Constraints on Large Extra Dimensions

Stefan Kowalczyk, Quinten Krijger, Maarten van de Meent,
Jorn Mossel, Gerben Schooneveldt and Bart Verouden

Institute for Theoretical Physics
University of Amsterdam, The Netherlands

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Abstract

Recently, new theories have been developed to solve the hierarchy problem (*i.e.* why gravity is so much weaker than electromagnetism). These theories use a higher dimensional spacetime. We review the theoretical framework of these scenarios, their implications in short-range gravity and collider experiments, and finally their cosmological and astrophysical effects. We also place constraints on large extra dimensions by using presented experimental data.

Contents

1 Introduction

In our world we are used to specify events by telling where and when they occur in our four dimensional spacetime. This seems beyond questioning. In 1919, however, a Polish mathematician named Theodor Kaluza challenged the obvious - he suggested that the universe might *not* actually have three spatial dimensions; it might have *more*. This may sound bizarre. In reality it is very plausible. To see this, we can study a clothesline between two trees. When looking from a far distance, you will easily see the long horizontal extent of the line, but the thickness will be difficult to discern. Now imagine that an ant would live on it. Would the poor animal then have just one dimension in which to walk in? No! In reality the clothesline *does* have thickness. By using binoculars you could zoom in on the line and see it. The line has two different dimensions. One direction is long, extended, and easily visible. The other is 'curled up' and harder to see. This idea was made explicit and refined by the Swedish mathematician Oskar Klein in 1926. He realized that the spatial fabric of our universe may have both extended and curled-up dimensions. These curled-up dimensions are called compactified.

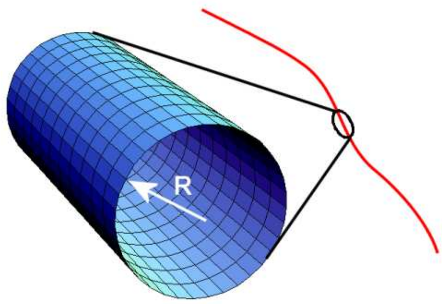

Figure 1: A curled-up dimension: from a distance we see a one-dimensional line. If we zoom in on it we see a two-dimensional area.

If such extra dimensions exist, it seems clear that ordinary light and matter cannot travel in these extra directions. Otherwise we ourselves would be able to move in such a direction, which we know we cannot, or we would be able to observe light entering or leaving our 3-dimensional space, resulting in an apparent energy conservation violation. This gives a significant constraint on the size of such extra dimensions, which would have to be smaller than the shortest wavelength photon observed in experiments (10^{-18} m). Such photons would otherwise be able to fit in the extra dimensions and disappear from our ordinary world, taking their energy with them.

In the more traditional theories it was thought that the compactification of the extra dimensions would be on the scale of the Planck length (10^{-35} m). This would make it all but impossible to ever find direct empirical evidence of their existence. This changed when in 1998 Arkani-Hamed, Dimopoulos, and Dvali (ADD) [1] [2] [3] postulated a scenario, which uses the string inspired brane world hypothesis. In this model the world that we see is a 3-dimensional brane (the higher dimensional equivalent of a membrane in 3-dimensional space) in a higher dimensional bulk. Light and matter are confined to the brane, whilst only gravity is allowed to propagate in the bulk.

The motivation for this comes from string theory, where all particles are described as vibrational modes of one-dimensional objects known as strings. These strings can be either ‘open’ (much like a string on a guitar) or ‘closed’ (like a rubber band). It turns out that gravitons are solutions of closed strings, while photons and such are all solutions for open strings, which in this scenario would have their ends attached to the 3-brane (see figure 2). Tension in the open strings keeps the photons caught on the brane, while the closed string gravitons are free to roam the entire bulk.

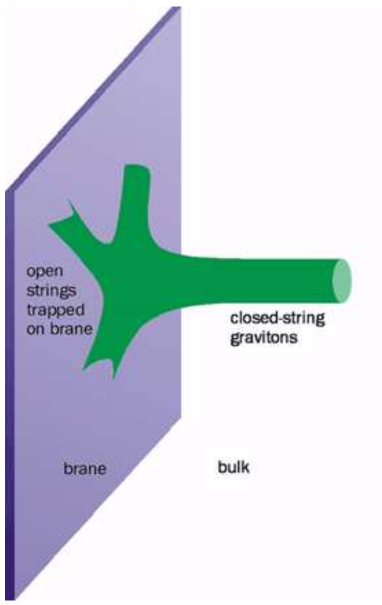

Figure 2: The string theory view of a body trapped on the brane. Matter corresponds to open strings, the ends of which are stuck on the brane. Graviton emission can occur when open strings collide and form closed strings that are free to go into the bulk. This picture was taken from http://physicsweb.org/

This scenario in fact allows for much larger extra dimensions than the Planck length, since gravity, the only force affected, has only been tested down to scales of a millimeter or so. This has aroused the attention of experimental physicists, because now it might be possible to find proof of such extra dimensions by looking for deviations in the behavior of gravity at short distances or energy conservation violations caused by gravitons disappearing into the bulk.

In this paper we will look at the theoretical implications of the existence of flat large extra dimensions, and how these can be used to obtain constraints on the size and number of extra dimensions from experimental observations. To keep things simple we assume that all extra dimensions are of the same size. Of course there are also some other theories (the most important of them was proposed by Randall and Sundrum (RS-type) and uses strongly warped extra dimensions), but these will not be considered in this paper. A nice book [4] has been written by Brian Greene (an expert in string theory); although mainly about string theory, it also contains relevant information about extra dimensions.

2 Theory

2.1 Gravitation in Extra Dimensions

Since in our model gravity is the only force which is able to propagate in the extra dimensions, it is important to know how gravity behaves in higher dimensional spaces.

Infinite extra dimensions

Before we can study how gravity behaves in compactified dimensions, we must investigate how gravity would behave in an *infinite* d-dimensional space. In our familiar 4-dimensional space-time gravity obeys Newton's law:

$$F = G_4 \frac{m_1 m_2}{r^2}. \tag{1}$$

This law has the property that any integral over a closed surface of the component of the gravitational field (F/m) normal to the surface, is always equal to the enclosed mass times some constant $(4\pi G_4)$. If we demand that gravity has this property in any space, then we find (observing the fact that the gravitational field around a point mass must be isotropic) that the gravitational force will be given by the following equation:

$$F_d = 4\pi G_4 \frac{m_1 m_2}{Vol[S_r^{d-2}]}, \tag{2}$$

where $Vol[S_r^n]$ is the volume of a n-dimensional sphere with radius r. We know that $Vol[S_r^n] = r^n Vol[S^n]$. Therefore we can rewrite equation 2 as follows:

$$F_d = G_d \frac{m_1 m_2}{r^{d-2}}, \tag{3}$$

where G_d is some new constant equal to $\frac{4\pi G_4}{Vol[S^{d-2}]}$. The corresponding potential function is given by:

$$\begin{cases} \Phi(r) = -\frac{G_d}{(d-3)} \frac{M}{r^{d-3}} & \text{for } d \neq 3 \\ \Phi(r) = G_d M \ln r & \text{for } d = 3 \end{cases}$$

This shows that there cannot be any extra dimensions, which are infinite, since we know that, at least at macroscopic scales, gravity obeys an inverse square law. But to illustrate the major impact of this result, we will investigate how this affects planetary orbits.

Consider a planet orbiting a star. An orbit always is limited to a single two-dimensional plane. Therefore we can always describe the planet's movement by only two variables. For these we will take the distance r between the two centers of masses and the angle ϕ.

In any such system there are two conserved quantities: the energy E and the angular momentum L:

$$E = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\dot{\phi}^2 + m\Phi(r), \tag{4}$$

and

$$L = mr^2\dot{\phi}. \tag{5}$$

We can eliminate $\dot{\phi}^2$ by using equation 5:

$$E = \frac{1}{2}m\dot{r}^2 + \big(\frac{1}{2}\frac{L^2}{mr^2} + m\Phi(r)\big). \tag{6}$$

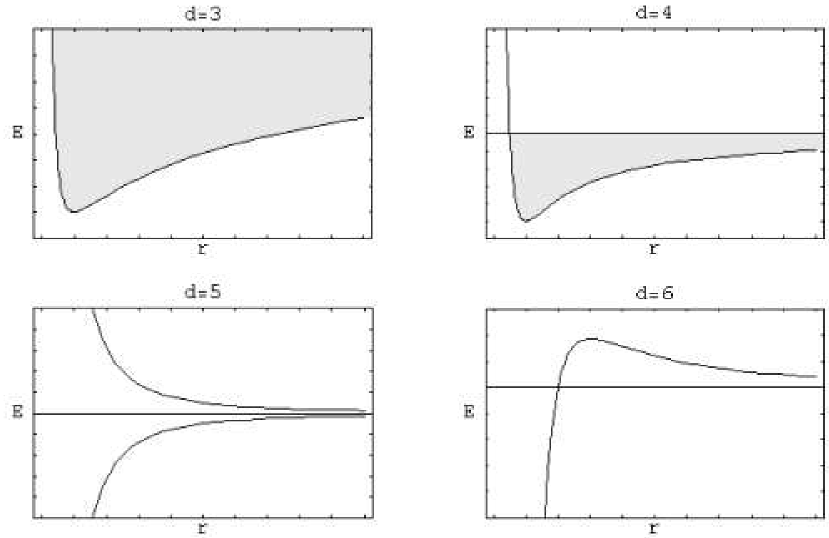

Figure 3: The potential functions for an orbiting planet in d dimensions, plotted for $d = 3$, $d = 4$, $d = 5$ and $d = 6$. The gray parts indicate stable orbits. For $d = 5$ and $d = 6$ there are no truly stable orbits.

