Signatures of Extra Dimensions

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June 29, 2005

Abstract

The possible effects of extra spatial dimensions as described in the ADD model might be detectable in future colliders such as the LHC at CERN. This article reviews two detectable consequences of these extra spatial dimensions at high energy collisions: black hole production and missing energy. Also included is a theoretical background about both compactified extra spatial dimensions and Kaluza Klein reduction in order to give the reader some insight in the concept of extra spatial dimensions.

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1 Theory

1.1 Introduction

What are extra dimensions and why would we even need them?

Through the ages people have tried to understand and describe the world around them. Physicists in particular try to describe nature in the simplest way possible. Describing forces plays a crucial role in describing nature.
In the late 17th century Isaac Newton was the first to create a model in which he explained the attractive forces between massive objects, hence making a model for the *gravitational force*[1]. Almost a century later Charles Augustin de Coulomb studied the interacting forces of electrically charged objects, creating what we now know as Coulomb's law, describing the *electrical force*. As time passed a few more forces of nature were discovered. Untill now, all the forces that have been discovered are:

force:	**describes:**
gravity	objects with masses
electromagnetism	charged objects
strong nuclear force	how the nucleus is tied together
weak nuclear force	responsible for e.g. beta-decay

It took the whit of James Clerk Maxwell to take several forces of nature[2] and put them together into one model: *electromagnetism*.
So if the forces of nature are all known, the dream of a physicist is to unify all the existing forces into one model. There is only one problem, which is that gravity is much weaker than all the other forces. This problem is known as the hierarchy problem. In order to unify the forces of nature we must find a way around the hierarchy problem. This is where extra dimensions come into play.
The first attempt to unify gravity with electromagnetism through extra dimensions was proposed by Theodor Kaluza. His idea of introducing an additional dimension to our four-dimensional world[3] was a radically new one. It was later refined and worked out explicitly by Oscar Klein.
To illustrate the idea of these extra spacial dimensions one can visualize the following scenario: When walking on a tight rope one can only go back and forth along the rope, hence having only one degree of freedom (1D). An ant however is not restricted to only walk on the top of the rope, it can also go around it, hence having 2 degrees of freedom (2D). In other words, how do we know a line is not a cylinder with a very small radius? Or in general, how do we know there are only four dimensions? The answer is: We don't! This gives a whole array of new possibilities to unify the forces of nature and creating one big mother-theory, or at least explaining why gravity as so much weaker than the other forces, in other words solving the hierarchy problem.

[1] Nowadays gravity is described by Albert Einstein's general theory of relativity.
[2] In this case the forces were the electrical and magnetic (Lorentz) force.
[3] We think of our world to have three spacial dimensions and one dimension of time, adding up to a four-dimensional world.

1.2 Kaluza-Klein Reduction

So how does it work?

To get a feel of how we can use extra dimensions to solve some of our problems, let's look at a simple model first. We're going to work out a way to unify the Coulomb force and Newton's gravity. The idea is to think of our world to have one extra compactified spatial dimension.[4] Einstein's relativity gives us the four-momentum of 4 dimensional particle[5]:

$$\vec{p} = \left(\frac{E}{c},\, p_x,\, p_y,\, p_z\right)$$

with norm $p \equiv \|\vec{p}\|$

$$p^2 = -\frac{E^2}{c^2} + p_x^2 + p_y^2 + p_z^2$$

If we want to add an extra compactified dimension we get an extra component p_c in our four- or rather five-momentum. Now because the extra dimension is compactified we can think of it as a circle with radius R. Quantum mechanics tells us that the allowed values of the wavelength ($\sim$ energy) along this circle should be quantized.

$$n\lambda = 2\pi R\,, \quad n \in \mathbb{Z}$$

plugging in the de Broglie wavelength $\lambda = \frac{h}{p}$ gives us:

$$p_c = \frac{n\hbar}{R} \tag{1}$$

Let's consider two massive 4D particles both sitting still,

$$\vec{p}_1 = \left(\frac{E_1}{c},\, 0,\, 0,\, 0\right) \qquad \text{and} \qquad \vec{p}_2 = \left(\frac{E_2}{c},\, 0,\, 0,\, 0\right)$$

with $E_1 = m_1c^2$ and $E_2 = m_2c^2$. Now we also write Newton's gravity as

$$F(r) = G_{\mathrm{N}}\frac{m_1m_2}{r^2} = \frac{G_{\mathrm{N}}}{c^2}\frac{-\vec{p}_1\cdot\vec{p}_2}{r^2} \tag{2}$$

If we assume that higher dimensional momenta remain Lorentz invariant we can add the extra compactified dimension, so the the two five-momenta will be

$$\vec{p}_1 = \left(\frac{E_1}{c},\, 0,\, 0,\, 0,\, \frac{n_1\hbar}{R}\right) \qquad \text{and} \qquad \vec{p}_2 = \left(\frac{E_2}{c},\, 0,\, 0,\, 0,\, \frac{n_2\hbar}{R}\right) \tag{3}$$

The inner product of these two vectors is

$$-\vec{p}_1\cdot\vec{p}_2 = m_1m_2c^2 - n_1n_2\frac{\hbar^2}{R^2}$$

We obtain the following expression for the attractive force:

$$F(r) = G_{\mathrm{N}}\frac{m_1m_2}{r^2} - \frac{G_{\mathrm{N}}\hbar^2}{c^2R^2}\frac{n_1n_2}{r^2}$$

[4]We call a dimension *compactified* if it is curled up like the dimension around the tight rope. An infinite dimension would be the one along the tight rope.

[5]We use the so-called Minkowski metric with signature $(-+++)$

The second term suspiciously resembles Coulomb's law. If we want to make this expression match Coulomb's law we need to set the radius of the extra dimension at $R = \frac{\hbar\sqrt{4\pi\epsilon_0 G_{\mathrm{N}}}}{ce} \approx 2 \cdot 10^{-34}\,\mathrm{m}$, which is very very small.[6] The formula for the attractive force now reads

$$F(r) \;=\; G_{\mathrm{N}}\frac{m_1 m_2}{r^2} - \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2} \;=\; F_g + F_e \tag{4}$$

with $q_1 = n_1 e$ and $q_2 = n_2 e$. So there it is, we found a way to get two forces into one "model" via Kaluza-Klein reduction. If you think there's something fishy about the previous calculation I can only say you're right. We used some quasi-relativistic ways to keep things simple, but nonetheless it gives a good qualitative notion of how powerful extra dimensions can be. In the next section we will be doing some calculations using *large* extra dimensions, which might be easier to "see" or detect.

1.3 The Hierarchy Problem

What's the problem?

To get a glance at how weak gravity is let us take a look at the following example. If you hold a magnet close enough to an iron marble the magnet has no hard time at all pulling it up. This indicates how much stronger electromagnetism is: the tiny magnet beats a colossal object —the earth— in attracting the marble. If we wish to unify gravity with the other forces there should not be such a big discrepancy between the scales of the regions on which gravity and electromagnetism work, but there is. As we just saw, this discrepancy is not just huge, it's *astronomical*. There are more (qualitative) problems, but this esthetic problem is the one we are dealing with now.

1.3.1 Higher Dimensional Gravity

How would gravity behave in a higher dimensional space?

A way of explaining why gravity is so weak is to say that it propagates in more than only three spatial dimensions. These extra dimensions should be so small that we cannot "see" them. But before we start talking about gravity in these small compactified dimensions, we need to know how it behaves in general in higher dimensional spaces.

Let's see what we already know about gravity. We know that in 4D (3 spatial + 1 time) Newton's law reads:

$$F(r) = G_N \frac{m_1 m_2}{r^2} \tag{5}$$

The gravitational field $\vec{\Phi}$ of just one mass m_1 is given by:

$$\vec{\Phi} = G_{\mathrm{N}} \frac{m_1}{r^2}\hat{r} \tag{6}$$

[6]In this case R is just about the same size as the Planck length.

We can also invoke Gauss' theorem:

$$\begin{aligned}
\oint \vec{\Phi} \cdot d\vec{s} &= \text{constant} \cdot m_{encl.} \\
&= 4\pi G_{\mathrm{N}}\, m_1 \\
&= \int_{\text{sphere}} \Phi_r \; ds \\
&= \Phi_r \int_0^{2\pi} d\phi \int_0^{\pi} d\theta \; r^2 \sin\theta \\
&= \Phi_r \, 4\pi r^2 \\
&= \Phi_r S_3(r)
\end{aligned}$$

where $S_3(r)$ is the surface area of a sphere with radius r in our 4 dimensional world —the 3 in $S_3(r)$ stands for the number of spatial dimensions. With this result we can obtain a more general expression for Newton's gravity in 4D:

$$F_4(r) = \Phi_r m_2 = 4\pi G_{\mathrm{N}} \frac{m_1 m_2}{S_3(r)} \tag{7}$$

We see that in 4D the surface area is proportional to the radius as $S_3(r) \propto r^2$. In d spatial dimensions this relation becomes $S_d(r) \propto r^{d-1}$. What we want to do now is find the proportionality constant Ω_d in

$$S_d(r) = \Omega_d r^{d-1} \tag{8}$$

so we can work out the gravitational force for d spatial dimensions:

$$F_{d+1}(r) = 4\pi G_{\mathrm{N}} \frac{m_1 m_2}{S_d(r)}$$

To get Ω_n we can use the following trick. We can integrate some function in two ways, in cartesian coordinates and in polar coordinates. In both cases the answer should be the same. The function we use is a Gaussian e^{-r^2}. Ω_n is the part of the polar integral which is independent of r. We will first work out the *Cartesian* integral.

$$\begin{aligned}
\int_{\text{all space}} d^d r \, e^{-r^2} &= \int_{-\infty}^{\infty} dr_1 \, e^{-r_1^2} \; ... \; \int_{-\infty}^{\infty} dr_d \, e^{-r_d^2} \\
&= (\sqrt{\pi}\,)^d
\end{aligned}$$

Now let's see what the *polar* integral has to offer us.

$$\int_{\text{all space}} d^d r \, e^{-r^2} \quad = \quad \int_0^{\infty} dr \, e^{-r^2} \; \int_0^{2\pi} d\phi \int_0^{\pi} d\theta \; ... \; \cdot \text{Jacobian}$$

but the Jacobian is proportional to r^{d-1} so the integral is

$$\begin{aligned}
&= \int_0^\infty dr\, r^{d-1} e^{-r^2} \int_0^{2\pi} d\phi \int_0^\pi d\theta \; ... \\
&= \int_0^\infty dr\, r^{d-1} e^{-r^2} \; \Omega_d \\
&= \frac{\Omega_d}{2} \int_0^\infty dx\, x^{\frac{d}{2}-1} e^{-x} \qquad \text{(subst. } x \equiv r^2\text{)} \\
&= \frac{\Omega_d}{2} \, \Gamma(\tfrac{d}{2})
\end{aligned}$$

