (62) with $v_{esc} = c$. This gives: $R_H = \frac{2G_N M}{c^2}$, more commonly written in terms of the Planck mass $M_p \equiv \sqrt{\frac{\hbar c}{G_N}}$ instead of G_N:

$$R_H = \frac{2\hbar M}{M_p^2 c} \tag{63}$$

3.1.3 Hawking radiation

In 1974, Steven Hawking published the theoretical argument for the existence of the radiation of black holes [5]. He showed that a black hole can emit thermal radiation, called Hawking radiation. As we have just learned, black holes are sites of immense gravitational attraction into which surrounding matter is drawn by gravitational forces. Classically nothing can ever escape the black hole. However, using quantum mechanics and classical gravity Hawking concluded that black holes emit particles in a thermal spectrum. He explained the emission of particles as follows: the vacuum surrounding the black holes is filled with virtual pairs of particles, one with positive energy and one with negative energy. Virtual particles are particles which can exist only for a short amount of time. Creation out of nothing is possible as long as the uncertainty principle is not violated. In the presence of the strong gravitational field of the black hole the virtual pairs are ripped apart to form a real pair. Now one particle, the one with negative energy, will fall into the black hole while the other is emitted. This cannot be inverted, because real particles only have positive energies.

A black hole radiates particles as a black body with a specific temperature. Before calculating this temperature, called the Hawking temperature, the acceleration due to gravity at the horizon of the black hole needs to be determined. This can be done by equalising the resulting force and the gravitational force with $R = R_H$. This gives:

$$a = \frac{G_N M_P^4 c^2}{4\hbar M} = \frac{c^3 M_p^2}{4\hbar M} \tag{64}$$

From this we see that T depends only on G_N and M through the acceleration. Assuming $T \sim a$ T can be found by dimensional analysis. The equation for T is then:

$$T = \frac{\hbar c^3}{32\pi k M G_N} \tag{65}$$

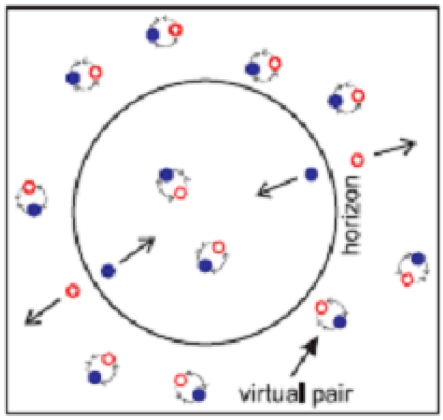

Figure 5: An impression of pair creation at the Schwarzschild radius of a black hole. A virtual pair is created and then ripped apart. Consequently, one particle falls into the black hole and the other is radiated as Hawking radiation [5].

From this formula we can conclude that when the mass of a black hole decreases as its temperature increases.

3.1.4 Lifetime

We have just discussed that from the vitual pair the particle with the negative energy will fall into the black hole. Since energy equals mass, the black hole will lose mass when the particle falls in. Due to the Hawking radiation the black hole will steadily lose mass. Therefore the black hole has a certain lifetime which can be calculated. First we need the luminosity of the radiation of the black hole:

$$luminosity = \frac{dE}{dt}\frac{1}{A} \tag{66}$$

We can assume that the luminosity obeys the Stephan-Boltzmann Law: $luminosity = \sigma e T^4$ with $e = 1$ for a black body, (66). When combining this law with $E = Mc^2$ and $A = 4\pi R_H^2 = \frac{16\pi G_N^2 m^2}{c^4}$, we find the following equation:

$$\int \frac{16\hbar^4 c^6 t}{32^4 \pi^3 k^4 G^2} dt = \int M^2 dM \tag{67}$$

After integrating (67) we acquire an integration constant, as usual. When the mass is zero the lifetime has to be zero too. This is only the case when the integration constant equals zero. Rewriting (67) gives a lifetime of:

$$\tau = \frac{M^3 32^4 \pi^3 k^4 G^2}{48\hbar^4 c^6} \tag{68}$$

3.1.5 Specific heat

The specific heat can be calculated from (65). Assuming that the total energy of a black hole is $E = Mc^2$ (65) can be rewritten as $E = \frac{\hbar c^5}{32\pi k G_N T}$. Now taking the derivative to T we obtain a formula for the specific heat:

$$C_v = -\frac{\hbar c^5}{32\pi k G_N T^2} \tag{69}$$

The result for the specific heat is negative, which is very surprising. When energy is removed from the black hole T increases in stead of decreases as we are used to with respect to all other objects in nature.