This can be interpreted as the energy equation for a one-dimensional particle with mass m in a potential $\frac{1}{2}\frac{L^2}{mr^2} + m\Phi(r)$. This potential has been plotted for cases $d = 3$, $d = 4$, $d = 5$ and $d = 6$ (figure 3).

As one can see the potential has a minimum in the first two cases, indicating the existence of a stable circular orbit. In the last case the potential shows only a maximum. This tells us that the possibility of a circular orbit exists, but that the tiniest disturbance will cause the planet to either fall onto the sun, or be flung out of the solar system. For d larger than six the plot will look similar to the one for $d = 6$. The plot for $d = 5$ shows no stable positions either. In this case the potential function takes the following form:

$$\frac{\frac{1}{2m}L^2 - \frac{1}{2}G_d Mm}{r^2}.$$

Whether a system uses the upper or the lower line depends on the sign of this expression. In the first case the planet will ultimately end up in the sun and in second the planet will disappear from the solar system. Circular orbits are possible, but only when the above expression is equal to zero and even then they would not be stable. This shows us that in a higher dimensional space our solar system would *not* be able to exist!

Compactified Extra Dimensions

Now that we know how gravity behaves in infinite spaces, we can investigate how it behaves if some of the dimensions are compactified. The best way to tackle this problem is what is known as the *method of images*. What one basically does is to unroll the curled-up dimension, so that you have an infinite space that repeats itself with a period of $2\pi R$ (where R is the compactification radius), see figure 4. To find an expression for the force that one particle 'feels' from a second particle, we just calculate the force it feels from all the 'images' of the second particle using equation 3 and add those together.

Consider two massive particles with masses m_1 and m_2 in a space with the ordinary three extended spatial dimensions and n extra compactified dimensions, all with radius R.

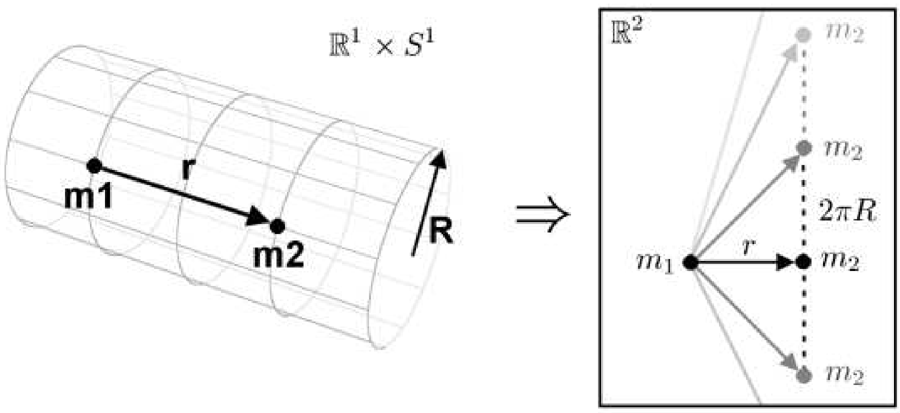

Figure 4: Using the method of images makes it easier to calculate the force between two masses. The idea is to unroll the curled-up dimension to get a flat geometry, which we are familiar with. Since $\mathbb{R}^3 \times S^1$ is a bit difficult to draw, the figure shows $\mathbb{R}^1 \times S^1$.

The expression for the force between the two particles now becomes (by using the method of images):

$$F = \sum_{i_1 \in \mathbb{Z}} \cdots \sum_{i_n \in \mathbb{Z}} G_{4+n} \frac{m_1 m_2}{(r^2 + \sum_{j=1}^{n} (2\pi R i_j)^2)^{\frac{n+2}{2}}} \frac{r}{\sqrt{r^2 + \sum_{j=1}^{n} (2\pi R i_j)^2}}. \quad (7)$$

Here r is the ordinary distance in our 3-dimensional space. The last factor is to compensate for the fact that not all the images work in the same direction. From the symmetry of the situation you can tell that in the end only a net force in the direction of $\vec{r}$ will remain. Each contribution to the total force is multiplied by $\frac{r}{\sqrt{r^2+\sum_{j=1}^{n}(2\pi R i_j)^2}}$, which is the fraction of the force working in the direction of $\vec{r}$.

In the case that $r << R$ the term with $i_j = 0$ is for all j much larger than all the other terms. We therefore regain equation 3.

$$F = G_{4+n}\frac{m_1 m_2}{r^{n+2}}, \qquad \text{for } r << R.$$

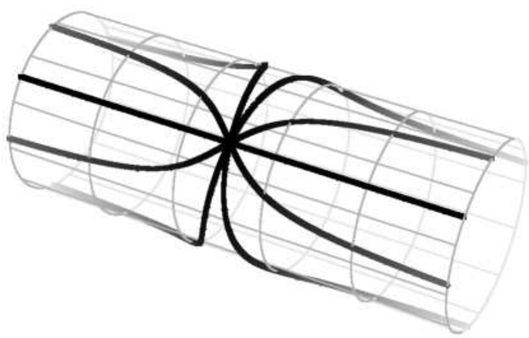

Figure 5: With this picture one can visualize how gravity behaves in $\mathbb{R}^1 \times S^1$. A gravitational field is shown on a cylinder. At longer distances the field becomes parallel and reduces back to the case of one-dimensional gravity.

On the other hand, for $r >> R$ the terms become so closely packed, that we may exchange the sums for integrals. What we get is

$$F = G_{4+n} m_1 m_2 \int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} \frac{r}{(r^2 + \sum_{j=1}^{n}(2\pi R i_j)^2)^{\frac{n+3}{2}}} di_1 \cdots di_n$$

$$= G_{4+n} m_1 m_2 (\frac{r}{2\pi R})^n \frac{r}{r^{n+3}} \int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} \frac{1}{(1 + \sum_{j=1}^{n} x_j^2)^{\frac{n+3}{2}}} dx_1 \cdots dx_n.$$

We note that the multiple integral is just a constant for each n.

$$F = G_{4+n}\frac{K_n}{R^n}\frac{m_1 m_2}{r^2}, \tag{8}$$

where

$$K_n = \frac{1}{(2\pi)^n} \int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} \frac{1}{(1 + \sum_{j=1}^{n} x_j^2)^{\frac{n+3}{2}}} dx_1 \cdots dx_n = \frac{1}{2^{n+1}\Gamma(\frac{n+3}{2})\pi^{\frac{n-1}{2}}}.$$

In the limit that the dimensions are tiny, gravity again obeys an inverse square law. This in fact allows us to express the gravitational constant G_4, in terms of the gravitational constant in the bulk G_{4+n} and the radius of the compactified dimensions R,

$$G_4 = G_{4+n} \frac{K_n}{R^n}. \tag{9}$$

A logical question to ask is: how much will gravity differ from the inverse square law found in equation 8 if r approaches R? This is an important question to ask if you want to verify such a theory experimentally.

Let us try to answer this question for the case $n = 1$. In this case a computer algebra program like *Mathematica* is able to calculate both the sum of equation 7 and the inverse square law found in equation 8. Therefore we can get the relative difference. The result has been plotted in figure 6. It looks like an exponential function, in section 3 we will see it is one indeed. From the plot one can read that the force is approximately twice as strong as the inverse square law predicts, at a distance of about two times R.

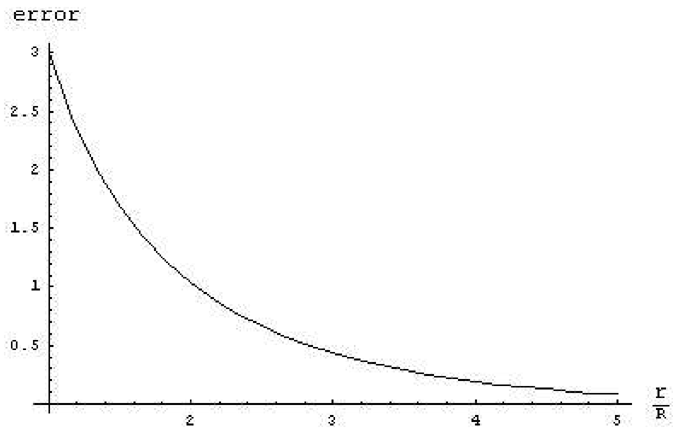

Figure 6: Relative deviation from the inverse square law. It looks like an exponential function. This is important for testing gravity at short distances.

2.2 Kaluza-Klein Reduction

Consider a particle moving around on a circle with radius R. Then its wave function will be a periodic function with period $2\pi R$, which implies that the function can be written as

$$\Psi(x) = \sum_{n\in\mathbb{Z}} c_n e^{i\frac{nx}{R}}.$$

This is guaranteed by Fourier analysis. If we let the momentum operator $\frac{\hbar}{i}\frac{\partial}{\partial x}$ operate on this function, we find the following expression for the momentum:

$$\frac{\hbar}{i}\frac{\partial}{\partial x}\Psi(x) = \sum_{n\in\mathbb{Z}} c_n \frac{\hbar}{i}\frac{\partial}{\partial x} e^{i\frac{nx}{R}} = \sum_{n\in\mathbb{Z}} c_n \frac{\hbar\, n}{R} e^{i\frac{nx}{R}}. \tag{10}$$

We note that our function apparently was an eigenfunction of the momentum operator. We conclude that $p = \frac{n\,\hbar}{R}$, $n \in \mathbb{Z}$, which means that momentum is quantized, and that the distance between two momentum levels is inversely proportional to the radius R. This is also the way momentum would behave in a single curled-up dimension.