Here $\Gamma(\frac{d}{2})$ is the Euler-gamma function. We obtain the following expression for the proportionality constant:

$$\Omega_d = \frac{2(\sqrt{\pi}\,)^d}{\Gamma(\frac{d}{2})} \tag{9}$$

A more esthetic way of writing the $d+1$ dimensional gravitational force is

$$F_{d+1}(r) = G_{d+1} \frac{m_1 m_2}{r^{d-1}} \tag{10}$$

with

$$G_{d+1} \equiv \frac{4\pi G_{\mathrm{N}}}{\Omega_d} = \frac{2\;\Gamma(\frac{d}{2})}{(\sqrt{\pi})^{d-2}}\, G_{\mathrm{N}} \tag{11}$$

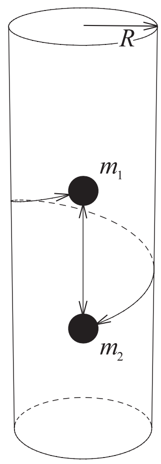
Let's check if this is correct for $d = 3$:

$$G_{3+1} = \frac{2\;\Gamma(\frac{3}{2})}{(\sqrt{\pi})^{3-2}}\, G_{\mathrm{N}} = \frac{2\frac{\sqrt{\pi}}{2}}{\sqrt{\pi}}\, G_{\mathrm{N}} = G_{\mathrm{N}} \qquad \checkmark$$

So that's it, we know how gravity behaves in a higher dimensional space. However, we know that gravity is proportional to $\frac{1}{r^2}$ so how can this result be right? This result can only be right if the extra dimensions are small enough. So it's time to start using *compactified* extra dimensions.

How small do the extra dimensions have to be?

We will start off by adding only one extra compactified dimension and try to generalize our strategy to n compactified dimensions.
Again we can treat the extra dimension as if it were a circle with radius R. The distance between two massive particles in the regular spatial dimensions is given by $\vec{r}_0$. We can choose our coordinate system along the direction of $\vec{r}_0$ so we get relatively simple system of a cylinder with radius R. As we can see in the picture gravity can also go around the cylinder. In general these contributions cannot be neglected. If we would "unroll" the cylinder (see Figure 1) we get a clearer view on things. We see that the gravitational force no longer just has a component in the $\vec{r}_0$ direction, but also in $\vec{r}_i$ with $i \in \mathbb{Z}$.



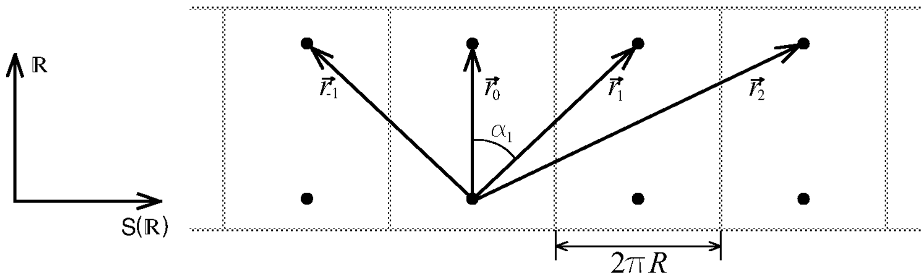


Figure 1: If we unroll the cylinder we get this band.

Every contribution to the attractive force along $\vec{r}_i$, for some i, can be written as a superposition of a force in the $\mathbb{R}$-direction and one in the $\mathrm{S}(\mathbb{R})$-direction.[7] The total force will be:

$$\begin{aligned}\vec{F} &= \sum_{i\in\mathbb{Z}} \langle \vec{F}_{r_i}\,,\, \hat{e}_{\mathbb{R}}\rangle\, \hat{e}_{\mathbb{R}} \;+\; \langle \vec{F}_{r_i}\,,\, \hat{e}_{\mathrm{S}}\rangle\, \hat{e}_{\mathrm{S}} \\ &= \sum_{i\in\mathbb{Z}} \cos\alpha_i\, \|\vec{F}_i\|\, \|\hat{e}_{\mathbb{R}}\|\, \hat{e}_{\mathbb{R}} \\ &= \sum_{i\in\mathbb{Z}} \cos\alpha_i\, F_{r_i}\, \hat{e}_{\mathbb{R}}\end{aligned}$$

where

$$\cos\alpha_i = \frac{r}{r_i} = \frac{r}{\sqrt{r^2 + (i\,2\pi R)^2}} \qquad (r \equiv r_0)$$

and

$$F_{r_i} = G_{4+1}\frac{m_1 m_2}{r_i^3} = G_{4+1}\frac{m_1 m_2}{\left(r^2 + (i\,2\pi R)^2\right)^{\frac{3}{2}}}$$

The contributions in the $\mathrm{S}(\mathbb{R})$-direction cancel, so the sum over the second inner products had to be zero.
That's it for one extra dimension. If we generalize this result to n extra dimensions[8] we get the following formula:

$$F(r) = G_{4+n} m_1 m_2 \left(\sum_{i_1\in\mathbb{Z}} \cdots \sum_{i_n\in\mathbb{Z}} \frac{r}{\left(r^2 + (i_1 2\pi R)^2 + \ldots + (i_n 2\pi R)^2\right)^{\frac{3+n}{2}}} \right) \quad (12)$$

Let's check if this formula gives the good results in the limits $r \ll R$ and $r \gg R$. We'll start with $r \ll R$. The denominator in F will be:

$$\left(r^2 + (i_1 2\pi R)^2 + \ldots + (i_n 2\pi R)^2\right)^{\frac{3+n}{2}} \approx r^{3+n}$$

$$\Rightarrow \quad F(r) = G_{4+n}\frac{m_1 m_2}{r^{2+n}} \qquad (r \ll R) \quad (13)$$

Working out the case where $r \gg R$ is a bit more subtle. Because the steps in the sums are relatively small we'll pretend they are infinitesimal. In other words

[7] The unit-vectors in these directions are respectively $\hat{e}_{\mathbb{R}}$ and $\hat{e}_{\mathrm{S}}$

[8] Previously we used d+1 for the total number of dimensions. Henceforth we will be working with the number of extra dimensions n (just substitute: $d = 3 + n$).

we convert the sums into integrals:

$$F(r) = m_1 m_2 G_{4+n} \int_{-\infty}^{\infty} di_1 \dots \int_{-\infty}^{\infty} di_n \frac{r}{\left(r^2 + (i_1 2\pi R)^2 + \dots + (i_n 2\pi R)^2\right)^{\frac{3+n}{2}}}$$

How do we get our Newtonian gravity out of this? We will just have to evaluate this integral. Let's start with a nice substitution: $x_m \equiv \frac{2\pi R}{r} i_m$ with $m = 1, \dots, n$ and so the Jacobian is $\left(\frac{r}{2\pi R}\right)^n$:

$$= m_1 m_2 G_{4+n} \int_{-\infty}^{\infty} di_1 \dots \int_{-\infty}^{\infty} di_n \frac{r}{r^{3+n}\left(1 + x_1^2 + \dots + x_n^2\right)^{\frac{3+n}{2}}} \left(\frac{r}{2\pi R}\right)^n$$

$$= \frac{m_1 m_2}{r^2} \frac{G_{4+n}}{R^n} \cdot \{\text{some integral only dependent on } n\}$$

This looks almost like Newton's gravity. If we call the n-dependent integral $\mathcal{I}_n$ and evaluate it we get

$$\mathcal{I}_n = \frac{1}{2^{n+1}\,(\sqrt{\pi})^{n-1}\,\Gamma(\frac{3+n}{2})}$$

and so the gravitational force will be

$$F(r) = \frac{m_1 m_2}{r^2} \frac{G_{4+n}\mathcal{I}_n}{R^n} \qquad (r \gg R) \tag{14}$$

If this is correct it means that Newton's constant must be

$$\begin{aligned} G_{\mathrm{N}} &= \frac{G_{4+n}\mathcal{I}_n}{R^n} \\ \Rightarrow \quad R_n &= \left(\frac{G_{4+n}\mathcal{I}_n}{G_{\mathrm{N}}}\right)^{1/n} \end{aligned} \tag{15}$$

Thus, in the two limits Equation 12 gives the correct results. That's nice, but the interesting part is the case in which $r \approx R$. We would want to use Equation 14, however we made an approximation which is only valid if $r \gg R$. To work around this problem we need to apply a correction which follows from the Euler-Maclaurin Integration Formulas.[9] We worked out the case of one extra dimension, $n = 1$ (see Figure 2). We can see that as r approaches R the higher-dimensional gravity grows stronger than the classical one quite rapidly.

It's nice to see that gravity grows stronger at short distances, but we cannot yet see it grows *astronomically* stronger as required in order to solve the hierarchy problem. Maybe one extra dimension (with a small radius) is not enough. In the next section we will compute the size and number of the extra dimensions required for solving the hierarchy problem.

[9]We will not to work this out explicitly, because it involves a very long calculation with a lot of derivatives and stuff. In stead we'll just look at the results in Figure 2.

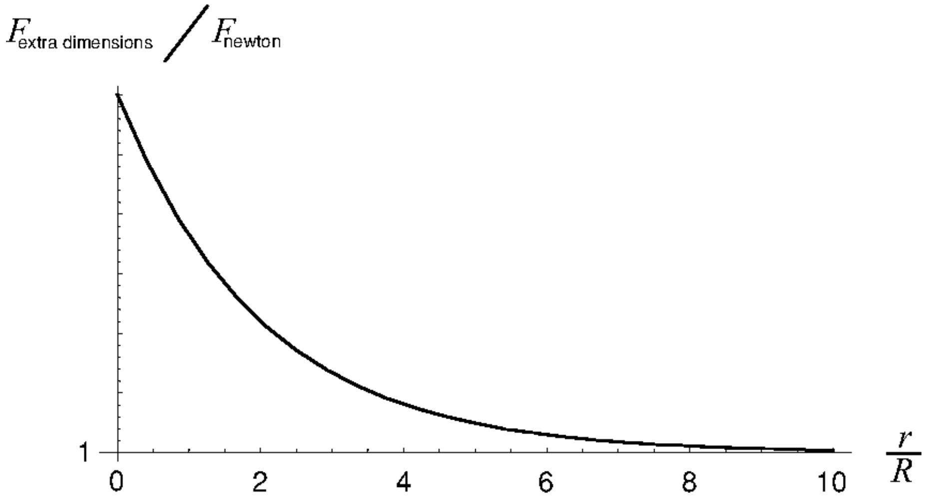


Figure 2: The difference between Newton's gravity and Equation 12.