3.1.6 Entropy

A black hole which does not possess entropy would violate the second law of thermodynamics. When mass falls into the black hole, the entropy of this mass is lost, however, since the total entropy always has to increase, the black hole has to have an entropy increase to compensate for the loss of entropy of the infalling mass. The entropy of a black hole has a special feature, namely that the black hole entropy is the maximal entropy that can be squeezed within a fixed volume.
The entropy can be calculated with the use of the first law of the thermodynamics: $dS = \frac{dE}{T}$. Now we can fill in the already known total energy ($E = Mc^2$) and (65) to get integrals over the mass and entropy. When worked out, we obtain:

$$S = \frac{16\pi k G_N}{\hbar c} M^2 \tag{70}$$

Expression in Planck length and Schwarzschild radius gives:

$$S = \frac{4\pi k l_p^2 c^4 R_H^2 M_p^4}{\hbar^4} \tag{71}$$

3.1.7 Properties in extra dimensions

It is time to study the properties that are different in extra dimensions.
Let us start with the escape velocity. Again we equalise the kinetic energy to the energy that is required to take an object off a surface to infinity. However the latter energy differs in several dimensions because the gravity in extra dimensions is different. The gravity in extra dimensions equals: $F_g = -\frac{G_N^{(D)} Mm}{r^{n+2}}$, in which $G_N^{(D)}$ is the gravitational constant in d dimensions.
Thus we come to the following equation:

$$-\int_{R}^{\infty} -\frac{G_N^D Mm}{r^{n+2}} dr = \frac{1}{2} m v_{esc}^2 \tag{72}$$

Working out the integral and arranging to v results in the escape velocity:

$$v_{esc} = \sqrt{\frac{2G_N^{(D)}M}{(n+1)R_H^{n+1}}} \tag{73}$$

Another equation for the escape velocity also has consequences for the Schwarzschild radius. That is why we will calculate this again. We equal the speed of light to (73):

$$R_H = \left(\frac{2G_N^{(D)}M}{(n+1)c^2}\right)^{\frac{1}{n+1}} \tag{74}$$

The Planck mass in four dimensions can be expressed in the Planck mass in d dimensions and the volume of the extra dimensions: $M_p^2 = (M_p^{(D)})^{n+2}V$. With this we can rewrite R_H as follows:

$$R_H = \left(\frac{2\frac{\hbar c}{(M_p^{(D)})^{(n+2)}V}^{(D)}M}{(n+1)c^2}\right)^{\frac{1}{n+1}} \tag{75}$$

The Hawking radiation also differs in extra dimensions. Hawking calculated an expression for the temperature which a black body would have if it radiates the same as a black hole with mass M does. To estimate this so-called Hawking temperature we use dimensional analysis. We obtain:

$$k_B T = \frac{(n+1)}{(4\pi)} \frac{\hbar c}{R_H} \tag{76}$$

or by filling in the Schwarzschild radius:

$$k_B T = C \frac{G_N^{(D)} M_{BH}}{\left(\frac{2G_N^{(D)}M_{BH}}{(n+1)c^2}^{\frac{1}{n+1}}\right)^{(n+2)}} \tag{77}$$

with C a constant.
We already know from the calculations in four dimensions that power equals the luminosity times the area of the sphere. In more dimensions: $luminosity = \sigma^{(D)} T_{(D)}^4$. We can use the area as computed in chapter one (14) for a $(d-1)$-dimensional sphere. However, we have now added the dimension of time to the spatial dimensions. Therefore we must replace $(d-1)$ with $(d-2)$. Furthermore, for further analysis it is useful to define an expression for the number of extra dimensions which could exist next to the four known to us. We therefore define that the total amount of dimensions equals the number of extra

dimensions and the four space-time dimensions, $d = 4 + n$. After integrating we obtain an equation for the mass. Leaving out the constants we get the following proportionality [6]:

$$\tau^{(D)} \sim \frac{1}{M_P^{(D)}} \frac{M}{M_P^{(D)}}^{\frac{n+3}{n+1}} \tag{78}$$