Now we will have a look at a combination of the normal 3+1-dimensional world with an extra, curled-up dimension (which we will label with a 5). We will start by considering a massless, relativistic particle moving in these five dimensions, and then explore the consequences it has for the four dimensional world.

This particle has momentum $p = (E/c, p_1, p_2, p_3, p_5)$ and it has to obey the relativistic equation of motion

$$||p||^2 = -E^2/c^2 + p_1^2 + p_2^2 + p_3^2 + p_5^2 = 0. \tag{11}$$

However, from 4-dimensional special relativity theory, we learn that, for a massive particle, the following equation must hold:

$$E^2 - \vec{p}^2 = m^2 c^4,$$

where $\vec{p}$ denotes the 3-dimensional momentum vector. If we now combine this equation with equation 11 and reinterpret the result, we can conclude that

$$p_5^2\, c^2 = m^2 c^4 \iff m = \frac{|p_5|}{c} = \frac{n\,\hbar}{R\,c},\ n \in \mathbb{N}\cup\{0\}. \tag{12}$$

We started with a massless 5D particle, and we got a whole (infinitely large) collection of 4D particle with some (quantized) mass in return. As we shall see, this is not the only thing we might infer from a fifth dimension.

Next, we consider the gravitational force between two particles in our 5D world. From the previous section, we recall the 5D law of gravity (equation 8):

$$F_g(r) = -\frac{G_5}{8\,R}\,\frac{m\,m'}{r^2}.$$

If we now suppose our particles to have zero velocity, we can write $m\,m' = -p \cdot p'/c^2$ and tidy up the formula a bit by writing $G = G_5/8\,R\,c^2$, in order to obtain a formula that we suppose to be relativistically invariant:

$$F_g(r) = G\,\frac{p \cdot p'}{r^2}.$$

The reason we suppose this formula is relativistically invariant, is that we assume that also in more than 4 dimensions, momentum is Lorentz invariant. Proceeding from this point, we will again calculate the gravitational force, but this time our particles will have momentum in the fifth dimension. Using $p \cdot p' = -m\,m'\,c^2 + p_5\,p_5'$ and $p_5 = \frac{n\hbar}{R}$ we obtain

$$F_g(r) = G\,\big(-\frac{m\,m'}{r^2}c^2 + \frac{n\,n'}{r^2}\,\frac{\hbar^2}{R^2}\big) = -\frac{G_5}{8\,R}\,\frac{m\,m'}{r^2} + \frac{G_5\,\hbar^2}{8\,c^2\,R^3}\,\frac{n\,n'}{r^2}. \tag{13}$$

If we look at this expression with a little more care, we discover something rather amazing: we might, in fact, consider the 5D gravitational force as the sum of *two* forces, one of which is always attractive (namely the first term with the masses), and another one, which can be either attractive or repulsive, depending on the signs of the momenta in the fifth direction. In our 4D world, we know only two fundamental forces that fall off by $1/r^2$, one of which is always attractive and one of which can be either attractive or repulsive, namely the force of gravity and the electromagnetic (Coulomb) force. So we will try to identify the terms in equation 13 with these two forces.

However, there appears to be a small problem. If one wants the parameters to be in agreement with the well known experimental facts and relative strengths of these forces, one would have to adjust the G and R in the formula in a slightly inconvenient way. Let us calculate them. First of all, we would like the $G/8\,R$ to be equal to G_4. On the other hand, in order to make the second term correspond to the Coulomb force, we need $\frac{G_5\,\hbar^2}{8\,c^2\,R^3} = \frac{e^2}{4\,\pi\,\varepsilon_0}$, where e is the elementary charge unit. By demanding so, we find

$$R = \sqrt{\frac{4\,\pi\,\varepsilon_0\,G_4\,\hbar^2}{e^2\,c^2}} = 1.9 \times 10^{-34}\,\mathrm{m}, \tag{14}$$

which is rather small; in fact, it is about the Planck length. That would not be a problem per se, but it is one, if one is looking for *large* extra dimensions, as we are: small extra dimensions cannot solve the hierarchy problem.

2.3 The Hierarchy Problem

It looks like there are (at least) two different fundamental energy scales in nature. The electroweak scale $m_{EW} \sim 1$ TeV and the Planck scale $M_{Planck} = G_N^{-1/2} \sim 10^{16}$ TeV (setting $c = \hbar = 1$). This is a very big difference. Why is gravity so much weaker than electromagnetism?

This is called the hierarchy problem. In 1998 the ADD-scenario was proposed to solve this problem. In fact it does not solve it completely, because a new fundamental scale (namely R) is introduced. It gives just a different perspective. Their idea was as follows. Take the electroweak scale as the only fundamental (short distance) scale. The observed weakness of gravity on distances > 1 mm is due to the existence of $n \geq 2$ new compact spatial dimensions. The Planck scale is not a fundamental scale. That this scale is so big is just a consequence of the large size of the extra dimensions. Only gravitons can freely propagate in the new dimensions.

In this ADD-scenario the claim is that gravity is so weak, because it spreads its power over more than 4 dimensions.

Fundamental Constants in 4+n Dimensions

Three fundamental constants that are important, are the constant of gravity G_4, Planck's constant $\hbar$ and the speed of light c. From Newton's law we find in 4 dimensions for the unit of G_4, that $[G_4] = \frac{\mathrm{m}^3}{\mathrm{kg\,s}^2}$. Form the uncertainty principle we get $[\hbar] = \frac{\mathrm{m}^2\,\mathrm{kg}}{\mathrm{s}}$ and of course $[c] = \frac{\mathrm{m}}{\mathrm{s}}$. A combination with unit length, as well as with unit mass can be made from these constants. We call them Planck's length and Planck's mass. First we will try to find out the Planck length: $[L_p] = \mathrm{m}$. We do not want kilograms, so we try $G\hbar$. That gives $[G\hbar] = \frac{\mathrm{m}^5}{\mathrm{s}^3}$. Dividing by c^3 and taking a root gives the following result for the Planck length in 4 dimensions:

$$L_{Planck} \equiv L_4 = \sqrt{\frac{G_4\,\hbar}{c^3}} \approx 10^{-35} \text{ m}. \tag{15}$$

What about the Planck mass? By following a similar strategy we get without much difficulty

$$M_{Planck} \equiv M_4 = \sqrt{\frac{\hbar\,c}{G_4}} \approx 10^{-8} \text{ kg}. \tag{16}$$

Consider now 4+n dimensions. The constant of gravitation has not only a different numerical value, but also a different unit. From Newton's law in 4+n dimensions (see equation 3) we find

$$[G_{4+n}] = \frac{\mathrm{m}^{n+3}}{\mathrm{kg\,s}^2}.$$

Again, we want to construct the fundamental length L_{4+n} and the fundamental mass M_{4+n}.

The rules of the game are as follows. We must find α, β and γ such that the unit of $G_{4+n}^{\alpha}\,\hbar^{\beta}\,c^{\gamma}$ equals meter or kilogram. We are going to work out the mass. Writing this equation out will give us three equations (one for kg, one for m and one for s) with three unknowns (α, β and γ):

$$\left(\frac{\mathrm{m}^{n+3}}{\mathrm{kg\,s}^2}\right)^{\alpha} \left(\frac{\mathrm{kg\,m}^2}{\mathrm{s}}\right)^{\beta} \left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{\gamma} = \mathrm{kg}^1.$$

We just have to solve the following system:

$$\begin{cases} \alpha(n+3)+2\beta+\gamma=0 \\ -\alpha+\beta=1 \\ -2\alpha-\beta-\gamma=0 \end{cases}$$

The only solution is

$$\alpha=\frac{-1}{n+2}, \quad \beta=\frac{n+1}{n+2}, \quad \gamma=\frac{1-n}{n+2}.$$

In this way we find for the Planck mass in 4+n dimensions the following:

$$M_{4+n}=G_{4+n}^{\frac{-1}{n+2}}\,\hbar^{\frac{1+n}{n+2}}\,c^{\frac{1-n}{n+2}} = \sqrt[n+2]{\frac{h^{1+n}}{c^{n-1}\,G_{4+n}}}. \tag{17}$$

For the Planck length in 4+n dimensions, we have to solve

$$\begin{cases} \alpha(n+3)+2\beta+\gamma=1 \\ -\alpha+\beta=0 \\ -2\alpha-\beta-\gamma=0 \end{cases}$$

By solving this system we get

$$L_{4+n}=G_{4+n}^{\frac{1}{n+2}}\,\hbar^{\frac{1}{n+2}}\,c^{\frac{-3}{n+2}} = \sqrt[n+2]{\frac{G_{4+n}\,\hbar}{c^3}}. \tag{18}$$

It would be nice to find a formula which relates the Planck mass in 4 dimensions (M_4) to the Planck mass in 4+n dimensions (M_{4+n}). We have already found a relation between G_4 and G_{4+n} in equation 9. By using this we can rewrite our formula for M_4:

$$M_4=\frac{\hbar}{c}\sqrt{\frac{c^3}{\hbar\,G_4}}=\frac{\hbar}{c}\sqrt{\frac{c^3\,R^n}{\hbar\,K_n\,G_{4+n}}}.$$

On the other hand, from our formula for M_{4+n} we can eliminate G_{4+n}, and plug it into this equation. The result looks complicated:

$$M_4=\frac{\hbar}{c}\sqrt{\frac{c^3\,R^n\,M_{4+n}^{n+2}}{\hbar\,K_n\,c^{1-n}\,\hbar^{n+1}}}.$$

When we tidy this up we find:

$$M_4^2=\frac{c^n}{\hbar^n}\,2^{n+1}\Gamma(\frac{n+3}{2})\pi^{\frac{n-1}{2}}\,R^n\,M_{4+n}^{n+2}. \tag{19}$$

This equation can be used to analyze graviton emission into extra dimensions.