1.3.2 The ADD Model

What are the Planck and electroweak *scales*?

A more precise way of saying gravity is weaker than the other forces is to say that their *scales* differ. The fundamental scale for the forces in the Standard Model is the scale of electroweak (EW) symmetry breaking:[10]

$$M_{\text{EW}} \sim 1 \text{ TeV}$$

Gravity's fundamental scale is the so-called Planck mass (or Planck scale):

$$M_{\text{Pl}} = \frac{1}{\sqrt{G_{\text{N}}}} \sim 10^{16} \text{ TeV}$$

Gravity needs about 10^{16} more "stuff" to work than electromagnetism does.

What does the ADD model stand for?

The following scenario to explain (or alter) the hierarchy problem was first proposed by Nima Arkani-Hamed, Savas Dimopoulos and Georgi Dvali, ADD for short. In this model the assumption is that the EW scale is the only fundamental scale at (very) short distances. If we would invoke compactified extra dimensions we see that the Planck mass depends on the number n and size R of these extra dimensions. The idea is to set

$$M_{4+\text{n}} \sim M_{\text{EW}} \sim 1 \text{ TeV}$$

where $M_{4+\text{n}}$ is the Planck mass at n extra dimensions. But what is $M_{4+\text{n}}$? We shall have to do some dimensional (unit) analysis to calculate this. Let's start with the simple case of the regular Planck mass M_{Pl} in 4D. The units of our fundamental constants are:

[10] Here we use *natural* units, which means we set $c = \hbar = 1$.

$$\begin{aligned}[c] &= \mathrm{m\,s^{-1}} \\ [\hbar] &= \mathrm{m^2 kg\,s^{-1}} \\ [G_\mathrm{N}] &= \mathrm{m^3 kg^{-1} s^{-2}}\end{aligned}$$

The unit of the the Planck mass should be kilograms. Using only c, $\hbar$ and G_N we get:

$$[M_\mathrm{Pl}] = \left[\sqrt{\frac{c\,\hbar}{G_\mathrm{N}}}\right] = \mathrm{kg}$$

Thus we set

$$M_\mathrm{Pl} = \sqrt{\frac{c\,\hbar}{G_\mathrm{N}}} \approx 10^{-8}\ \mathrm{kg} \sim 10^{16}\ \mathrm{TeV} \tag{16}$$

Now let's calculate $M_{4+\mathrm{n}}$. The unit of Newton's constant in $4+n$ dimensions is $[G_{4+n}] = \mathrm{m^{3+n} kg^{-1} s^{-2}}$. To get $M_{4+\mathrm{n}}$ we must solve the following equation:

$$[c^i\,\hbar^j\,{G_{4+n}}^k] = \mathrm{kg}$$

$$\Leftrightarrow \quad (\mathrm{m\,s^{-1}})^i\,(\mathrm{m^2 kg\,s^{-1}})^j\,(\mathrm{m^{3+n} kg^{-1} s^{-2}})^k = \mathrm{kg}$$

By using some algebra we get i, j and k as functions of n which yield:

$$M_{4+\mathrm{n}} = \left(\frac{c^{1-n}\,\hbar^{1+n}}{G_{4+n}}\right)^{\frac{1}{2+n}} \tag{17}$$

but Equation 15 already told us that

$$R_n = \left(\frac{G_{4+n}\mathcal{I}_n}{G_\mathrm{N}}\right)^{1/n}$$

If we combine this equation with Equations 16 and 17 (and again apply some algebra) we get an expression for the radius R_n of the compactified dimensions:[11]

$$R_n = \left(\frac{M_\mathrm{Pl}^2}{{M_{4+n}}^{n+2}}\left(\frac{c}{\hbar}\right)^n \frac{1}{2^{n+1}\,(\sqrt{\pi})^{n-1}\,\Gamma\left(\frac{3+n}{2}\right)}\right)^{1/n} \tag{18}$$

or in orders of magnitude (and in *natural* units):

$$R_n \sim \left(\frac{M_\mathrm{Pl}^2}{{M_{4+n}}^{n+2}}\right)^{1/n}\ \mathrm{TeV}^{-1} \tag{19}$$

This is great! Now we know how large the extra dimension have to be, given the number of extra dimensions n and the higher dimensional Planck scale M_{4+n}.

So how large do these extra dimension have to be?

Now we can actually set $M_{4+n} \sim M_\mathrm{EW} \sim 1$ TeV so the compactification radius R_n will be of order:

$$R_n \sim 10^{\frac{32}{n}}\ \mathrm{TeV}^{-1} \qquad \text{or in SI units:} \qquad R_n \sim 10^{\frac{32}{n}-19}\ \mathrm{m} \tag{20}$$

[11] We assume that all the extra dimensions have the same size and are compactified on a torus.

number of extra dim. n	1	2	3	4	5	6
size of extra dim. R_n (TeV^{-1})	10^{32}	10^{16}	10^{10}	10^{8}	10^{6}	10^{5}
R_n in standard units (meters)	10^{13}	10^{-3}	10^{-9}	10^{-11}	10^{-13}	10^{-14}

The case of one extra dimension cannot be correct for it implies deviations in Newton's gravity over distances as big as our solar system. We can safely say that is excluded. However, for the cases where $n \geq 2$ everything is still possible.[7]

Look Ahead

In the following sections we will discuss some of the possible effects that extra spatial dimensions have on particle physics. Two of those possible effects, measurable at high energy particle collision, namely: missing energy due to escaping gravitons and mini black hole creation, will be discussed in detail.

2 Missing Energy

2.1 Interdimensional gravitons

In the ADD model all particles except gravitons are confined to the four dimensional brane. The gravitons on the other hand, are allowed to propagate into the extra spatial dimensions. Thus, it is possible that a graviton created at a particle collision carries energy away from the brane into the extra dimensions. This will result in a nett loss of total energy, because the total energy before the collision is bigger than the total energy after the collision. The energy lost in this process is known as "missing energy".
The detection of missing energy will confirm the existence of extra spatial dimensions. Detecting this missing energy can be done in several ways. One of them is to simply measuring all the energy before the collision, and then subtract the total energy measured after the collision. An other way of detecting missing energy is by detecting the graviton that escaped off the brane. In the ADD model, all the extra dimensions are compactified. Therefore, the escaped graviton will repeatedly pass through the brane, where it, or rather the jet created by the graviton, will be observable. The jet is created because gravitons alone are unstable particles that decay into other particles. The observable particles within the jet are referred to as final state particles.

2.2 Kaluza Klein states

A graviton propagating outside the brane will acquire Kaluza Klein modes. The energy spacing between the Kaluza Klein modes is proportional to $1/R$ where R is the radius of the extra dimensions. For extra dimensions with a radius R of between 10^{-3}m and 10^{-15}m the energy spacing between the Kaluza Klein modes varies from 1 MeV to 100 MeV. Because the energy spacing is very small compared to the energy of the escaping gravitons[12], the spectrum of the Kaluza

[12]The energies of the colliding particles at LHC are ~ 1 TeV. It is therefore likely to expect that the gravitons created at such collisions carry energies of the same order of magnitude, or

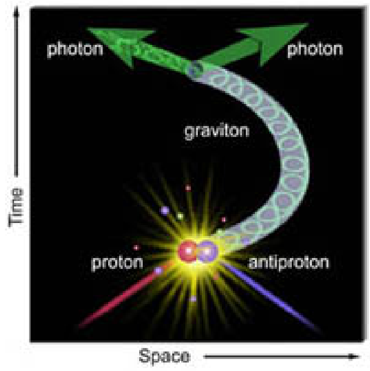


Figure 3: An example of an escaping graviton due to proton antiproton collision.[1]

Klein gravitons will appear continous. This spectrum breaks down at a certain ultraviolet cutoff above which yet unknown quantum gravity effects will take place. Since the higher dimensional planck scale in the ADD model M_{4+n} is $\sim$ 1 TeV, it's natural to expect this cutoff, M_S, to be of the same order of magnitude: $M_S \sim M_{4+n}$.
In a particle collision with an energy of $\sim$ 1 TeV for example, there are as many as 10^4 Kaluza Klein modes available for each extra dimension. Should we consider the case where the energy spacing between adjacent Kaluza Klein modes is maximized, then the radius $R \sim 1fm$ of the extra dimensions is minimized. Because the given size of the extra dimensions corresponds to a total number of 7 extra dimensions, the total number of exitable Kaluza Klein modes is $10^{7\times4} = 10^{28}$. Because of the huge number of Kaluza Klein modes available for each extra dimension, the gravitational interaction in the extra dimensions is enhanced tremendously, despite the fact that gravity is weak compared to other interactions.[21]

2.3 Measuring missing energy

Unfortunately, not all the missing energy is due to escaping gravitons from the brane. For example, experimental conditions like holes in the detector, result in more than a third of the events in CDF[13] having large missing energy. In order to distinguish missing energy caused by escaping gravitons from missing energy caused by other processes, we simulate the experiment. The constraints imposed by the simulation upon the obtained data will allow us to separate the possible events that mimic the process of missing energy due to escaping gravitons, from the process of escaping gravitons itself. Figure 4 displays the result of a simulation of missing energy processes.
In a collider, apart from the process that produces Kaluza Klein gravitons, there are many other processes which do not involve missing energy that can occur during a collision. These collisions are not interesting from our point of view, thus making them unwanted collisions. It is convenient to choose the so called

somewhat less, but certainly much more than $\sim$ 100 MeV.
[13]The CDF is a detector of the Tevatron experiment at Fermilab