The entropy in extra dimensions can be calculated with the help of the first law of the thermodynamics, just as we did in four dimensions. From the formula for the temperature (76) we can get an equation for M with respect to T. After integrating with respect to S and M we obtain a formula for the entropy [6]:

$$S^{(D)} = S(\frac{R}{R_H})^{(\frac{n}{n+1})} \tag{79}$$

Now we have the Schwarzschild radius, the Hawking temperature, the lifetime and the entropy in extra dimensions. To compare these with the ones in four dimensions we need all the equations in terms of the Planck mass; for those equations, see [6]. Then we can obtain the proportionality with respect to the mass and the Planck mass for all the equations. Comparing those proportionalities for four dimensions and extra dimensions gives the following results [6].
For the Schwarzschild radius we see that:

$$R_H < R_H^{(D)} \tag{80}$$

This means that a black hole -when its not too big- will be larger in the world of extra dimensions than in the 'normal' world.
For the Hawking temperature we acquire:

$$T > T^{(D)} \tag{81}$$

The Hawking temperature is smaller in extra dimensions.
For the lifetime we achieve:

$$\tau < \tau^{(D)} \tag{82}$$

This means that a black hole will live longer in extra dimensions.
Finally, for the entropy we attain:

$$S < S^{(D)} \tag{83}$$

3.2 Black hole production in particle colliders

Motivated by string theory and in attempts to solve the hierarchy problem many ideas have been proposed that assume the existence of extra dimensions in the universe. One of these ideas, formed in 1998, no longer treated the extra dimensions as Planck-length-size dimensions. It was postulated that extra dimensions could be as large as 1 mm if the fields of matter are confined to the four dimensional surface of a brane and only gravity alters as a result of the extra dimensions [9]. In earlier performed measurements the gravitational interactions and forces were measured down to a certain length scale, which matched the upper limit of the size of the so-called “large extra dimensions”. If dimensions of these sizes would exist we would encounter a different gravitational behaviour on scales smaller than the size of the extra dimensions. In other words the gravitational force would then have a different dependence on r and, as the amount of extra dimensions increases, the fundamental Planck scale will be lowered down to the TeV scale, solving the hierarchy problem. These values that will soon be accessible in high energy colliders [6].

An important and interesting consequence of lowering the Planck scale is the production of TeV-mass black holes, a result of the modification of the gravitational forces on scales smaller than the radius of extra dimensions. This phenomenon might appear in the next generation of particle accelerators. As P. Kanti explains in [8] , when the impact parameter b of a collision is smaller than or equal to the Schwarzschild radius, that corresponds to the centre-of-mass energy $\sqrt{s}$ of the two particles, strong gravitational effects will rule, resulting in the formation of a black hole. The existence of extra dimensions is of great advantage for the production of black holes. Not only does it result in a lowered Planck scale, “it also allows the Schwarzschild radius to be significantly increased, thus making the condition $b < R_H$ distinctly easier to satisfy” [9].

Similar to the 4-dimensional case, black holes are formed when matter on the brane caves in under gravitational forces. If the Schwarzschild radius is much larger than the radius of the extra dimensions the black hole will act like a 4-dimensional object, obeying the classical description and forces. The influence of the extra dimensions is then negligible. In the case that the Schwarzschild radius is smaller than the radius of the extra dimension, the small black hole will be a higher dimensional object behaving in a drastically different manner than the mentioned case [6], [8], [section 3.1].

3.2.1 The cross-section

The cross-section of the production of black holes is given by the geometrical cross-section

$$\sigma \sim \pi b^2 \sim \pi R_H^2 \tag{84}$$

with b the impact parameter, which gives the distance between the nullplanes of the particles. See figure six.
According to Hossenfelder [5] the accuracy of the formula for the cross-section has been argued, but seems to be useful for energies up to about $10M_p^{(D)}$ with $M_p^{(D)}$ the new fundamental scale ~ 1 TeV . The produced black hole will have an angular momentum, unless

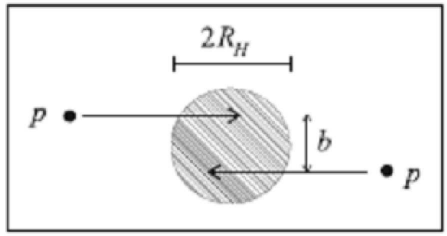

Figure 6: Impression of a particle collision of two particles p. When these particles have sufficient energies, they will be able to come within the $2R_H$ that matches their energy. The impact parameter b should be sufficiently small for a black hole to be formed [**?**].

the impact parameter is zero and the particles collide head on. Generally, an angular momentum will arise of

$$J = \frac{1}{2}Mb \tag{85}$$

Hossenfelder also mentions that when two point particles collide head on a black hole will always be formed, due to General Relativistic arguments. However, when the energies of the particles are small, the uncertainty relation causes the wave functions of the particles to be stretched out and the production of a black hole will be impossible.

The amount of energy and matter trapped within the event horizon of the black hole need to be known to calculate the mass of the formed black hole. The cross-section in high energy limits is governed by large impact parameters, $b \leq R_H$, and therefore Giddings and Thomas [7] conclude that the mass should be in the order of the centre-of-mass energy

$$< M > \leq \sqrt{s} \tag{86}$$

From this we can see that the cross-section increases with s to a power in which the number of dimensions contributes.