Graviton Emission into Extra Dimensions

In the ADD-scenario the claim is that the electroweak force is more fundamental than gravity. There is an important difference between these forces. Electroweak forces have been probed at distances of order m_{EW}^{-1}. On the other hand gravitational forces have not been probed at distances of order M_{Pl}^{-1}. Gravity has yet not been measured on sub-millimeter scales.

The idea from the ADD-scenario to take $M_{4+n} \sim m_{EW}$. By demanding that R be chosen to reproduce the observed M_4 we can find numerical values for R, for different values of n, using formula 19,

$$M_4^2 \sim R^n M_{4+n}^{n+2}. \tag{20}$$

We take $M_{4+n} = m_{EW} \approx 1$ TeV and express R in terms of TeV^{-1}. We found already $M_4 \approx 10^{16}$ TeV and therefore R is given by

$$R \sim 10^{\frac{32}{n}} \text{ TeV}^{-1}.$$

We use 10^{-16} TeV^{-1} $\leftrightarrow$ 10^{16} TeV $\leftrightarrow$ 10^{-35} m to write R in meters:

$$R \sim 10^{\frac{32}{n}-19} \text{ m}. \tag{21}$$

Now we can give estimates of R for different n:

n	1	2	3	4	5	6
R (in m)	10^{13}	10^{-3}	10^{-8}	10^{-11}	10^{-13}	10^{-14}

The case of $n = 1$ is empirically excluded, since it would imply deviations from Newtonian gravity over solar system distances. For all $n \geq 2$, however, nothing yet can be concluded, because there are no experimental results about gravity on sub-millimeter scales. In contrast to gravity, the other forces from the Standard Model have been accurately measured at very small distances.

We will now take a closer look at a collision process, in which gravitons can be produced and emitted into the extra dimensions. Let E be the total energy in the center of mass system. We will show that the total number of Kaluza-Klein graviton-excitations is of order $(ER)^n$.

As we found earlier their momentum is quantized: $p = \frac{\hbar}{R}k$. By using $E^2 = p^2c^2$ (and setting again $c = \hbar = 1$) we see that the total energy squared is of the form

$$E_{total}^2 \sim \frac{k_1^2}{R^2} + \frac{k_2^2}{R^2} + \ldots + \frac{k_n^2}{R^2} \tag{22}$$

Conservation of momentum in each compactified extra dimension, gives that after a collision we can expect two gravitons in the extra dimensions. The momentum in the n dimensions was zero before the collision, so it must be zero after the collision (but we are just making an approximation so we do not need this factor two). The energy of the produced Kaluza-Klein gravitons must be smaller than the total energy. Therefore equation 22 becomes:

$$ER \gtrsim \sqrt{k_1^2 + k_2^2 + \ldots + k_n^2} \tag{23}$$

The righthand side scales as a (hyper-)volume and therefore we find that the number of graviton-excitation that could be produced in a collision is approximated by $(ER)^n$.

$$N(graviton\, excitations) \sim (ER)^n \tag{24}$$

This can be a relatively large number. Knowing the number of possibilities is not enough for a complete analysis. We must also know the probability that a single graviton will be produced to say something sensible about the process. In particle physics we usually do not speak about probabilities, but about cross sections. Can we estimate the cross section σ for the production of a single KK-particle? A cross section is an area and since the Planck length is the only fundamental quantity with dimension length it looks that our cross section is of order L^2_{Planck} (this kind of argumentation is strange, but works surprisingly often in physics). By looking back at the formulas that we found we see that

$$\sigma(single\, graviton) \sim L^2_{Planck} = G_4 = \frac{1}{M_4^2}. \tag{25}$$

Now we can give a rough estimate of the total cross section:

$$\sigma(total) \sim \frac{1}{M_4^2}(ER)^n. \tag{26}$$

We rewrite this by using the relation between the Planck mass in 4 dimensions and the Planck mass in 4+n dimensions (equation 20):

$$\sigma(total) \sim \frac{1}{E^2}(\frac{E}{M_{4+n}})^n. \tag{27}$$

This shows that the probability of creating a Kaluza-Klein graviton is independent of the radii of the extra dimensions! It depends only on the total energy in the center of mass system and the fundamental mass in 4+n dimensions.

3 Constraints from Short-Range Tests of Gravity

Although gravity was the first of the fundamental forces of nature to be captured in a mathematical model, the inverse square law is the most inaccurately tested one. Until quite recently the inverse square law had only been tested on distances larger than 1 cm. This is mainly due to the weakness of gravity, compared to the other fundamental forces. At short ranges electromagnetic forces and acoustic vibration outweigh gravity by many orders of magnitude.

Recently, there has been a renewed interest in tests of gravity on short-range scales due to developments in theoretical particle physics. Many recent models trying to unify gravity and the other fundamental forces have predicted effects on gravity in the sub-millimeter range. One of those effects is the deviation from the inverse square law, which we discussed earlier in section 2.1.

In these experiments, the deviation from the inverse square law is modeled as a Yukawa interaction. The potential energy due to gravity and an additional Yukawa force is given by:

$$V(r) = -G_4 \frac{m_1 m_2}{r} [1 + \alpha \exp(-r/\lambda)], \tag{28}$$

where α is the strength of the new interaction relative to gravity and λ is the range of the interaction. The Yukawa potential is usually used to describe interactions mediated by massive particles (like the strong nuclear force). In fact deviation from the inverse square law found in section 2.1 may be seen as such an interaction. As shown in section 2.2 gravitons traveling into the bulk manifest themselves as particles with mass in our three dimensional world. Naturally the gravitons with the lowest mass - and thus the longest range interaction - will be the first to be encountered when probing gravity at shorter ranges. This gives us a range λ of R, the radius of the extra-dimensions and a relative strength α of $2n$. (The graviton with the lowest mass is one traveling into the bulk with only one quantized unit of momentum. For every extra dimension this momentum can be in two directions, giving us a total of $2n$ particles and an equal number of extra Yukawa interactions).

To date, none of the experimental groups have reported any results showing any significant deviations from the inverse square law. In the absence of any such deviations results are reported as what part of the $\alpha - \lambda$ parameter space their experiment has eliminated. Thus reports are usually shown in plots like the one in figure 9.

3.1 Problems in Testing Gravity at Very Short Distances

The most apparent problem is the weakness of gravity as a force. Moreover, it rapidly decreases as the experimental setups become smaller, because as one always needs to use test masses, an experiment has a minimum range at which it can measure gravity. If you want to measure gravity at a shorter range, you will have to downscale your experiment. This causes a significant decrease in the maximum measurable force. Since, when you scale down by a factor A, both test masses will scale down by a factor A^3 reducing the gravitational force between them by a factor A^6. This is hardly compensated by the fact that the minimum range is also reduced by a factor A increasing the maximum measurable force by a factor A^2, reducing the total decrease in strength to a factor A^4.

In the case of Yukawa forces there is another problem. Because the strength of these forces decreases exponentially with the distance, the force from a Yukawa interaction on a single particle will be dominated by nearby mass, while the effect from more distant mass can be neglected. In an experiment this means that only a part of the test masses effectively contributes to the strength of the Yukawa interaction, while normal inverse square law gravity uses the entire mass. This reduces the effective relative strength of the Yukawa interaction. To minimize this problem the test masses of experiments designed to detect Yukawa interactions are ideally heavy thin plates concentrating as much mass as possible at the closest range.

Because of the weak nature of gravity it becomes extremely important to eliminate all background effects. These included electrostatic en magnetic effects. Further problems are caused by acoustic and seismic vibrations. When these effects have been sufficiently suppressed the next problem in line is thermal noise.

As experiments approach the 1 μm mark a new problem surfaces. At this ranges Casimir and Van der Waals forces quickly become dominant over gravity making it increasingly difficult to distinguish forces of gravitational strength. This will be the next big hurdle to take if experiments are to probe gravity at ranges below 1 μm.

3.2 Experiments Measuring Short-Range Gravity

Currently there are two types of experiments to measure gravity at short distances: classical gravitation (low frequency) and high frequency experiments. In this section we shall look at one (simple) example of both of them.

Classical Gravitation Experiments

This type of experiments, using torsion balances, has been around for over two centuries. One of the first persons to use such a technique was Cavendish, who used a torsion balance to measure gravity at a range of 10 cm in 1798. Nowadays there are two experimental groups using torsion balance techniques:

- One group at the University of California at Irvine using a 2-5 cm cryogenic torsion pendulum [5]
- The Eöt-Wash group at the University of Washington [6].

We will now take a closer look at the latter.