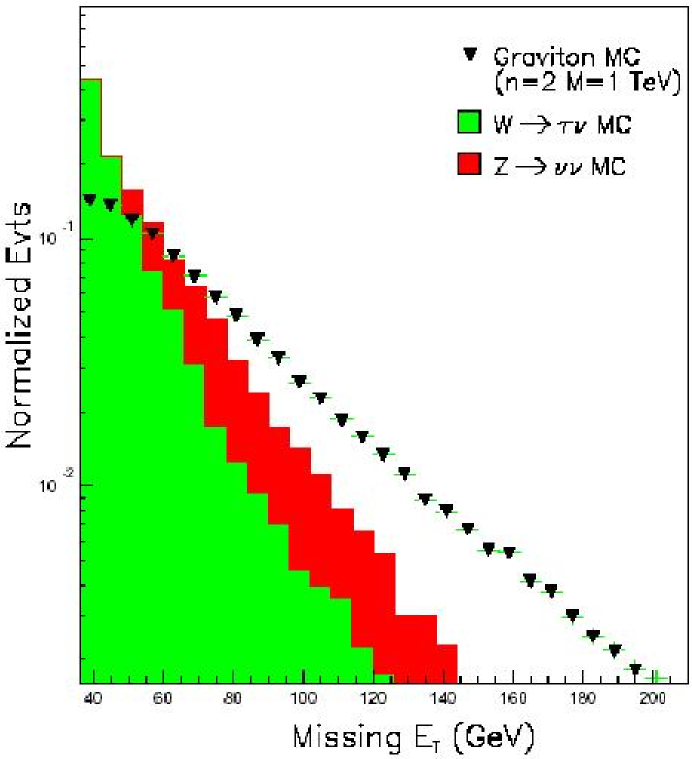


Figure 4: Events with missing energy as predicted by the standard model (in grey and black) and from a graviton signal (points). For large missing energy it is more likely that the missing energy is due to escaping gravitons.[2]

acceptance cuts in such a way that the number of unwanted collisions relative to the number of wanted collisions is minimal. The signal received from the unwanted collisions is known as the background signal.
As an example we will examine the following collision.

$$e^+e^- \rightarrow f\bar{f}\breve{E}$$

where f can be either a muon or a quark. To keep things short, we will only discuss the situation where f is a muon. Because Kaluza Klein gravitons couple to the energy momentum tensor, they can be added to any vertex or line in a Feynman diagram. Thus, the possible ways for a graviton to be created in the electron positron collision are the following: see figure 5.
We start our analysis by imposing the following constraints:

C1 We require that the events present a missing transverse momentum bigger than 10 GeV

C2 The muons or jets should have a transverse momentum bigger than 5 GeV

C3 The muons or jets be observed in the region $|\cos\theta| < 0.98$, where θ is the muon or jet polar angle

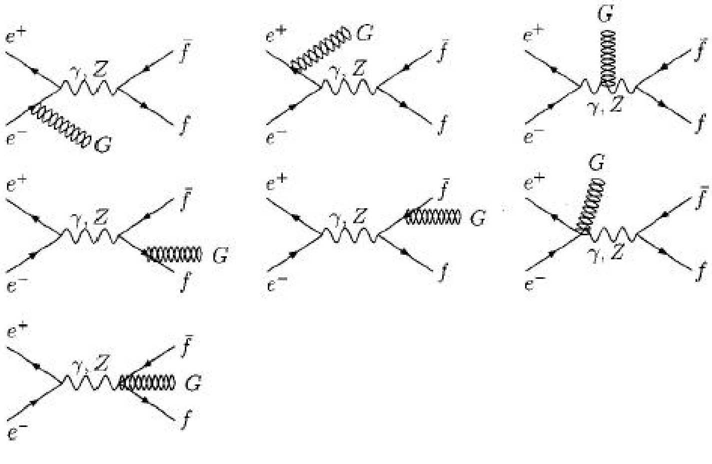


Figure 5: Feynman diagrams contributing to the Kaluza Klein graviton radiation process $e^+e^- \to f\bar{f}\breve{E}$.[12]

C4 The muons are required to be separated by $\Delta R > 4$.

n	2	3	4	5	6	7
$\sigma^{signal}_{\mu\mu\breve{E}}(fb)$	55.1	17.2	6.08	2.27	0.888	0.357

Table 1: Total signal cross section in fb where $b = 10^{-24}cm^2$ for different number of extra dimensions, using a center of mass energy of 500 GeV and $M_{pl(4+n)} = 1$ TeV after applying the acceptance cuts (C1)-(C4).[12]

In table 1 the signal cross section drops quickly as n increases. We know $M_{pl}^2 \sim R^n M_{pl(4+n)}^{n+2}$ from 19, so

$$R \sim \sqrt[n]{\frac{M_{pl}^2}{M_{pl(4+n)}^n}}$$

This means that R gets smaller for a larger number of extra dimensions (n). As we already know, the energy spacing between the Kaluza Klein modes is $\sim \frac{1}{R}$ so for larger n the Kaluza Klein modes aquire higher energies. The chance of a collision where gravitons will be exited in the nth Kaluza Klein mode will decrease as n increases.
For comparison, the total cross section for the standard model background is $\sigma^{back}_{\mu\mu\breve{E}} = 73.6fb$. As we can see in table 1 the signal cross sections are very small, especially for large n, compared to background cross section. The signal cross section is proportional to the chance for a collision where a graviton, together with two muons, is created, while background cross section is proportional to the chance for a collision where background radiation is created. In order to

investigate the Kaluza Klein gravitons, it is therefore necessary to maximize the signal cross section relative to the background cross section.

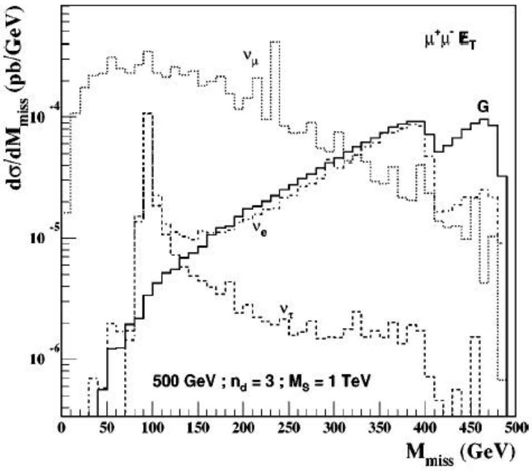


Figure 6: Missing invariant mass M_{miss} spectrum originated from the Kaluza Klein graviton radiation (solid line) and the standard model contributions to the muonic channel divided in the neutrino flavors: ν_e (dot-dashed), ν_μ (dotted) and ν_τ (dashed). $\frac{d\sigma}{dm}$ is proportional to the probability for a graviton with mass m to be created.[12]

In figure 6 we see the Kaluza Klein graviton radiation has the largest differential cross section[14] for $M_{miss} > 320$ GeV. We choose the next cut (C5) to be: $M_{miss} > 320$ GeV. Due to the fact that emitted Kaluza Klein gravitons carry momentum in the brane directions, the final state jets or muons are not expected to be back to back because the total momentum is conserved. This means the angle between the final state muons ($\cos\theta_{\mu\mu}$) is not close to -1.
As figure 7 displays, the Kaluza Klein graviton signal prefers the region where the angle $\cos\theta_{\mu\mu}$ between the final state muons is small due to the fact that the cosine of $\cos\theta_{\mu\mu}$ is close to 1. Therefore, the next cut (C6), is chosen as followed: $\cos\theta_{\mu\mu} > 0$

n	2	3	4	5	6	7
$\sigma^{signal}_{\mu\mu\breve{E}}(fb) : (C1) - (C4)$	55.1	17.2	6.08	2.27	0.888	0.357
$\sigma^{signal}_{\mu\mu\breve{E}}(fb) : (C1) - (C6)$	18.7	7.46	3.05	1.27	0.537	0.230

Table 2: The total signal cross section for the muonic channel with cuts (C1)-(C4) compared to (C1)-(C6).[12]

After applying (C5) and (C6) the signal cross sections are almost reduced three

[14]The cross section is the area within which the particles have to meet in order to come close enough to collide in a certain manner. Therefore, the chance on a particular collision is proportional to the cross section.

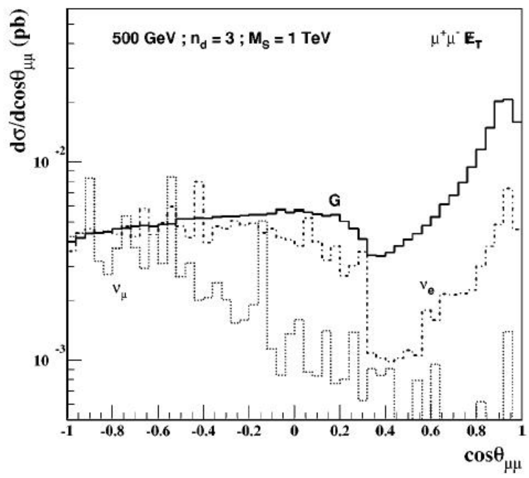


Figure 7: Distribution of the cosine of the angle between the final state muons originated from the Kaluza Klein graviton radiation and the standard model contributions as in figure 6.[12]

times in size, while the background cross section is reduced a lot more! The standard model background is reduced to $\sigma^{back}_{\mu\mu\breve{E}} = 3.24fb$ which is more than 20 times as small as $\sigma^{back}_{\mu\mu\breve{E}}$ without (C5) and (C6). Therefore, these cuts enhance the signal relative to the background.[12]

2.4 Graviton effects

It will not surprise the reader that, when investigating extra spatial dimensions as described in the ADD model, gravitons are an important subject of study. Therefore, we will now spend some more time on these interesting particles. To be more specific, in the following section we will discuss the following two possible effects of gravtions, one of which has already been introduced in the section Interdimensional gravitons.
-Monojet creation due to graviton decay
-Drell-Yan process
The first effect is a direct graviton effect, while the second is a virtual graviton effect. Also, direct graviton effects depend directly on M_{4+n}, while virtual graviton effects depend directly on M_S.
An exited graviton that is allowed to propagate by itself after the collision, for example a Kaluza Klein graviton that propagates in the extra dimensions, is what we refer to as a normal graviton. It is relatively stable, and more important, it satisfies it's equations of motion.
A normal graviton as described above is created at the following collision, where a quark antiquark collision results in the creation of a normal graviton and a Kaluza Klein graviton.

$$q\bar{q} \rightarrow g + G_{KK}$$

At this process, a monojet is created. The final state particles in the monojet

can be detected by a non conservation in transverse momentum. Also, the monojet will lead to an amplification of the tail of transverse energy spectrum. Apart from collisions where normal gravitons are excited, it's also possible for a so called virtual graviton to be created. Virtual, in a sense, means that if we look at the Feynman diagram that describes the collision, the graviton is not allowed to leave the diagram. For example, all the Feynman diagrams that are shown in figure 5, display normal gravitons that are allowed to leave the diagram. Virtual gravitons on the other hand, are stuck between the two vertices. What is meant by "not allowed to leave the diagram" is that the gravitons are not able to live by themselves. This is a direct consequence of the fact that the virtual gravitons do not satisfy their equations of motion[15]. Thus, they are only allowed to exist in some sort of intermediate state, which is what the line connecting the two vertices in the Feynman diagram stands for.
Virtual graviton effects take place at the Drell-Yan process, mentioned above. Note that this is just one of the many processes where virutal graviton effects can take place. The Drell-Yan processes in the presence of large extra dimensions is shown in figure 8.