To achieve black hole production in particle colliders, energies must exceed the TeV scale, a condition that can only be met in hadron colliders. A proton consists of components called partons. To assess proton-proton collisions we must consider all possible combinations of partons within the protons that will produce enough energy to produce a black hole. The cross-section of a proton-proton collision forming a black hole will therefore contain so called parton distribution functions $f_i(x)$, which represent the chance of finding a parton with a fraction x of the total momentum of the proton. Thomas and Giddings [7] have deceived that the cross-section for such a collision then becomes

$$\sigma_{pp\rightarrow BH} \equiv \sum_{i,j} \int_{\tau_m}^{1} d\tau \int_{\tau}^{1} \frac{dx}{x} f_i(x) f_j(\frac{\tau}{x}) \sigma_{ij}(\tau s) \tag{87}$$

In this function $\tau = x_i x_j$ is the squared fraction of the parton-parton centre-of-mass energy, x is the parton momentum fraction, $\sqrt{s}$ is the centre-of-mass energy, $\sqrt{\tau_m s}$ is the minimum centre-of-mass energy for which the parton collision will produce a black hole. The mass of the black hole is assumed to be around $\sqrt{\tau s}$ [7], [8].

3.2.2 Black hole factories

The Large Hadron Collider (LHC) is now being developed and testing will begin there in September 2007. This collider, with a centre-of-mass energy of 14 TeV presents us with the first opportunity for the production of black holes if the Plack mass is indeed of TeV scale [5]. The number of black holes produced in the LHC N_{BH} in one year equals

$$N_{BH} = L\sigma_{pp\to BH} \tag{88}$$

with $L = 10^{33}cm^{-2}s^{-1}$ the estimated luminosity for the LHC [5]. With a centre-of-mass energy of 14 TeV this gives a N_{BH} of 10^9 per year, which results in a black hole production frequency of 1 Hz [5]. In this case the LHC would become a “black hole factory”. This process has encouraged many physicists to study the topic of black holes with TeV masses. However, as the authors in [11] explain, the production of black holes in next generation colliders like the LHC could be very unlikely for they claim that the minimal amount of energy required for the production of black holes will be significantly increased as a result of the generalized uncertainty relation.

3.2.3 Black hole production in cosmic rays

As described above, the consequences of the existence of extra dimensions on the Planck mass open up the opportunity for the production of black holes. This effect could not only be apparent in particle colliders but also in high energy collisions of cosmic ray particles with particles in the earth’s atmosphere. The energies of these Ultra High Energetic Cosmic Rays (UHECRs) have been measured up to 10^8 TeV, a value which seems extremely large. However, the particles in the cosmic rays collide with inert particles in our atmosphere. In general, particles in these cosmic rays have an energy of about 10^7 TeV, which can be converted to a centre-of-mass energy of $\sqrt{2Em} = 100$ TeV, an energy sufficient for the study of black holes with TeV masses [5], [8].

According to astrophysics only massive black holes exist, for just objects heavier than several sun masses are able to complete all stages of stellar evolution [9]. However fascinating these creations are, it seems that small black holes are far more interesting for modern physics, due to their relation to extra dimensions and their radically modified behaviour. Hawking radiation, for example, which was clarified in the previous section, is a process only perceivable from small black holes.

As we have seen, the addition of dimensions to the four dimensions known to man could result in a considerably lower Planck mass on small scales, opening up the possibility of producing TeV mass black holes in future particle colliders. By increasing the experimental energies in colliders, the Schwarzschild radius of the black holes will grow. As a result increasingly more processes in collider experiments will fall within the event horizon and therefore become invisible. The creation of black holes could consequently mean the end of particle physics [7]. On the other hand, by examining the possible production and evaporation of these collision produced black holes, which will be discussed in the next section, knowledge of the dimensionality of our universe can be gained, maybe even revealing the dimension's structures or adding to information about the quantum gravity effects apparent on small scales.

3.3 The decay of sub-millimeter black holes

In the previous section the production of black holes in particle colliders is discussed. This section will evaluate the decay of the produced black hole. It will also assess the signatures in particle colliders which inform us a black hole was formed. In the final part of this section we will look into methods by which the dimensionality of space can be calculated from the mass and temperature of the formed black hole.

3.3.1 Decay stages

When a black hole has been formed, it will start decaying immediately. The black hole will undergo four stages, in which the black hole will lose mass by emitting gravitational, gauge and Hawking radiation. Gravitational radiation is the emission of gravitons onto the brane as well as into the bulk. When the black hole emits gauge radiation, it emits force carrying particles onto the brane. When a particle one of a pair of virtual particles gets sucked into the black hole the other one seems to be emitted by the black hole onto the brane as well as into the bulk, this process is called Hawking radiation [13]. The energy of the emitted Hawking radiation is determined only by the temperature of the black hole.