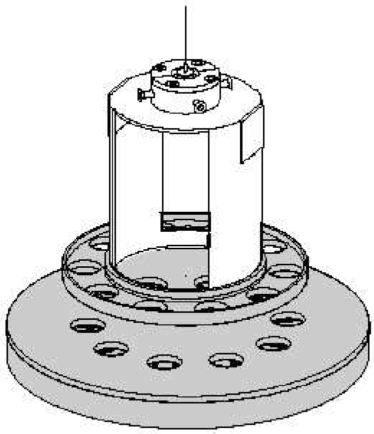

Figure 7: Scale drawing of the Eöt-Wash experimental setup. (source: Ref. [6])

The setup of this experiment (also see figure 7) consists a 1 mm thick aluminium disk with ten evenly spaced holes suspended from a torsion fiber acting as detection mass and a stack of two copper disks with similar holes in them serving as a source mass. When the source mass is rotated, breaking the symmetry, a torque is induced on the detection mass via a gravitational interaction. The lower of the two copper disks, which is slightly thicker, is offset by 18 degrees. This largely cancels out the torque caused by ordinary inverse square law gravity, leaving only the torque due to a possible Yukawa force, which effectively only 'feels' the top disk. The source rotates approximately once every two hours, torquing the detection mass ten times per revolution. The angular offset of the detection mass is measured by using an optical readout.

To shield against electrostatic backgrounds a piece of 20 μm thick beryllium-copper foil is stretched out between the source and detection mass. To further improve conduction the detector, source and foil are coated with a thin layer of gold. To exclude magnetic backgrounds the materials used are selected to be specifically non-magnetic. Vibrations due to the source drive can easily be recognized as the measured signal is ten times as frequent as the rate of rotation. External vibrations (such as footsteps) are suppressed by 'normal' dampening methods. Since thermal noise is of a smaller magnitude than other effects, no special steps (such as cryogenic cooling) were taken to reduce it.

This experiment has measured gravity down to distances of about 200 μm. No anomalous deviations from standard gravity were found. The constraints on α and λ resulting from this experiment are shown in figure 9 as Washington.

Future improvements to this setup include thinner disks made of denser materials to increase or maintain the test mass while reducing the scale of the experiment. The hole arrangements have been further optimized to measure Yukawa interactions and cancel standard gravity. Better isolation and a thinner electrostatic shield should allow for smaller mass separations. The projected limits for this technique are shown in figure 10.

High Frequency Resonance Techniques

While experiments using torsion balances are attractive from a thermal noise point of view because of the low operating frequencies and high mechanical quality factors, their sensitivities are usually limited by low frequency noise, which also limits the test mass separations. Recently, there have been several groups developing techniques using high frequency resonance techniques, which have shown promise of operating at smaller test mass separations. The three experimental groups using high frequency resonance techniques are:

- The group of J.C. Price at the University of Colorado, Boulder.([7]-[9])
- The group of S. Schiller at the University of Düsseldorf. [10]
- The group of A. Kapitulnik at Stanford University.[11]

We shall take a closer look at the torsional oscillator experiment from the Colorado group.

The source mass is a 305 μm thick plate, which is driven at a natural resonance frequency of the detection mass of about 1 kHz. This induces a torque on the detection mass via a gravitational interaction. The detection mass itself consists of a 195 μm thick tungsten torsion oscillator (see figure 8). At the resonance mode of interest the two rectangular areas counter rotate. The benefit of operating at a resonance frequency is twofold. First it gives an optimal response of the detector, which is measured using an inductive device on the second rectangle. Secondly it reduces the effects of vibrational noise.

Between the source and detection mass there is a 60 μm stiff conducting plate of gold-plated sapphire wafer acting as an electrostatic shield and also as a shield against acoustic vibrations. Naturally all materials used in this experiment are non-magnetic to minimize magnetic backgrounds. To further reduce acoustic backgrounds the experiment is performed in vacuum. To reduce backgrounds due to external vibrations the test mass and shield mount

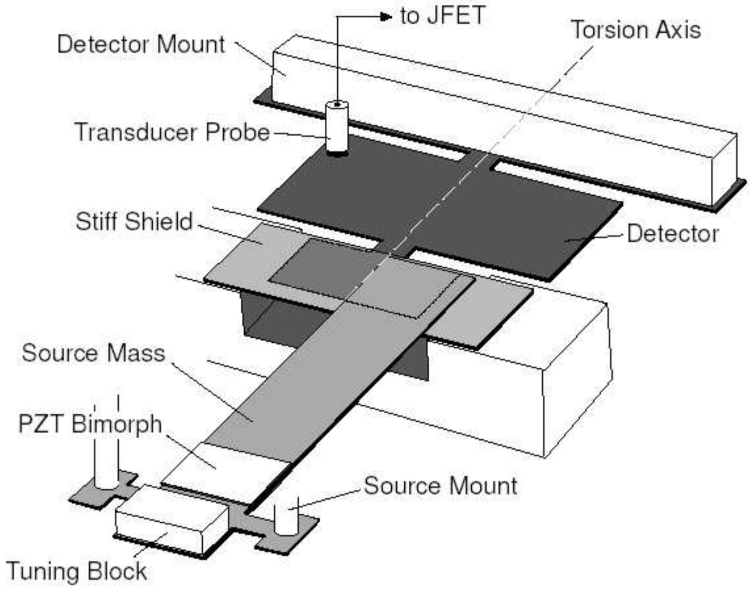

Figure 8: Central components of Colorado experiment. Figure is to scale. (source: Ref. [9])

are suspended from vibration isolation stacks consisting of solid brass disks connected by wires under tension. This system reduces external vibrations at 1 kHz by 20 orders of magnitude.

With electromagnetic, acoustic, and vibrational backgrounds sufficiently suppressed, the thermal noise becomes the limiting factor. Thermal noise is reduced by a process of annealing. Before installation the detector is heated to about 1300 degrees Celsius. When it gently cools down it will naturally end up in a (from an energy perspective) stable state, which greatly reduces thermal noise.

In the current experiment the minimum test mass separation is approximately 100 μm, mainly due to the thickness of the electrostatic shield. To date this experiment has yielded no signal above thermal noise. From this we get the limits shown in figure 9 as the curve marked Colorado. Currently thinner test masses and a thinner shield are developed possibly reducing the test mass separation by a factor 2, yielding the limit shown in figure 10. Plans exist for a cryogenic version of this experiment operating at 4 K, which could increase sensitivity by another factor 40.

3.3 Results and Constraints on Extra Dimensions

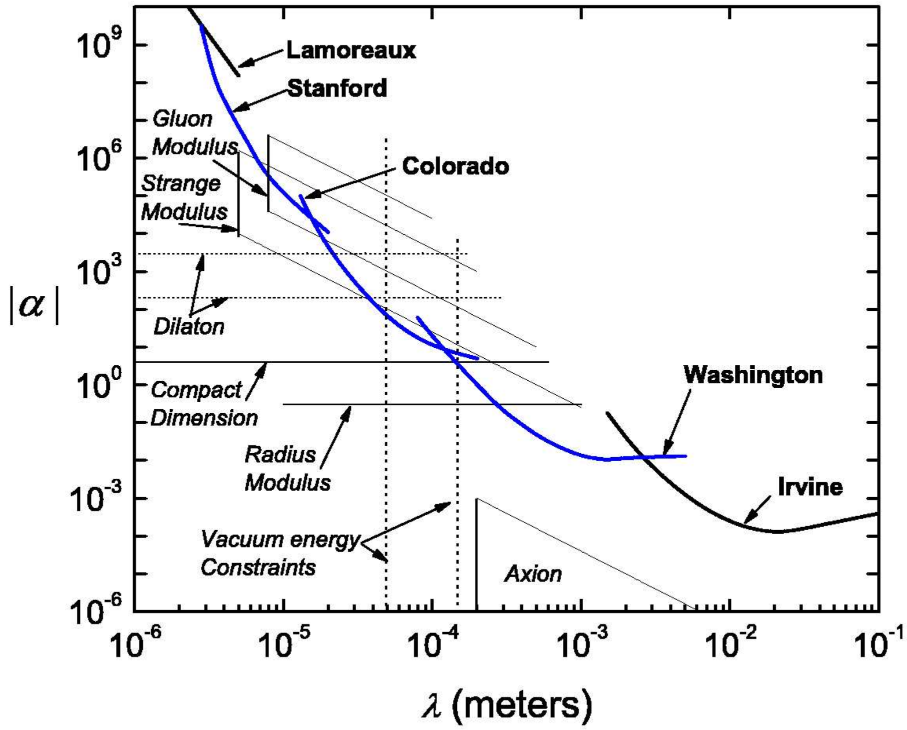

Figure 9: α-λ Parameter space for deviation from the inverse square law. The area above and to the right of the bold curves is currently excluded by experimental results. This plot was taken from ref. [12]

The results of all current experiments are shown in figure 9. Black curves are published results and blue lines are results available in preprints. Also shown are some of the expected effects from different new theories. The one we are interested in is the line labeled 'compact dimensions'. This line shows the strength of an extra Yukawa interaction in the case of two extra dimensions. This excludes values of R larger than about 150 μm. The limits for more dimensions are a bit more stringent, because the relative strength of the Yukawa interaction should be larger.

The red curves in Figure 10 show the results as they are expected from future improvements to the experimental setups. The projected limit for R in the case of two extra dimensions is about 35 μm.