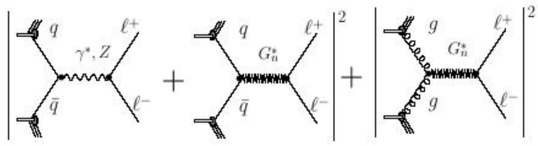


Figure 8: Feynman diagrams for the modified Drell-Yan production in the presence of large extra dimensions.[21]

Both processes noted above have already been studied at Tevatron Fermilab with the CFD and DØ detectors. We will discuss the results obtained at Tevatron Fermilab **ref** shortly. Results from the monojet searches in Run 1 are summarized in the next table.

Experiment and channel	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$
DØ monojets, $K=1$	0.89	0.73	0.86	0.64	0.63	0.62
DØ monojets, $K=1.3$	1.99	0.80	0.73	0.66	0.65	0.63
CDF monojets, $K=1$	1.00		0.77		0.71	
CDF monojets, $K=1.3$	1.06		0.80		0.73	

Table 3: Individual 95% CL lower limits on the fundamental Planck scale M_D (in TeV) in the ADD model from the CDF and DØ experiments. Ordering of the results is chronological.[21]

It is possible to compute the chance of a certain collision to take place from the accompanying Feynman diagram. Figure 9 shows both the
calculated events per transverse energy of the monojet, as well as the measured events per transverse energy of the monojet.

[15] Virtual gravitons don't satisfy $E^2 = m^2c^4 + p^2c^2$

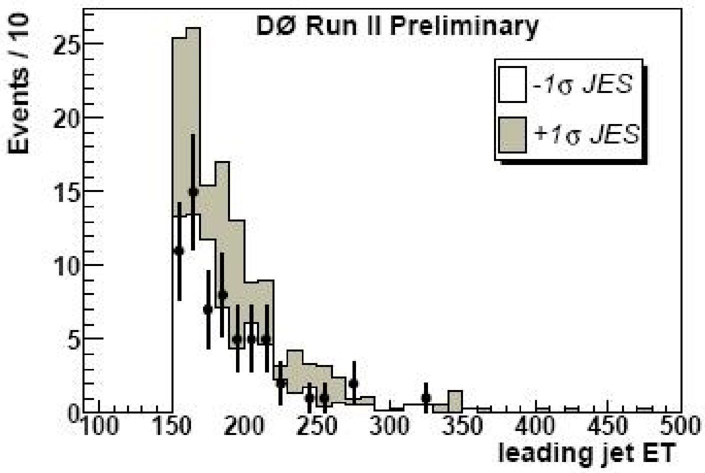


Figure 9: Transverse energy of the leading jet in the DØ monojet analysis. Points are data, while the histogram is the expected background radiation. The shadow band corresponds to the change in the jet energy scale (JES) of +/- one standard deviation.[21]

At Tevatron Run 2 the Drell-Yan process was investigated using both the CDF and the DØ detectors. Figure 10 shows the invariant mass spectra of the produced dielectrons[16] for two different topologies. As can be seen from figure 10, the Kaluza Klein graviton contribution to the signal is dominant in the region where the signal exceeds the expected background, rather than the interference term which causes virtual graviton exchange.[21]

3 Black holes

A lot of new theories in physics suggest extra dimensions or need extra dimensions to be valid. Until now there is no evidence for the existence of extra dimensions but the investigation of mini black holes produced in particle colliders could give insight in the unexplored world of extradimensional physics. If the fundamental planck scale is of order a TeV, as the case in some extra-dimensions scenarios, future colliders such as the Large Hadron Collider (LHC) will be black hole factories. In this section we investigate the behavior of black holes in more dimensions and the possibility of creating them at the LHC.

3.1 Black holes in general

A black hole is a body which has such a great mass that not even light can escape from it due to the gravitational field surrounding the black hole. Before we will deal with black holes in accelerators we will take a look at black holes in general.

[16]Indeed, in this case the leptons in figure 8 are electrons.

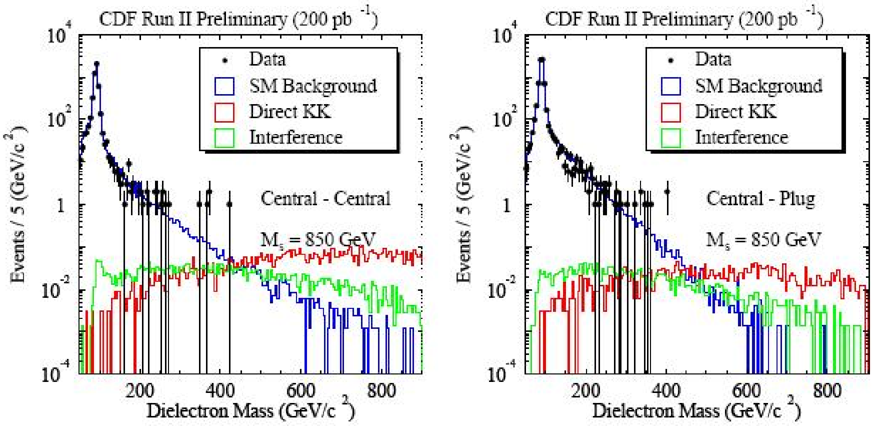


Figure 10: Dielectron mass spectrum from the CDF search for large extra spatial dimensions via virtual graviton effects. Left: central-central, right: central-forward combinations. Again, points are data, while the solid line is the sum of the standard model backgrounds. Also shown in the plots are effects of virtual graviton exchange due to interference and direct terms for $M_S = 850$ GeV.[21]

3.1.1 Black holes in 4 dimensions

The radius within which the gravitational force is so strong that nothing can escape the black hole is called the Schwarzschild radius. To derive the Schwarzschild radius we equate the potential energy to the kinetic energy of a particle with vertical velocity c.

$$\begin{aligned} R_S &= \frac{2G_{\rm N}M_{BH}}{c^2} \\ &= \frac{2\hbar M_{BH}}{cM_{\rm Pl}} \end{aligned} \tag{21}$$

In the last step we used $M_{\rm Pl} = \sqrt{\frac{\hbar c}{G_4}}$. Combining the formula for the Schwarzschild radius with the formula for the acceleration due to gravity, $a = \frac{MG_{\rm N}}{r^2}$, will give us the acceleration due to gravity at the horizon of a black hole:

$$a_{horizon} = \frac{c^4}{4G_{\rm N}M_{BH}}$$

Hawking showed that a black hole radiates particles with a specific temperature T. This temperature is proportional to the acceleration [3]. Using the fundamental constants $\hbar$, k_B and c, we can get a formula for the temperature by dimension analysis, up to a numerical constant.

$$
\begin{aligned}
[T] &= K \\
[a_{horizon}] &= \frac{m}{s^2} \\
[\hbar] &= \frac{kgm^2}{s} \\
[k_B] &= \frac{kgm^2}{s^2 K} \\
[c] &= \frac{m}{s}
\end{aligned}
$$

We need to satisfy the following equation:

$$[T] = [a_{horizon}]^\alpha [\hbar]^\beta [k_B]^\gamma [c]^\delta$$

This gives a system of equations which can be solved. We get:
$\alpha = 1 \qquad \beta = 1 \qquad \gamma = -1 \qquad \delta = -1.$
Now we can write down an explicit formula for the Hawking Temperature in 4 dimensions. The numerical constant is $(8\pi)^{-1}$.

$$T = \frac{\hbar c^3}{8\pi G_{\mathrm{N}} M_{BH} k_B} \tag{22}$$

Note that the temperature of the black hole increases as the mass decreases. A black hole gets hotter as it decays, which causes the decay-process to accelerate.

We know from thermodynamics that the integrating factor of the entropy is the inverse of the temperature. Using the first law of thermodynamics, $\mathrm{d}E = T\mathrm{d}S$, we find [19]

$$
\begin{aligned}
S &= \int \frac{c^2}{T(M_{BH})} \mathrm{d}M \\
&= \frac{4\pi G_{\mathrm{N}} k_B M_{BH}^2}{\hbar c} \\
&= k_B \pi \frac{R_S^2}{L_{\mathrm{Pl}}^2}
\end{aligned} \tag{23}
$$

In the last step we used equation 21 and $L_{\mathrm{Pl}} = \sqrt{\frac{G_{\mathrm{N}}\hbar}{c^3}}$.
If you consider a black hole in a canonical way, the energy density of a black hole is given by the Stefan-Boltzmann law. To compute the luminosity of a black hole we have to multiply the energy density by the surface area of the black hole.

$$
\begin{aligned}
L &= 4\pi R_S \sigma T^4 \\
&= \frac{\sigma \hbar^4 c^8}{16^2 \pi^3 G_{\mathrm{N}}^2 k_B^4} * \frac{1}{M_{BH}^2}
\end{aligned} \tag{24}
$$

Note that the luminosity of a black hole is inversely proportional to the square of the mass.

3.1.2 Black holes in (4+n) dimensions

The black holes that will possibly be formed at the LHC are smaller than the radius of the extra dimensions. The topology of the object can be assumed to be spherical symmetric in 4+n dimensions. We will use a semi-classical way of deriving the properties of the black hole. Our approach is valid if $M_{BH} \gg M_{4+n}$. [19] [20]

To derive the schwarzschild radius of a (4+n)-dimensional black hole, we have to equate the (4+n)-dimensional potential energy needed to take a particle away from the surface of a black hole to infinity, to the kinetic energy of a particle with vertical velocity c. The (4+n)-dimensional potential energy is given by

$$U = \frac{1}{n+1} * \frac{G_{4+n} M_{BH} m}{R_S^{n+1}} \tag{25}$$

The kinetic energy in (4+n) dimensions is $\frac{1}{2}mc^2$ because the particle moves in a two dimensional plane. Now it is easy to see that the (4+n)-dimensional schwarzschild radius is given by

$$\begin{aligned} R_H &= \left(\frac{2G_{4+n} M_{BH}}{(n+1)c^2} \right)^{\frac{1}{n+1}} \\ &= \left(\frac{2}{n+1} \frac{M_{BH}}{M_{4+n}^{n+2}} \right)^{\frac{1}{n+1}} \frac{\hbar}{c} \end{aligned}$$

In the last step we used $M_{4+n} = \sqrt[n+2]{\frac{\hbar^{n+1}}{c^{n-1} G_{4+n}}}$. Setting $\hbar$ and c equal to 1, which is often done in theoretical physics, gives us a nice formula for the schwarzschild radius in 4+n dimensions.

$$R_H = \left(\frac{2}{n+1} \frac{M_{BH}}{M_{4+n}} \right)^{\frac{1}{n+1}} \frac{1}{M_{4+n}} \tag{26}$$

If we assume that $M_{4+n} = 1$ TeV, and $M_{BH} = 5$ TeV, which is reasonable for the experiments at the LHC, we may calculate the value of the schwarzschild radius as an function of n. [20] These values are given in table 4. From the values for R_H can be concluded that the two particles in a particle collision have to come closer than 10^{-4} fm to form a black hole.