The first stage is called the balding phase. In this phase the black hole looses hair. This means that by emitting classical gravitational (gravitons) and gauge radiation (force carrying particles) it looses its multipole moments, which are the coefficients of an expansion of the potential [7].

As mentioned in section 3.2.1, when two particles collide into each other to form a black hole, they will generally not crash head on. Therefore the black hole will get extra angular momentum and the radius will be asymmetric. The multipole moments correspond to this excess angular momentum and the asymmetric radius. The energy of the radiation is related to the oscillation frequency of the multipole moments. The Schwarzschild radius determines both the time the balding phase takes and the oscillation frequency of the multipole moments by $\tau_b >\sim r_h$ and $\omega \sim 1/r_h$ [7].

Giddings and Thomas suggest by calculation that gravitational radiation is more abundant than gauge radiation in the balding phase and in the large mass limit the gauge radiation can be neglected. During the balding phase the mass is decreased by 16% by emission of gravitational energy in the four dimensional case. This percentage should be an indication of the amount of mass lost in case of extra dimensions [7]. However, when the initial mass of the black hole is in the vicinity of the fundamental Planck mass, the above approximation of the decrease in mass might need quantum corrections. Also, a more substantial part of the radiation can then be gauge radiation.
Gauge charges in the black hole resulting from the initial particles will be shed by Schwinger emission during or just after the balding phase [7]. A spinning Kerr solution, which in general relativity gives the curvature of space-time around a spinning body with mass, like a black hole [21], now represents the black hole [7].

After the black hole has lost its hair and gauge charges it will start evaporating during two phases, emitting semi-classical Hawking radiation. The particles are radiated along the brane and into the bulk (gravitons only). However, the amount of particles emitted into the bulk is negligible compared to the amount emitted along the brane.

This approximation can be explained by noting that the wavelength of the radiation is given by $\lambda = 2\pi/T_H$. Consequently, the radius of the black hole is smaller than the wavelength of the radiation. The black hole can thus be seen as a point source which in majority radiates s- waves. Therefore the black hole will evenly decay into bulk particles and brane particles. Since the amount of brane particles is a lot higher than the amount of bulk particles, the majority of the decay products will be Standard Model particles which can be detected [10].

Other simplifications are made, namely it is assumed that the emitted particles are only Standard Model particles [7] and that there is no recoil [14]. According to Newton's third law, when the black hole emits a particle, the black hole will feel a force from that particle. This becomes important at the end of the evaporation when the black hole is light. However, the recoil effects are neglected. These approximations simplify the evaluation of the decay of the black hole.

The first phase of evaporation is the spin-down phase. In this phase the intrinsic angular momentum, spin, is lost by emitting Hawking radiation. This happens by emitting particles with spin ~ 1 and with an energy of $E \sim 1/r_h$ [7]. During the spin-down phase in four dimensions, about a quarter of the total mass is lost. When the black hole has gone through the balding phase and the spin-down phase it is round and smooth and characterised merely by its mass [13].

The Schwarzschild phase is the second evaporation phase, during which the remaining mass evaporates. This phase takes longer than the spin-down phase. The majority of the particles emitted during the Schwarzschild phase have energies $1/r_h$ of order of the Hawking temperature T_H. The entropy of the initial black hole is determined by the amount of particles emitted in all phases [7].

It is not yet known what will happen after the evaporation phase. When the black hole reaches the mass in n dimensions the so-called Planck phase will start. When this happens a semi-classical description is not sufficient and a theory of quantum gravity is indispensable. In this last phase the black hole is assumed to decay totally by radiating Planck or string scale particles, or the decay stops and a stable relic is left [5]. Most experts say that nothing can be said about the Planck phase until there is a theory of quantum gravity, but they assume that the black hole will evaporate completely. This being said, another possibility, namely that of a black hole relic, is assessed in more detail in the next subsection.