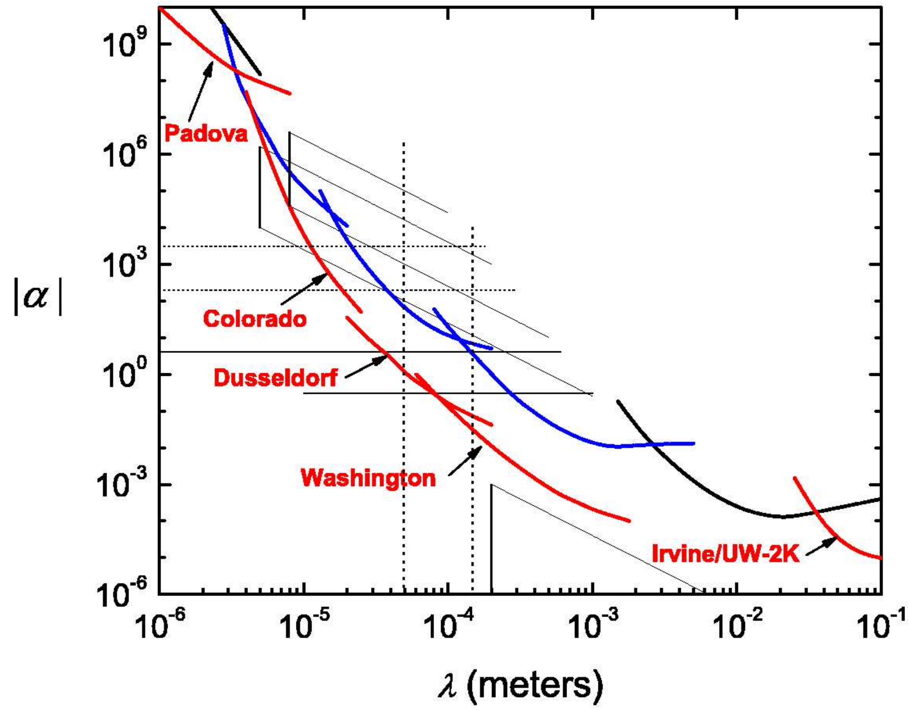

Figure 10: Project limits on short-range Yukawa force for currently running experiments. This plot was taken from ref. [12]

4 Colliders, Cosmology and Astrophysics

In this section ideas and methods used in particle colliders, cosmology and astrophysics will be presented, as well as constraints on the size of large extra dimensions obtained from experiments in these fields. Colliders provide relatively weak bounds, but the results are more reliable than from cosmology and astrophysics.

4.1 Constraints from Particle Colliders

A possibility of experimental signatures of large extra dimensions is the appearance of Kaluza-Klein excitations (which we have studied in section 2.3) at high energy particle colliders.
Particle colliders are in fact the finest microscopes that we have ever build. Why not use them to explore large extra dimensions?

The large extra dimensions that are felt only by gravity can reveal themselves through the emission of gravitons into the bulk. This emission is another way of describing the process of graviton 'evaporation', an apparent loss of energy.

At the moment the greatest energies obtained in particle colliders are $\sim$ 1 TeV. Bounds on the radii of the extra dimensions can be found by analyzing high energetic collisions, thereby looking for a violation of the law of conservation of energy (or momentum). When we find something like this, it is a very acceptable possibility that some energy, in the form of KK gravitons, has escaped into the bulk.

We will present an overview of the experiments done with colliders that are relevant for testing the ADD model and providing bounds on R. There are two classes in which we can divide the governing processes at accelerators. In the first class we have the direct production of Kaluza-Klein gravitons and in the second the so called virtual graviton exchange processes. We will explore these two in more detail.

Direct Production of Kaluza-Klein Gravitons

An important process of this kind is the collision of a proton and an antiproton (or in general a particle and its antiparticle). This will produce a jet of particles plus a graviton (G_{KK}), which will be emitted into the bulk.

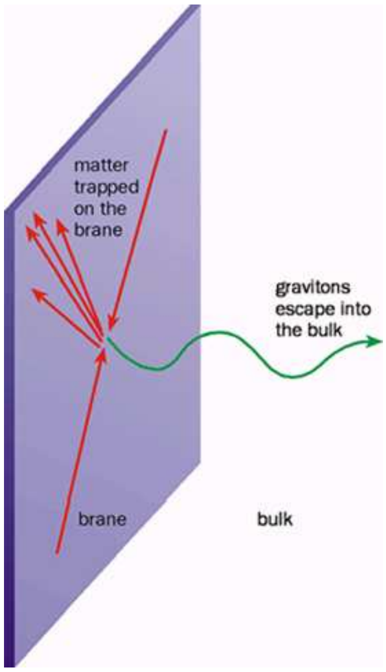

Figure 11: A collision between a proton and an antiproton can produce, for example, a single jet of matter particles plus graviton emission into the bulk. Such collisions might be seen in high-energy physics experiments. This picture was taken from http://physicsweb.org/

In a formula:

$$proton + antiproton \rightarrow \ jet\ of\ particles + G_{KK}.$$

There can be many different kinds of particles in the jet. We do not specify them, because in different collision different particles are made. This kind of $p\bar{p}$ collisions have been studied at the Tevatron collider (at Fermilab, near Chicago) and give certain constraints. From the DØ detector (run I) we get 1.1 TeV as a lower bound for the fundamental mass in $4 + n$ dimensions. This result is almost independent of n. More details can be found in [13].

Another process in which Kaluza-Klein gravitons are produced is the electron-positron annihilation. The idea of electron-positron colliders (like LEP II at CERN) is to let an electron annihilate with a positron, thereby emitting a photon:

$$e^+e^- \rightarrow \gamma + \not{E}_T \,,$$

where $\not{E}_T$ stands for missing energy. We can check afterwards if the energy of the emitted photon equals the total energy before the collision.

If there are really extra dimensions and the collision is energetic enough to produce a Kaluza-Klein graviton, then this graviton will be emitted into the bulk and again we will be left with an apparent loss of energy. Experiments done at CERN (LEP II electron-positron collider) exclude a fundamental Planck mass in 4+n dimensions smaller than 1.45 TeV for $n = 2$ and 0.6 TeV for $n = 6$. For a summary of LEP results on graviton emission, see [14].

Virtual Exchange of Kaluza-Klein Gravitons

These processes are more model dependent than direct KK graviton production. Reaction formulas are of the form:

$$particle + antiparticle \rightarrow \; e^+e^- \; or \; \gamma\gamma.$$

More specific, Kaluza-Klein gravitons are formed and subsequently decay to other degrees of freedom on our brane:

$$q\,\overline{q} \rightarrow \; G_{KK} \rightarrow \; e^+e^- \,,$$

where q is a particle and $\overline{q}$ its antiparticle. Results from the Tevatron collider for the lower limit are ~ 1 TeV. A more extended explanation of this rather advanced topic can be found in [15] and [16].

Intermezzo: Production of Micro Black Holes

Is it really realistic to create micro black holes?
Is it dangerous to create black holes in a laboratory?
How will you know if you have created one?
What could you do with it if you made one?

These are questions that speak to the imagination.

We will look the process of the creation of micro black holes in elementary particle collisions. As we have seen, in the ADD-scenario the observed weakness of gravity at long distances is due the existence of extra sub-millimeter dimensions. In the standard 4-dimensional theory we must have very high energies to let gravity compete with electromagnetism. This can be seen from the laws:

$$F_{EM} = \frac{1}{4\pi\varepsilon_0}\frac{q_1\;q_2}{r^2} \qquad F_{Gravity} = G_4\frac{m_1\;m_2}{r^2}$$

Let us consider two protons. How much more powerful is the electromagnetical force between them? The mass of a proton is 1.66 10^{-27} kg. We can take a distance of one meter (in does not matter). Filling in the numbers gives:

$$F_{EM} \approx 9 \times 10^9 (1.6 \times 10^{-19})^2 \approx 2.3 \times 10^{-28}\,\mathrm{N}$$

and

$$F_{Gravity} \approx 6.67 \times 10^{-11} (1.66 \times 10^{-27})^2 \approx 1.8 \times 10^{-64}\,\mathrm{N}.$$

The electromagnetic force between two protons is $64 - 28 = 36$ orders of magnitude stronger than the gravitational force. What we can do with accelerators, is to make the mass of the protons bigger by speeding them up. If we want to let gravity be as strong as electromagnetism we must accelerate the protons to 10^{18} times their rest mass, and since the rest mass is 931 MeV this is approximately 10^{18} GeV. The only problem is that 10^{18} GeV is some 15 orders of magnitude beyond the experimental borders. If there are no extra dimensions we will probably never be able to produce black holes in a laboratory. With extra dimensions things are very different. Our fundamental mass is of order 1 TeV, depending on the number and the radii of the extra dimensions. This means that if we accelerate a proton to 1 TeV gravity is as strong as electromagnetism.

When $F_{Gravity} > F_{EM}$ strange things happen. For example two protons can stick together, held together by gravity witch is stronger than electromagnetic repulsion. The argument that light will not be able to escape when gravity is stronger than electromagnetism, is a very naive one, because the coulomb force does not work on photons. Nonetheless calculations carried out by experts, imply that it should be possible to create micro black holes in this way (i.e. by letting particles collide at extremely high speeds). If M_{4+n} is not very high this may happen in the nearby future. The Large Hadron Collider (LHC) is under construction and will be operational within a couple of years. Expectations are that the LHC will work at about 7 TeV. Machines producing even higher energies of $10-100$ TeV seem already technically possible. The limitation is the cost. Gravity is either really weak, or is strong but diluted by extra dimensions. In the latter case we will find microscopic black holes in colliders!