Table 4: Black hole horizon radii for different values of n

n	1	2	3	4	5	5	7
R_H (10^{-4} fm)	4.06	2.63	2.22	2.07	2.00	1.99	1.99

The temperature of a (4+n)-dimensional black hole can be derived in a way similar to the 4 dimensional case. We won't give the derivation here. An explicit derivation can be found in [19].

$$T_H = \frac{n+1}{4\pi} \frac{1}{R_H} \tag{27}$$

In the (4+n)-dimensional case the temperature is inversely proportional to $M_{BH}^{\frac{1}{n+1}}$. As the parton collision energy increases, the resulting black hole gets heavier and its decay products get colder. We also see that higher dimensional black holes are hotter.

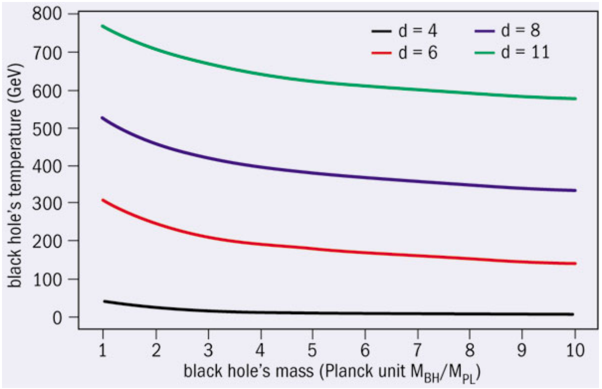


Figure 11: The temperature of a black hole as a function of its mass for different numbers of extra dimensions. NB: here d is the number of dimensions, i.e. n+4. [9]

Now we can derive the entropy. Again we are using the first law of the thermodynamics.

$$S_H = 2\pi \frac{n+1}{n+2} \left(\frac{2}{n+1} \frac{M_{BH}}{M_{4+n}} \right)^{\frac{n+2}{n+1}}$$
$$= 2\pi \frac{n+1}{n+2} (M_{4+n} R_H)^{n+2} \tag{28}$$

The entropy is a measure for the number of decay products, as we will see later in this article.

The luminosity of a (4+n)-dimensional black hole is given by the (4+n)-dimensional energy density times the (4+n)-dimensional volume of the black hole. We will qualitatively motivate that the energy density is proportional to $T_H^{(n+4)}$.

Remember the derivation of the energy density in 4 dimensions. The energy density is given by $\sum E_n * n_{pl}$, where E_n is the energy of a particle in a certain state n, and n_{pl} is the planck distribution. To determine E_n one has to check which wavelength of the particle fits in a 3 dimensional box with size L, to find $E_n = \frac{hc|\vec{n}|}{2L}$ with $|\vec{n}| = \sqrt{n_x^2 + n_y^2 + n_z^2}$. Converting the sum into an integral will give a formula depending on n^3. Changing variables to ν will give a formula depending on ν^3. Now, changing variables again to $x = \frac{h\nu}{k_B T}$ (the exponent of the boltzmann factor), and evaluating the integral gives the 4 dimensional stefan boltzmann equation, σT^4. This equation is proportional to T^4, because of the ν^3 plus an extra T from the jacobian. [22] [4]

Now we will give the analogue in 4+n dimensions. To determine E_n one has to check which wavelength of the particle fits in a (3+n) dimensional box with size L. This will give $E_n = \frac{hc|\vec{n}|}{2L}$, but now $\mid \vec{n} \mid$ is the absolute value of a (3+n) dimensional vector: $\mid \vec{n} \mid = \sqrt{n_1^2 + n_2^2 + \cdots n_{3+n}^2}$. Converting the sum into an integral gives a formula depending on n^{3+n} and changing of variables twice will give a formula depending of T^{4+n}, where the 'extra T' again comes from the Jacobian. This time we cannot evaluate the integral exact but we know it will just give a constant which we will call σ_{4+n}. Note that this constant is not the same at the 4 dimensional σ. So the (4+n) dimensional energy density is given by

$$\epsilon = \sigma_{4+n} T^{4+n}$$

The volume of a sphere in 4+n dimensions is given by

$$V_{4+n}(R) = \frac{2\pi^{\frac{n+3}{2}}}{\Gamma(\frac{n+3}{2})} R^{n+2}$$

as has been derived in section 1, equations 8 and 9. The luminosity is proportional to

$$\begin{aligned} L_H &\sim R_H^{n+2} T_H^{(n+4)} \\ &\sim M_{BH}^{-\frac{2}{n+1}} \end{aligned} \tag{29}$$

3.2 Black hole production

3.2.1 The experiment at the LHC

In the year 2007, scientists at the Large Hadron Collider (LHC) at CERN in Geneve, Switzerland might be able to produce mini black holes with the next generation of particle colliders. Two beams of accelerated particles will be "fired" against each other, each particle having a high kinetic energy of about 14 TeV in the center of mass (c.o.m.) system. With such high energies the particles will have such a small Compton-wavelength $\lambda_C \propto 1/E$ and the wave package will be tightly packed such that the probability of particles coming very close to each other will be high. Figure 12 shows a scheme of a particle

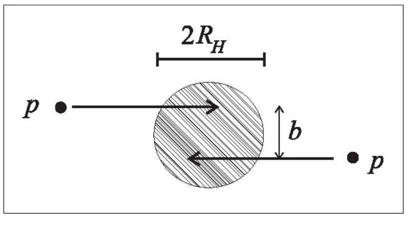


Figure 12: Particle Collision [19]

collision. As particles with very high energies collide, they can come closer than the Schwarzschild radius R_H, associated with their energy. If the impact parameter b is smaller than $2R_H$ a mini black hole will be formed. [10] [9]

3.2.2 Energy needed to form a black hole

The Schwarzschild radius R_H of a (4 + n) dimensional black hole with mass M is given by equation 26. We suppose the Planck mass M_{4+n} to be of order 1 TeV in (4 + n) dimensions. Thus, a black hole with mass of order M_{4+n} has a Schwarzschild radius

$$R_H \approx L_{\mathrm{Pl}} = 1/M_{4+n} \tag{30}$$

We want the Compton wavelength λ_C of the particles to be in the same order of magnitude. Assuming $\lambda_C \approx 1/E$ (and we can do that since by dimensional analysis we can show that the fundamental constants must be the same in both cases R_H and λ_C and no big extra factors occur) we need a c.o.m. energy of order M_{4+n} to get λ_C close to R_H. Thus the energy needed to form a black hole will be of order E = 1 TeV. The next generation of particle accelerators at the LHC will be able to produce such high energies in the coming years.[10] [14] [18]

3.2.3 Cross section

The cross section is the area within which the partons have to meet to come close enough to form a black hole. Arguments along the line of Thorne's hoop conjecture indicate that a black hole forms when partons collide at impact parameter b that is less than the Schwarzschild radius R_H corresponding to E [14]. This cross section can be approximated by the classical geometric cross section

$$\sigma(M) \approx \pi R_H^2 \tag{31}$$

and contains no small coupling constants. This approximation of the cross section has been and is still under debate, but it seems to be justified at least up to energies of $\approx 10M_{4+n}$ [19]. The cross section of a parton collision with c.o.m. energy $\sqrt{\hat{s}} \approx M_{4+n} \approx TeV$ is of order $(1TeV)^{-2} \approx 400pb$.[19] [14] [23]

3.2.4 Differential cross section

We will need the differential cross section $\frac{d\sigma}{dM}$ to be able to compute the number of black holes that will be formed with a certain mass M_{BH} at a c.o.m. energy $\sqrt{s}$. It is given by summation over all possible parton interactions, which is expressed in the functions $f_A(x_1, \hat{s}), f_B(x_2, \hat{s})$ giving the distribution of the partons within the protons, and integration over the momentum fractions. We will give the equation here but refer for further explanation to [19].

$$\frac{d\sigma}{dM} = \sum_{A_1, B_2} \int_0^1 dx_1 \frac{2\sqrt{\hat{s}}}{x_1 s} f_A(x_1, \hat{s}) f_B(x_2, \hat{s}) \sigma(M, d) \tag{32}$$

It is complicated to calculate this term. We will give a numerical evaluation of it, shown in Figure 13, left. By integrating equation 32 we get the total cross section. The plot in Figure 13, right, shows the total cross section of a black hole with mass $M_f = M_{4+n} = 1TeV$ as a function of the collision energy $\sqrt{s}$. The total (production) cross section at the LHC for black hole masses above 1 TeV ranges from 0.5 nb for $M_{4+n} = 2TeV$ and n = 7 to 120 fb for $M_{4+n} = 6TeV$ and n = 3.[19]

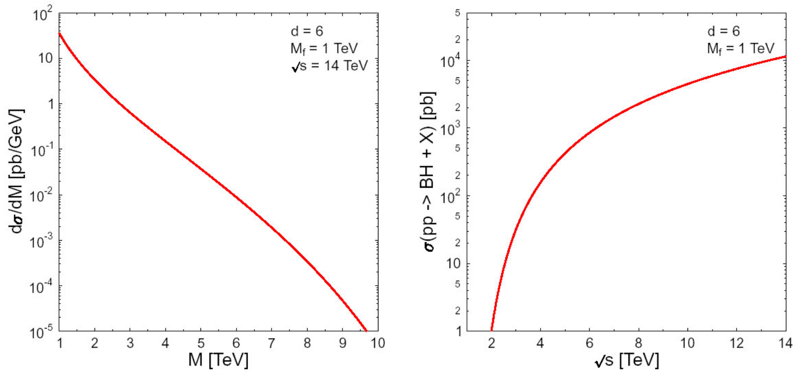


Figure 13: Left: Differential cross section for black hole production in proton-proton-collisions at the LHC for $M_{4+n} = 1TeV$, Right: Integrated total cross section as a function of the collision energy $\sqrt{s}$, both plots deal with the case d = 6 extra dimensions. [19]

3.2.5 Number of produced black holes per year

According to [11], the LHC will have a (estimated) peak luminosity of $30fb^{-1}/year$. This would result in a black hole production rate of $10^7/year$. According to [19] the luminosity is going to be about $L = 10^{33}cm^{-2}s^{-1}$, which, at a c.o.m. energy of 14 TeV, would give birth to about 10^9 black holes per year. This is about one black hole per second! [19] [11]