3.3.2 Planck phase

According to Hossenfelder [5] the difficulties in the Planck phase are linked to the black hole information paradox. When a particle in a pure quantum state is sucked into a black hole, the black hole's mass will increase. If we wait until the black hole has decreased to

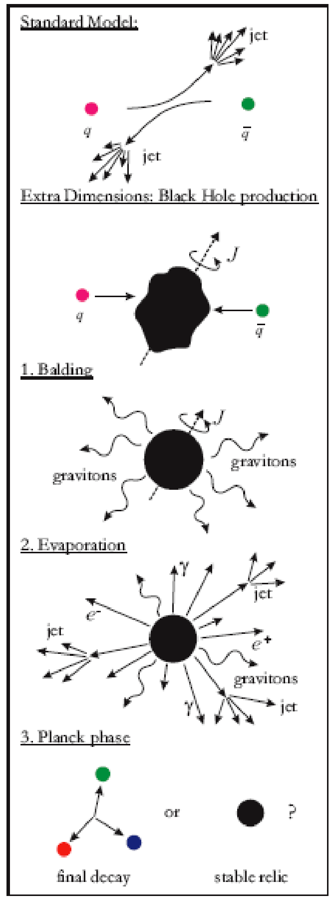

Figure 7: The different phases through which a black hole decays are depicted above. First the black hole is formed, after which the balding phase begins. The third picture portrays the evaporation phase, in which the spin is lost and the mass decreases. Finally, the black hole enters the Planck phase, which still remains a mystery.

its original mass by Hawking radiation, there will be a black hole of mass M and a mixed quantum state (thermal radiation) as opposed to the black hole of mass M and the pure quantum state we started with. A transformation from a pure state to a mixed state results in information loss. Thermal radiation has no properties except for temperature, whereas a pure quantum state has a lot more properties. Hence, there seems to be a loss of information, which according to Liouville's theorem is not possible. This is called the black hole information paradox [19].

Several solutions have been given, but all are yet to be proven. In the case of a sub-millimeter black hole a possible solution would be that the black hole does not decay completely but that a relic is left. Another possibility is that the information somehow escapes the black hole, but it is uncertain how this should happen. [5]

According to Hossenfelder [5], the diameter of the black hole should be a whole number of times half the wavelength ($1/2N$, with N an integer) of the emitted particles. As the atten-

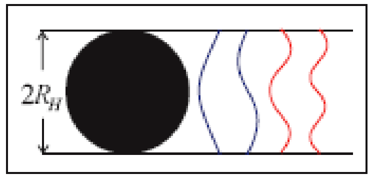

Figure 8: From Hossenfelder's article we learn that the wavelength of the radiation should precisely fit into the Schwarzschild diameter, according to $2R_H = 1/2N\lambda$.

tive reader will notice, this is contrary to what Dimopoulos and Landsberg [7] claim. The lowest energy state is reached when the diameter is half of the emitted particles' wavelength. The diameter decreases during decay, therefore the energy of the lowest energy state will increase. When the mass approaches the Planck mass, the lowest energy will be higher than the total energy of the system, and thus the particles cannot be emitted. The decay process will stop at this point and we are left with a black hole remnant. This remnant could be detected either as missing energy or if the relic is charged it could be detected by a detector [5].

However, most authors doubt the possibility of a relic as the final state of the black hole. Since there is no theory of quantum gravity, a lot can be possible and it is assumed that problems such as the information loss paradox can be solved once the quantum gravity theory is complete.

3.3.3 Decay particles

If black holes are formed in particle colliders, we can detect them by looking at the decay products. The production of a black hole is believed to give an explicit signal, due to the high amount of energy involved. We will detect a large quantity of emitted particles in comparison with Standard Model events and see a high sphericity due to the high number of radiated particles and because the black hole can be seen as a point radiator [7, 5]. When a black hole is produced other high energy events will get swallowed by the black hole, thereby giving a clear signal that a black hole is formed and reducing the background [5].

The average amount of particles emitted during the decay of the black hole, is the initial black hole mass divided by the energy of each emitted particle: $\langle N \rangle = \langle M_{BH}/E \rangle$. Dimopoulos and Landsberg [10] derived a formula for the average amount of radiated particles:

$$\langle N \rangle = \frac{2\sqrt{\pi}}{n+1}\left(\frac{M_{BH}}{M_P^{(D)}}\right)^{\frac{n+2}{n+1}}\left(\frac{8\Gamma(\frac{n+3}{2})}{n+2}\right)^{\frac{1}{n+1}} \tag{89}$$

This formula is consistent only when time evolution is neglected and when the initial mass of the black hole is far greater than the Hawking temperature. Time evolution can be ignored because most of the decay particles are emitted at the beginning when the initial mass and temperature have not significantly changed. The amount of radiated particles is shown as a function of $M_H/M_P^{(D)}$ in figure nine. [10] for different numbers of extra dimensions n.

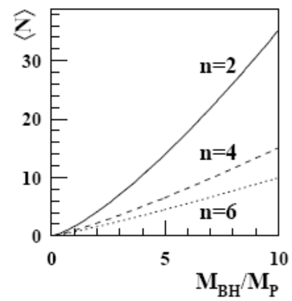

Figure 9: In this figure the number of emitted particles is depicted as a function of the mass of the black hole. The three curves show this number for two, four and six extra dimensions. This has been calculated with (89).