We will now try to answer the four questions which were presented at the beginning of this intermezzo.

It may be possible to create micro black holes if the ADD-scenario is correct and if the fundamental Planck mass is reachable by colliders. It is not dangerous to create such black holes. Theoretically their lifetime is of the order of 10^{-30} sec (they evaporate by emitting Hawking-radiation). Furthermore, if micro black holes can be created in a collider, then already billions and billions of them have been formed in the atmosphere by extremely relativistic particles. A correct answer to the question how we could detect such a black hole is: by looking at the Hawking-radiation it emits. Although this answer may be not completely satisfactory (or even completely not satisfactory), we will not discuss it any further. This would go beyond the purpose of this paper. At the internet (for example http://www.arxiv.org/) many articles about micro black holes can be found. If we would be able to make such a black hole, many research could be done on it. Besides discovering extra dimensions, we could test Hawking-radiation, thereby exploring the last stages of black hole evaporation. Other interesting things are quantum gravity and the information loss problem, which shall not be discussed here.

4.2 Cosmological Aspects

There are various types of cosmological constraints on the size of extra dimensions. Constraints arise from, among other things, inflation models and Big-Bang Nucleosynthesis (BBN), so we will start by shortly introducing these subjects.

In order to explain the extremely homogeneous cosmic microwave background (CMB) and the flatness problem, physicists believe that at a very early stage, the universe must have blown up exponentially (see for instance [17]). This is called inflation. Subsequently, to provide the energy needed for inflation, the inflaton field has been introduced and quite generally accepted. This inflaton field (that somehow just happened to be there already) must have decayed, thereby supplying the requisite energy. As a consequence, objects that otherwise would seem to be without causal relation (because they are too far away from each other to effectively influence each other within the age of the universe), can have been close enough to each other (before the inflation), so that they have come into equilibrium. This would explain the homogeneity of the cosmic microwave background (CMB). The flatness problem is solved, because inflation (as opposed to normal expansion) causes the curvature of the universe to diminish.

After that the inflaton field has decayed, the remaining energy has reheated the universe, a process accompanied by the copious production of all types of particles, including KK gravitons. Following this period of reheating, the universe cooled down again, and this time it cooled enough to make the existence of baryons, and later even small nucleons, possible. This process is called Big-Bang Nucleosynthesis (BBN), and the temperature range at which it could occur is quite narrow.

During this whole process of cooling and reheating and cooling again, but especially during the reheating epoch, lots of KK gravitons must have been produced. The existence of these KK gravitons has two consequences: they have absorbed energy, that thus could not be used for other purposes; and they will decay themselves as well, providing energy at sometimes 'unwanted' moments.

Many of the following constraints are in fact constraints on the so-called *normalcy temperature* T_*. This is the temperature below which physics has to be predominantly 'normal', *i.e.*, the exotic consequences of the extra dimensions must be negligible, and above which the evolution of the universe must be considered non-standard, and thus mainly unknown. Usually, an upper bound for T_* is derived, which depends on M_{4+n}. However, there must have been time and energy enough for the universe to evolve the way it has (think, for instance, of the nucleosynthesis), and this provides a model dependent lower bound. $T_* \gtrsim 100$ MeV may be considered quite safe and will be used to calculate the constraints on M_{4+n}, see [18].

It must be said, however, that assuming the compact dimensions to have other geometrical properties (for instance warped instead of flat), may often give very different results (generally much less stringent).

Expansion and Cooling

The energy density ρ in the universe has mainly decreased since the Big-Bang. Adopting the ADD-scenario, there are two possible ways of cooling: the normal (adiabatic) expansion, and 'evaporation' of gravitons into the extra dimensions. Observations show, however, that expansion must have been (and is) the dominant mechanism for most of the time, and so comparing the different rates of cooling might give some constraint on T_*.

The cooling rate due to expansion follows quite easily from the thermodynamic identity $\mathrm{d}U = T\,\mathrm{d}S - p\,\mathrm{d}V + \mu\,\mathrm{d}N$: we suppose $\mathrm{d}S$, p and $\mathrm{d}N$ to be zero (because of the reversibility of the process, the fact that massive particles do not have a (considerable) pressure and the negligible spontaneous production of particles, respectively), so we just keep $\mathrm{d}U = 0$. Then we plug in $U = \rho V$, so $\mathrm{d}U = \rho\,\mathrm{d}V + V\,\mathrm{d}\rho$, and, supposing a sphere, finally we add $V = 4/3\,\pi a^3$ and $\mathrm{d}V = 4\pi a^2 \mathrm{d}a$. After some rearranging, this leads to:

$$\frac{\mathrm{d}\rho}{\mathrm{d}t}\Big|_{expansion} \sim -3\rho H \sim -T^4 \frac{T^2}{M_P},$$

where H is the Hubble parameter $H = \frac{\dot{a}}{a} \sim T^2/M_P$. The cooling rate due to evaporation can be estimated by dimensional analysis and reads [3]:

$$\frac{\mathrm{d}\rho}{\mathrm{d}t}\Big|_{evaporation} \sim -\frac{T^{n+7}}{M_{4+n}^{n+2}}.$$

Since both mechanisms are temperature dependent, this leads indeed to a constraint upon T_*, since we want the expansion to dominate the evaporation for all time after $T = T_*$:

$$-T_*^4 \frac{T_*^2}{M_P} \gtrsim -\frac{T_*^{n+7}}{M_{4+n}^{n+2}} \iff T_* \lesssim M_{4+n}\left(\frac{M_{4+n}}{M_P}\right)^{\frac{1}{n+1}}. \tag{29}$$

Filling in $T_* = 100$ MeV and various values of n, we obtain the following results:

n	2	3	4	5	6
M_{4+n} (TeV)	10	1	0.2	0.07	0.03

So especially for high values of n, this bound is not very stringent. We will derive much more stringent bounds later on.

Over-closure of the Universe

The energy density of the gravitons ρ_{grav} at temperature T_* behaves like [3]:

$$\rho_{grav} \sim T_* \times n_{grav} \sim \frac{T_*^{n+5}}{M_{4+n}^{n+2}} M_P,$$

where n_{grav} is the number density of the gravitons, *i.e.* the number of gravitons per unit space. Due to cosmic expansion, this energy density will decrease, since the same amount of energy has to be spread out over a larger space. It appears, however, that there is a difference between radiation and (non-relativistic) matter: whereas the matter density decreases proportionally to the increase of the volume ($\rho_{matter} \sim (\frac{a}{a_0})^{-3} \sim T^{-3}$), the density of radiation decreases even faster: $\rho_{rad} \sim (\frac{a}{a_0})^{-4} \sim T^{-4}$. This is because of the red-shift: as space expands, the wavelength of a photon will expand with the same rate. Since gravitons can be considered as massive 4D particles, their density decreases as $\left(\frac{a}{a_0}\right)^{-3} \sim T^{-3}$. So, $\frac{\rho_{grav}}{T^3}$ is invariant. Now we can put a lower bound to ρ_{grav} by demanding that

$$\frac{\rho_{grav}}{T^3} \lesssim \frac{\rho_{crit}}{T^3} \sim 3 \times 10^{-12} \text{ TeV},$$

where ρ_{crit} is the critical density and $\sim 3 \times 10^{-12}$ TeV the present day value for the density temperature rate. The critical density is that density, for which the expansion of the universe leads to an equilibrium (that is, after an infinite time, the universe will have a positive, finite size and will neither expand nor contract). Another feature of the critical density is, that it will cause the universe to be flat. Since observations show that the energy density is lower than the critical density, this gives us

$$\frac{\rho_{grav}}{T_*^3} \sim (\frac{T_*}{M_{4+n}})^{n+2} M_P \lesssim 3 \times 10^{-12} \text{ TeV}$$

$$\rightarrow T_* \lesssim \left(\frac{3 \times 10^{-12} \text{ TeV}}{M_P}\right)^{\frac{1}{n+2}} M_{4+n}. \tag{30}$$

Now we can create a table similar to that in the previous section:

n	2	3	4	5	6
M_{4+n} (TeV)	760	32	3.9	0.9	0.3

Note that these constraints are much stronger than the ones in the previous table, in particular for $n = 2$, the bound is very strong.

Present Day Decay to Photons

KK gravitons cannot decay very easily, and consequently, they have large life times τ [3]:

$$\tau(E) \sim \frac{M_P^2}{E^3} \gtrsim \frac{M_P^2}{T_*^3} \sim 10^{-38}\ \mathrm{TeV}^{-1} \sim 10^8\ \mathrm{yr},$$

where E is the graviton's energy. That implies that a significant part of the gravitons formed in the early universe still exists and therefore decays. That means that we should be able to measure distortions in the diffuse gamma spectrum. These distortions, however have as yet not positively been identified, and therefore must be very small. This induces again a bound on T_* (as derived in [3]):

$$T_* \lesssim 10^{\frac{6n-15}{n+5}}\ \mathrm{MeV} \times \left(\frac{M_{4+n}}{\mathrm{TeV}}\right)^{\frac{n+2}{n+5}} \tag{31}$$

And this leads to by far the strongest constraints so far:

n	2	3	4	5	6
M_{4+n} (TeV)	18×10^3	398	32	5.2	1.3

These values, however, are extremely model dependent, and especially sensitive to the possible existence of other 'parallel' branes.