3.3 Black hole decay

3.3.1 Balding phase, evaporation phase, Planck phase

Decay of spinning black holes can be divided in three phases.
Phase 1: The *balding phase* in which the black hole loses its 'hair', that means lost of multipole moments through the emission of (classical) radiation.
Phase 2: The *evaporation phase*, evaporation through emission of Hawking radiation, which starts with a brief spin-down phase, giving away its angular momentum, followed by the *Schwarzschild phase*: the emitted particles carry signatures of mass, entropy and temperature of the black hole.
Phase 3: The *Planck phase*, when the mass of the black hole approaches M_{4+n} and the black hole's final decay takes place by emission of a few quanta with corresponding Planck-scale-energies. There are two possibilities for an end state: Either the black hole decays completely or some stable remnant is left, which carries away the remaining energy. Information is lost.[19] [14] [15]

3.3.2 The evaporation phase: Hawking radiation

In the past scientists thought that there was no way for energy to escape a black hole, they thought of a black hole as a massive object letting nothing escape the so-called Schwarzschild-radius. But then there was Hawking, who published a new theory in 1975. When taking quantum theory in account, black holes can radiate according to Hawking's theory because of random vacuum fluctuations

near the Schwarzschild-radius.
Virtual particle pairs are constantly being created near the horizon of a black hole. Normally, a particle-antiparticle pair annihilates quickly after creation. However, near the horizon of a black hole it's possible for one particle to fall into the black hole, while the other particle escapes as Hawking radiation. In this case the antiparticle with negative energy falling into the black hole takes energy of the black hole that escapes by means of the other particle. [19] [5] [17] [15]

3.3.3 Black body radiation

Hawking showed that the radiation of a black hole can be seen as radiation of a black body with a specific temperature T. This implicates the use of thermodynamics to calculate various properties of a black hole. The particle spectrum of the radiation of a black hole can be seen as a thermal spectrum (Planck / Maxwell-Boltzman distribution). Since we are dealing with black holes of very small mass i.e. only few particles, one could suggest that the distributions are not valid anymore. But since the black holes that are going to be produced have a mass of order TeV and emitted particles will have an energy of order 10 GeV, the spectra will keep their validity. Furthermore the radiated particles can be seen as massless because their total energy will exceed their mass energy by far. Therefore all kinds of particles of the Standard Model (SM) will be produced with the same probability. [19] [5]

3.3.4 Evaporation rate and life time of a (mini-)black hole

The luminosity of a black hole is the energy it loses per second through radiation.

$$L = -\frac{dE}{dt} \tag{33}$$

The energy of a black hole of mass M has amount $E = Mc^2$, therefore:

$$L = -c^2 \frac{dM}{dt} \tag{34}$$

Combining with equation 24 we get the evaporation rate:

$$\frac{dM}{dt} \approx -\frac{1}{15\pi 8^3} \frac{c^4 \hbar}{G_{\mathrm{N}}^2 M_0^2} \tag{35}$$

where M_0 is the initial mass of the black hole at the time being investigated. As the black hole loses mass, its temperature (see equation 22) increases and so does $\frac{dM}{dt}$ which results in an even quicker decay.
Integration over t reveals an estimate τ for the lifetime of a black hole.

$$\tau \approx 15\pi 8^3 \frac{G_{\mathrm{N}}^2}{c^4 \hbar} M_0^3 \approx 10^{59} \left(\frac{M_0}{M_\odot} \right)^3 Gyr \tag{36}$$

For a mini-black hole or a black hole with small mass, say $M_{BH} \approx M_{\mathrm{Pl}}$, $\frac{dM}{dt}$ should be calculated by using the micro canonical ensemble, applying the laws of statistical mechanics (see [19]) since, using Hawking radiation and the Stefan-Boltzman law for black body radiation, $\frac{dM}{dt}$ goes to infinity as M_{BH} approaches

0. $\frac{dM}{dt}$ can be calculated using the canonical ensemble, too, see [19]. Then the variation of the mass is given by

$$\frac{dM_{BH}}{dt} = \frac{4\pi^3}{45}\frac{M_{BH}^2}{M_{\rm Pl}^4} exp[-4\pi(M_{BH}/M_{\rm Pl})^2] \int_0^{M_{BH}} (M_{BH}-x)^3 exp[4\pi(x/M_{\rm Pl})^2]\, dx \tag{37}$$

For further explanation and derivation of this formula see [19].

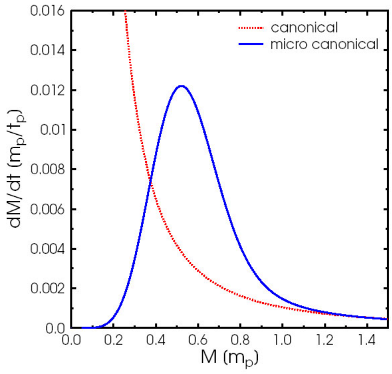


Figure 14: Canonical contra micro canonical. [19]

Figure 14 shows the canonical and micro canonical results. As we can see, the canonical evaporation rate diverges as M goes to 0 whereas the micro canonical drops back after having passed a maximum near $M_{BH} = M_{\rm Pl}$.

In (4 + n) dimensions we get for the evaporation rate (see [19]:

$$\frac{dM_{BH}}{dt} = \frac{\Omega_{3+d}^2}{(2\pi)^{d+3}} R_H^{2+d} \zeta(4+d) e^{-S(M)} \int_0^{M_{BH}} (M_{BH}-x)^{(3+d)} e^{S(x)}\, dx \tag{38}$$

where Ω_{d+3} is the surface of the (d + 3) dimensional unit sphere $\Omega_{d+3} = \frac{2\pi^{(\frac{d+3}{2})}}{\Gamma(\frac{d+3}{2})}$, $\zeta(4+d) = \sum_{j=1}^{\infty} \frac{1}{j^{d+4}}$ and $S(M) = 2\pi \frac{d+1}{d+2} (M_P R_H)^{(d+2)}$
Integration over t reveals a formula for the mass M as a function of t. We leave it out here and only give a plot of the function of the mass M for various d. [19]

3.3.5 black hole relics

It is not clear how the last stages of Hawking radiation would look like. If the black hole completely decays into statistically distributed particles, unitarity can be violated. To avoid the information loss problem two possibilities are left. Either some unknown mechanism will regain the information or the black hole will form a final stable remnant which keeps the information. One way to explain this remnant is the following. The spectrum of a black hole is quantized in discrete steps, only particles with a wave length that fits the horizon size can be emitted. Suppose now that the lowest energetic mode of such a particle exceeds the total energy of the black hole, then the remaining energy can't be emitted. Thus, there will always remain a remnant of the black hole.[19] [6].

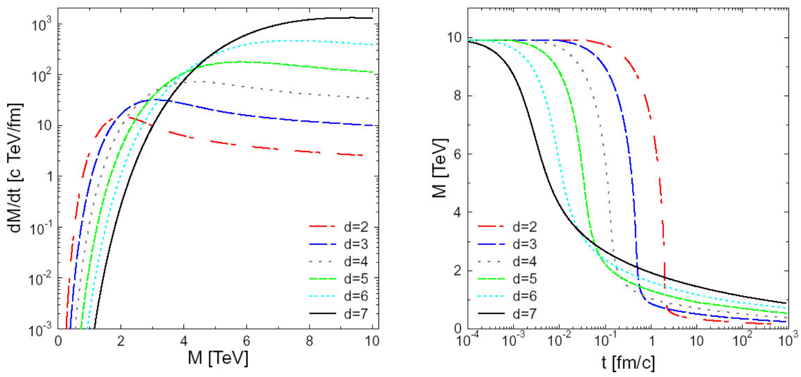


Figure 15: The left plot shows the evaporation rate as a function of the initial mass for various d, where d is the number of extra dimensions. The right plot shows the time evolution of a black hole with an initial mass of 10 TeV. [19]

3.4 Signatures of black hole decay

In this section we will give some spectacular experimental signatures associated with black hole production and decay.

3.4.1 Multiplicity

The number of decay products, which we will refer to as the multiplicity, is high compared to Standard Model processes [19]. The average multiplicity of particles produced in black hole evaporation is given by [11]

$$\begin{aligned} \langle N \rangle &= \left\langle \frac{M_{BH}}{E} \right\rangle \\ &= \frac{M_{BH}}{2T_H} \\ &= \frac{2\sqrt{\pi}}{n+1} \left(\frac{M_{BH}}{M_{4+n}} \right)^{\frac{n+2}{n+1}} \left(\frac{8\Gamma(\frac{n+3}{2})}{n+2} \right)^{\frac{1}{n+1}} \end{aligned} \tag{39}$$

Combining this equation with 28 shows us that the average number of particles produced in the process of black hole evaporation is of the order of S_H [14]. The particles have typical energies given by the Hawking temperature T_H, thus ranging over roughly 100 GeV - 1 TeV [13].

Figure 16 left shows the number of expected decay products of black hole events. For larger n, with a fixed mass, the temperature increases 27, which leads to evaporation into high energetic particles. Therefore the number of produced particles is relatively low and the multiplicity reduces. Figure 16 right shows the number of expected events with a certain event multiplicity for different n, for a black hole with $M_{BH} = 5$ TeV.