Approximating that the black hole will decay into Standard Model particles only, we can calculate the probability of a specific type of particle being emitted. There are about sixty Standard Model particles of which six are leptons, three are neutrinos and one is a photon. Thus the approximate percentage of emitted leptons is 10%, of neutrinos is 5% and of emitted photons is 2% [10]. The energy of one particle will be of order hundred GeV [7]. Most of the radiated particles will be emitted during the spin-down and Schwarzschild phase [7].

If new physics particles exist and they have a mass of about 100 GeV, i.e. a light Higgs particle, these could be radiated with a probability of 1% times the number of quantum degrees of freedom [12]. This method of finding new physics could be easier than by direct particle collision, because the probability of being produced is only slightly related to the particle's mass and the production rate could be higher [12].

According to Giddings and Thomas, at variance with what Dimopoulos and Landsberg say, about fifty particles will be emitted for a 10 TeV black hole in ten dimensions $(n = 6)$ in the spin-down and Schwarzschild phases. The energy of these particles will be along the scale of the Hawking temperature of the black hole at the start of the evaporation [7]. The percentages of radiated particles with different spin rely on the grey body factors. The percentages for emitted particles with spin 0, spin $\frac{1}{2}$, spin 1 and spin 2 are 42%:40%:16%:2% respectively in four dimensions. The number of hadrons emitted compared to the number of leptons and photons emitted is approximately 5:1 and 100:1, respectively [7].
Grey body factors cause objects to radiate differently from the Planck spectrum they would emit if they were totally black [20]. They also change the ratios of the emitted particle species. The grey body factors for more dimensions should be evaluated experimentally to acquire a more precise approximation of the above mentioned percentages in extra dimensions space [7]. Especially for energies in the low and moderate part of the emitted energy spectrum, which is where the majority of the particles are radiated, the grey body factors are significant and change the distinct black hole spectrum making it more difficult

to identify the event as black hole production [8].

Kanti [13] finds a remarkable fact by assessing the emission rates and grey body factors for all species of particles. The ratio of the different particles which are emitted, changes for various numbers of extra dimensions. This is caused by the dependence of the grey body factors and the emission rates on the number of extra dimensions and on the spin [13, 18]. For six extra dimensions, for instance, most of the particles emitted are force carrying particles. Kanti therefore concludes that the number of extra dimensions can be calculated by the amount and class of the emitted Hawking radiation [13].

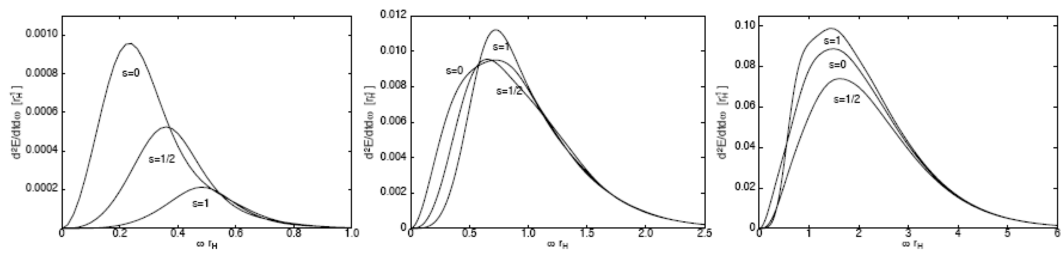

Figure 10: This image shows the different ratios of emitted particle species (spin 0, spin $1/2$ and spin 1 particles) for different numbers of extra dimensions. Left for $n = 0$, middle for $n = 2$ and right for $n = 6$ [13].

Giddings and Thomas assume that the black hole will evaporate completely in the Planck phase thereby emitting particles with energies of the order of the Planck or string scales. By detecting these particles we could acquire more knowledge of this last phase of black hole decay [7].

3.3.4 Calculating the dimensionality

Only about 5 % of the energy will be missing, because only 5 % of the decay products are neutrinos. Since there is such a small amount of decay particles, there are a lot of black holes produced that decay with no missing energy. It is easiest to work only with these events when the mass is calculated [10]. The mass can be determined by adding the energies of all the emitted particles [5, 14].

The initial temperature of the black hole is determined by fitting the Planck spectrum to the energy spectrum of the decay products. When quarks or gluons are emitted by the black hole, this will result in jets. It is predicted that this will occur a lot [10]. The temperature is best determined from electrons and photons in the final state (at the end phase of the jet) for they have very little background and even at high energies their resolution is still very high. By obtaining both the initial mass and temperature, it can be verified that the events detected are truly black hole decay products [10].