Cosmic Rays

Apart from the cosmic microwave background, we receive other radiation: the cosmic rays. Its energy spectrum is of an enormous width (roughly from 1 to 10^{10} GeV), and has a striking form: there appears to be a sort of hard to explain surplus beginning at energies of a few TeV (see figure 12). It has been suggested that this surplus is caused by the production of gravitons at particle collisions (mainly $pp \to pp + G_{KK}$) in the atmosphere [19]. These gravitons are not detected, and so lead to 'missing' energy.

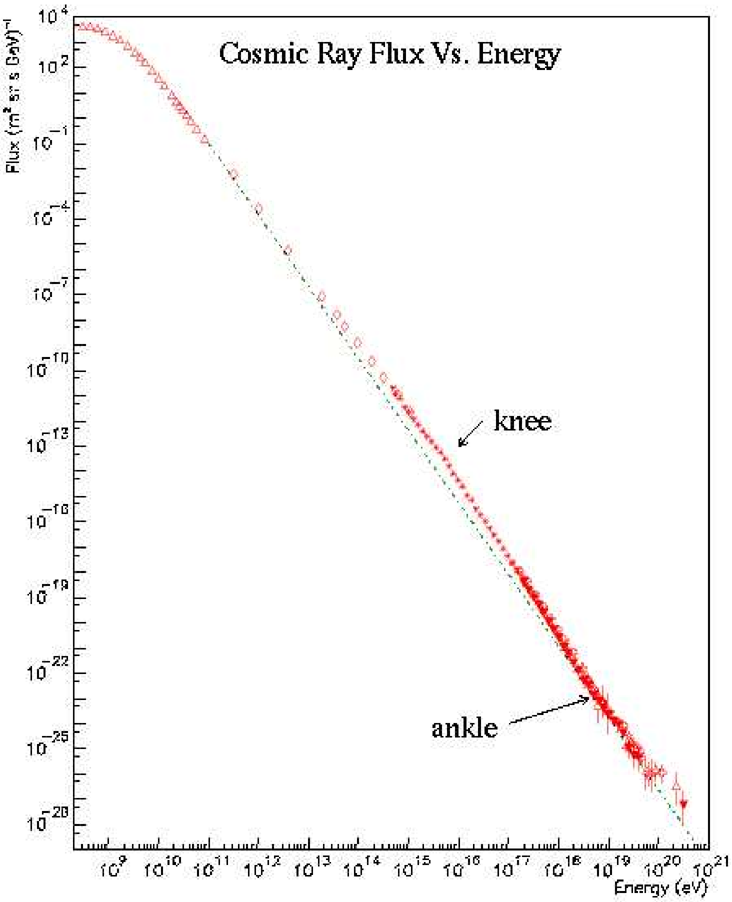

Figure 12: The cosmic ray spectrum $f(E)$. Note the surplus (the area to the right of the green line, indicated with 'knee') beginning at a few TeV. (source: http://astro.uchicago.edu/$\sim$ smoneil/pics/background/spec.gif)

Thus, a surplus of particles with (relatively) low energies is found. This means, however, that the problem is in fact shifted to regions of higher energies, where a corresponding shortage of particles should be found. But because of the decreasing accuracy of the measurements at higher energies and the partly unknown influences of other effects (such as extra-galactic particles), it is not possible to say something sensible about it. Fitting curves based on various energies and a different number of extra dimensions, leads to the suggestion that there will be four extra dimensions with energies of about 8 TeV, although other possibilities are by no means excluded. However, this is all still highly speculative and so should be investigated further.

4.3 Astrophysical Constraints

If the ADD picture is right, then there are a lot of Kaluza-Klein gravitons produced in systems like the sun or a supernova (because of the high temperatures of these systems). Energy will leak from the system into the bulk, because of these gravitons. The number of produced gravitons is constrained by observations. We will examine constraints provided by observations of the sun, red giants and a supernova 1987A.

The probability that a process where a KK graviton is produced will happen, is given by the corresponding cross section. In formula 27 we already saw that

$$\sigma \sim \frac{1}{E^2}(\frac{E}{M_{4+n}})^n.$$

In astrophysical applications temperature is the dominant factor of the energy of a system, E $\sim$ T. This suggests that the cross sections for each process contains the factor $T^n/M_{(4+n)}^{n+2}$, which already has the correct dimensions for a cross section. The cross sections of the processes we will study next, are in fact more complicated. A derivation of these cross sections will go beyond the purpose of this paper. Detailed cross sections and derivations of the listed bounds below can be found in [3].

The Sun and Red Giants

The core temperature of the sun is ~ 1 keV ($\sim 1.1 \times 10^7$ K). The relevant particles in equilibrium are electrons, protons and photons. In this situation the two most important processes which produce KK gravitons (G_{KK}) are:

- Gravi-Compton scattering: $\gamma + e \rightarrow e + G_{KK}$
- Photon fusion: $\gamma + \gamma \rightarrow G_{KK}$

The observed rate at which the sun releases energy per unit mass per unit time is: $\dot{\epsilon}_{obs} \sim 1\, erg\, g^{-1}\, s^{-1} \sim 10^{-45}$ TeV. This puts a constraint on the release of gravitons into the bulk. The Photon fusion gives the strongest bound:

$$M_{4+n} \gtrsim 10^{\frac{18-6n}{n+2}} \text{ GeV}. \tag{32}$$

From this we obtain the following bounds:

n	2	3	4	5	6
M_{4+n} (GeV)	30	1	0.1	0.02	0.006
R(m)	1	5×10^{-4}	1×10^{-5}	1×10^{-6}	2×10^{-7}

This is still a very weak bound if we compare it with the pervious results. The temperatures of red giants are somewhat larger, T $\sim$ 10 keV, but this also does not give us a strong bound.

SN1987A

In 1987 we observed (on earth) a supernova type II, SN1987A for short. According to the standard theory of type-II supernovae, most of the $\sim 10^{53}$ ergs of gravitational binding energy released during the core collapse, is carried away by neutrinos. This hypothesis was essentially confirmed by the measurement of neutrino fluxes from SN1987A by Kamiokande [20] and IMB [21]. If there are extra dimensions, then also KK gravitons, which are produced during a supernova, will carry away some energy. This can only be a limited fraction of the total energy, because otherwise it would contradict the neutrino observations. Therefore we can obtain a lower bound for M_{4+n}. After the core collapse of SN1987A the resulting neutron star had a core temperature of $\sim 30\,\text{MeV}$. This is significantly higher than the temperature of the sun, therefore we expect a stronger bound. The two most dominating processes in supernova are:

- Gravi-Primakoff process: $\gamma + \texttt{EM field of nucleus Z} \to G_{KK}$
- Nucleon-Nucleon Bremsstrahlung: $N + N \to N + N + G_{KK}$

The Nucleon-Nucleon Bremsstrahlung gives the strongest bound:

$$M_{4+n} \gtrsim 10^{\frac{15-4.5n}{n+2}}\ \text{TeV}. \tag{33}$$

This lead to much more stronger constraints:

n	2	3	4	5	6
M_{4+n} (TeV)	30	2	0.3	0.08	0.03
R(m)	1×10^{-6}	1×10^{-9}	6×10^{-11}	8×10^{-12}	2×10^{-12}

Other Possibilities

In a supernova core collapse, massive KK gravitons would be produced with average velocities $\backsimeq 0.5\,c$, therefore many of them are gravitationally retained by the supernova core. Thus, every neutron star would have a halo of KK gravitons which decay into photons and some other particles, on time scales $\backsimeq 10^9$ years. Observations from EGRET (Energetic Gamma Ray Experiment Telescope), of nearby neutron stars, lead to the stringent constraint $M_{(4+2)} \gtrsim 90$ TeV, for $n = 2$, as can be found in [22]. More exotic models predict even higher lower bounds for M_{4+n}.

The problem with the bounds obtained by astrophysics is that they are not very reliable. This is because of uncertainties in the observed data. Also there are different models which describe the same phenomenon. It seems that the higher the predicted lower bound, the less reliable it is.

5 Conclusions

We have seen that the ADD-scenario gives a different perspective to the hierarchy problem. Experiments can test this relatively new scenario. There is no evidence that extra dimensions exist. However, large extra dimensions cannot be excluded, but experiments do place constraints on their sizes. The results, which were discussed in this paper, are summarized in the plot below. The subject of large extra dimensions is still a very active field of research.

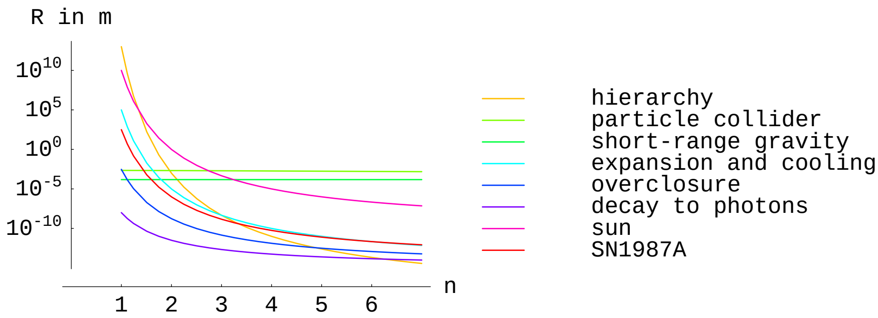

Figure 13: In this plot we present an overview of the discussed constraints on R.

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