The signatures for black hole production in particle colliders will, in summary, be the following.
- Very high production rates, possibly as high as 1 every second and very large total cross sections (see section 3.2.2).
- The number of extra dimensions and their geometry characterizes the rate with which the parton cross sections grow with energy (see section 3.2.1).
- The amount of emitted particles is high.
- The hadron to lepton ratio of the emitted particles will be about 5:1.
- There might be black hole remnants or particles emitted in the Planck phase with energies as high as the order of the fundamental Planck scale.
- The black hole will radiate like a point radiator and will have high multiplicity thus the events are highly spherical.

When all of these signatures are taken into account as well as the above mentioned approximations of establishing the initial mass through events with no missing mass and looking only at electrons and photons for the temperatures, it is possible to search for black hole production with very little background [7].

The number of extra dimensions can be calculated once we know the black hole's mass and temperature. We have evaluated the formula we need to obtain this number in section (3.1.7).

$$k_B T = C \frac{G_N^{(D)} M_{BH}}{\left(\frac{2G_N^{(D)} M_{BH}}{(n+1)c^2}^{\frac{1}{n+1}}\right)^{n+2}} \tag{90}$$

The methods mentioned above to determine the initial mass and temperature are based on approximations. It is not known how much these simplifications will effect the determination of the mass and the temperature of the black hole. Many authors are positive and state that the approximations are valid [7, 10], others are more critical and are searching for other methods to determine the dimensionality [14, 18]. Theoretical incertitudes include the time evolution, grey body factors, the events during Planck phase, the approximations that all particles are Standard Model particles, the assumption that no particles are radiated into the bulk and that there is no recoil [5, 14]. Experimental doubts result from the background and the ambiguity when the number of decay products is small.

Harris et al. [14] do not believe that the negligence of time evolution is correct and that (90) will lead to a correct result. Therefore, they propose a different model for calculating the number of extra dimensions. In this model they calculate the temperature of the black hole for each point in time a particle was emitted [14]. A simulation was done using this model to evaluate the precision with which the dimensionality and the fundamental Planck scale can be established [5]. This resulted in a precision of 15% of the calculation of the fundamental Planck mass and an accuracy of $\pm$ 0.75 when calculating the dimensionality of space [14].

As discussed in the previous section, if the fundamental Planck scale is of the order of a TeV the LHC could be a black hole producer. This means that by this time next year the existence of extra dimensions could be a fact.

4 Conclusion

An important goal of physicists today is to find a theory which unifies all theories of nature. To do so we must first find an explanation why gravity is so much weaker than other forces. In attempts to solve this so-called hierarchy problem physicists have come up with numerous extravagant theories and ideas. Some of them include an addition of extra dimensions to our universe, a model with which the gravity scale is lowered to TeV scale. Extra dimensions could be compactified or localised, changing the physical behaviour of matter and energy on scales the size of the extra dimension.

As we have analysed in the beginning of part 2 black holes act differently when adding dimensions to the four dimensions we know. In a world with extra dimensions the Schwarzschild radius of black holes turns out to be larger than in four dimensions. The dimensionality also influences the lifetime, Hawking temperature and entropy of a black hole.

In the four dimensional space-time continuum the Planck scale is much higher than the scale of all other forces of nature. In this case the production of black holes in colliders is impossible, due to the extremely high energies needed for formation, which cannot be achieved in experiments on earth. Adding extra dimensions could lower the fundamental Planck scale, hopefully even down to the TeV scale, making gravity stronger on small scales. If this is the case black holes might be produced by centre-of-mass energies not much larger than this scale in particle accelerators. Therefore, the observability of black hole production at future colliders depends strongly on the dimensionality of space.

Once black holes are formed they will start radiating immediately. The evaporation begins with balding, after which the black hole loses its spin. During the Schwarzschild phase the black hole sheds its mass until its mass equals the Planck mass. What follows is a much discussed topic and remains a mystery today. By evaluating the emitted particles and the spectrum they make, the temperature and mass of the black hole can be determined. By using these characteristics we could assess the dimensionality of the universe. However, the precision of these calculations must be scrutinized for they contain many approximations and assumptions.

Experimentally, black holes have yet to be formed. Next year the first particle collider will be launched with an energy scale sufficiently high to reach the fundamental Planck scale in more dimensions. Physicists will be on the look out to detect black holes signatures, proving their presence and consequently the existence of extra dimensions. Whether these black holes will truly be formed, only the (near) future can tell.

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