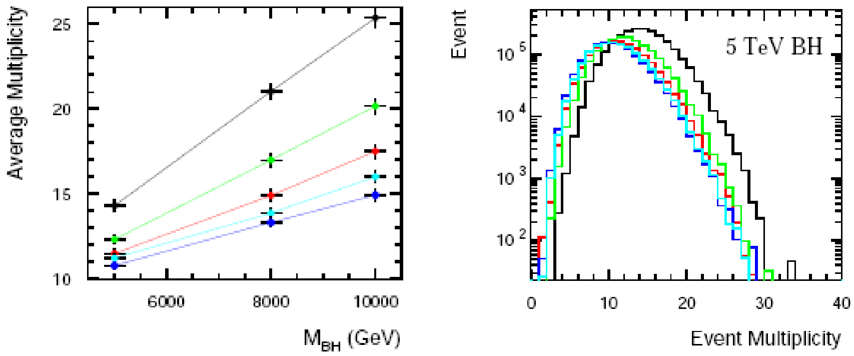


Figure 16: Left: The average multiplicity of black hole events for various numbers of extra dimensions n=2 (black), 3 (green), 4 (red), 5 (cyan), 6 (blue) [19]. Right: Typical multiplicity distribution for 100 fb^{-1} of integrated luminosity. [16]

3.4.2 Cut off high energy jets

In the standard model, we expect high energy collisions to be characterized by a large multiplicity of QCD[17] jets. The QCD-jets are created when one of the decay products of the particle collision is a color-charged particle (quark or gluon), since these particles can not exist isolated. Crossing the threshold for black hole production causes a sharp cut-off for QCD jets as those jets now end up as black holes instead. [19] [8]

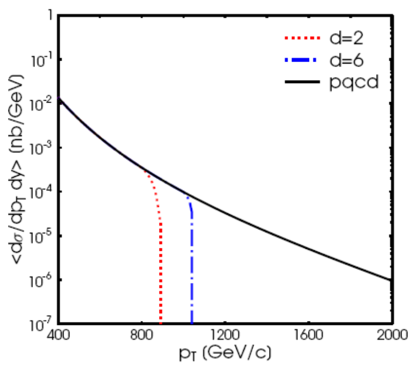


Figure 17: Cross-section for jets at high transverse momentum. $\sqrt{s} = 14$ TeV, $M_{4+n} = 1$ TeV. NB: d = n = number of extra dimensions [19]

The expected cross section for pertubative QCD jets is shown in figure 17. If there are extra dimensions the cross section drops at a certain energy. The cut-off is close to the (4+n)-dimensional plankmass but it's hardly sensitive to the number of extra dimensions. This cut-off is been considered as a smoking gun signature for black hole production.

[17] QCD stands for Quantum Chromo Dynamics

3.4.3 Sphericity and transverse energy of black hole events

High energy collisions normally are characterized by jets. When a black hole is created, is will evaporate by hawking radiation. Most of the hawking radiation is isotropic in the black hole's rest frame, due to thermal properties [19] [13]. Therefore these events would have a high sphericity, in contrast to standard model processes which are not spherical at all. Even the low multiplicity events tend to be rather spherical.
Figure 18 shows the expected sphericity of black hole events. For higher dimensions, the events become significantly less spherical [16].

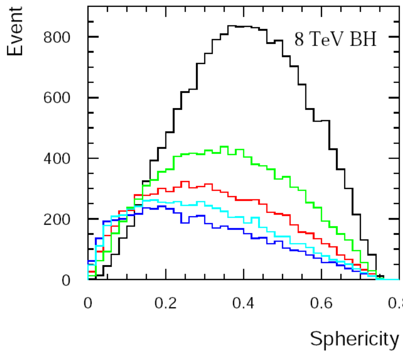


Figure 18: A typical set of expected distributions of the sphericity for 8 TeV black holes for n = 2 (black), 3 (green), 4 (red), 5 (cyan), 6 (blue). [16]

Due to the high sphericity a large component of the energy is in the transverse direction. The total visible transverse energy of a typical black hole event ($\sum p_T$) is between $\frac{1}{3}$ and $\frac{1}{2}$ of the total visible energy. The transverse energy increases as the black hole mass increases. See figure 19. [16] [15]

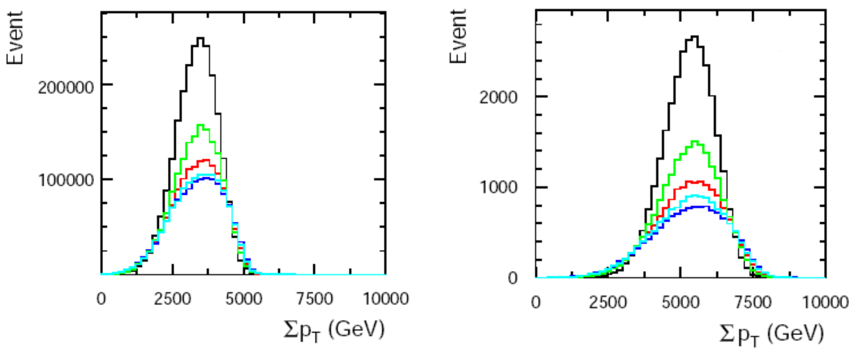


Figure 19: The distribution of $\sum p_T$ for various numbers of extra dimensions n=2 (black), 3 (green), 4 (red), 5 (cyan), 6 (blue), for 100 fb^{-1} of integrated luminosity. [16] Left: M_{BH} = 5 TeV. Right: M_{BH} = 8 TeV. [16]

3.4.4 High missing transverse energy

As a black hole decays, neutrinos can be produced. This will not always happen, but if it does, the neutrinos will have energies that can be as high as half the black hole mass. The energy of these neutrinos is considered as missing energy, $\not{p}_T$. Most standard model processes tend to have much lower missing transverse energy. Figure 20 left shows the distribution of transverse missing energy of standard model events, super symmetry events and black hole evaporation events in 2 and 6 extra dimensions. As we can see from the figure the transverse missing energy for black hole evaporation is much greater than for the other processes. A high missing transverse energy will be a clear signal for black hole production. [16]

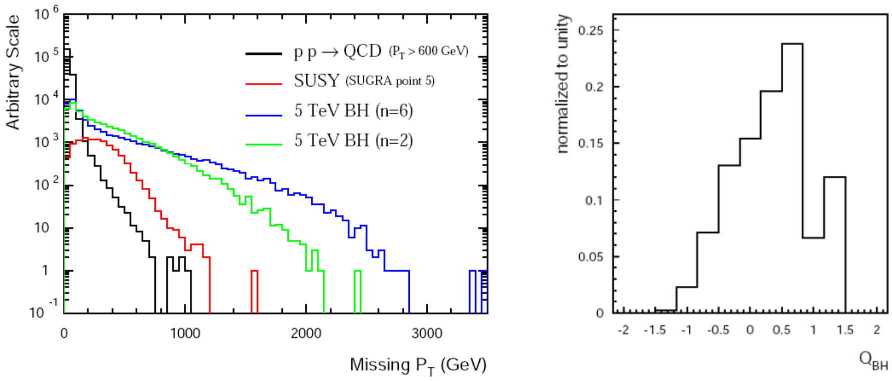


Figure 20: Left: The distribution of $\not{p}_T$. [16] Right: Distribution of the charge of a black hole. $M_{4+n} = 1$ TeV, n = 3. [16]

3.4.5 Black hole charge

Black holes are charged objects because they are formed of charged valence quarks. The charge of the black hole should be $\sim +\frac{2}{3}$ electron charge (e) and has a little energy-dependence. A black hole charge distribution is given by figure 20 right [23]. The remaining of the charge of the colliding protons, $\frac{4}{3}$ e, will disappear down the beam pipes.
To determine the black hole charge, $\langle Q_{BH} \rangle$, one has to measure the average charge of the leptons emitted by the black hole, $\langle Q_{lep} \rangle$. This charge should equal the black hole charge times a probability of emitting a charged lepton. [16]
There are two reasons to use leptons for the charge determination instead of other charged particles. First, the average lepton charge can be measured unambiguously. Second, because of the black hole thermal spectrum the light lepton are produced more often than other charged particles are. [3]

3.4.6 Ratio hadronic leptonic particles

A black hole decays roughly with equal probability to all 60 particles of the standard model. [14] [13], due to its thermal properties. The ratios of particles change a bit due to fact that the particles interact with each other, As well as the fact that the primary particles decay. The ratio of hadronic and leptonic activities we will eventually measure is roughly 5:1. This will be a clear signal of black hole decay [15].

4 Conclusion

Within a few years particle physics will possibly make a big breakthrough. Future colliders are going to be able to give particles enough energy to reveal new physics: mini black holes as well as Kaluza Klein gravitons might be produced and perhaps even detected at these colliders. Whether or not this is going to happen will decide if extra dimensions, as proposed in the ADD model, exist within our universe. The existence of these extra spatial dimensions will mean the end of short distance physics due to the creation of mini black holes at high energy colliders. On the other hand, it will also mean the beginning of the exploration of the geometry of the extra spatial dimensions.

Acknowledgments

We would like thank Jan de Boer, Nick Jones and Asad Naqvi for their effort, assistance and guidance.

References

[1] `http://www.eurekalert.org/features/doe/2003-10/dnal-ftl020904.php`.

[2] `http://www-cdf.fnal.gov/PES/kkgrav/kkgrav.html`.

[3] J. de Boer, N. Jones, A. Naqvi; private communication.

[4] `http://scienceworld.wolfram.com/physics/Stefan-BoltzmannLaw.html` and `http://scienceworld.wolfram.com/physics/PlanckLaw.html`.

[5] `http://library.thinkquest.org/C007571/english/advance/english.htm`.

[6] S. Alexeyev, A. Barrau, G. Boudoul, O. Khovanskaya, and M. Sazhin. Black hole relics in string gravity: Last stages of hawking radiation. *arXiv:hep-th/9906038*, January 2002.

[7] Nima Arkani-Hamed, Sava Dimopoulos, and Gia Dvali. The hierarchy problem and new dimensions at a millimeter. *arXiv:hep-ph/9803315*, March 1998.

[8] Tom Banks and Willy Fischler. A model for high energy scattering in quantum gravity. *arXiv:hep-th/9906038*, June 1999.

[9] Aurlien Barrau and Julien Grain. The case for mini black holes. *CERN courier*, November 2004.

[10] Bernard J. Carr and B. Giddings. Quantum black holes. *Scientific American*, April 2005.

[11] Savas Dimopoulos and Greg Landsberg. Black holes at the lhc. *arXiv:hep-ph/0412265*, June 2001.

[12] O.J.P. Eboli, M.B. Magro, P. Mathews, and P.G. Mercandante. Direct signals for large extra dimensions in the production of fermion pairs at linear colliders. *Physical Review D*, 64, July 2001.

[13] Steven B. Giddings. Black holes at accelerators. *arXiv:hep-th/0205027*, May 2002.

[14] Steven B. Giddings. Black holes in the lab? *arXiv:hep-th/0205205*, May 2002.

[15] Steven B. Giddings and Scott Thomas. High energy colliders as black hole factories: The end of short distance physics. *arXiv:hep-ph/0106219*, June 2002.

[16] C.M Harris, M.J. Palmer, M.A. Parker, P. Richardson, A. Sabetfakhri, and B.R. Webber. Exploring higher dimensional black holes at the large hadron collider. *arXiv:hep-ph/0411022*, November 2004.

[17] Steven W. Hawking. *The Universe in a Nutshell.* Bantan books, 2001.

[18] Stefan Hofmann, Marcus Bleicher, Lars Gerland, Sabine Hossenfelder, Sascha Schwabe, and Horst Stcker. Suppression of high-p_t jets as a signal for large extra dimensions and new estimates of lifetime for meta sable micro black holes - from the early universe to future colliders. *arXiv:hep-ph/0111052 v2.*

[19] Sabine Hosenfelder. What black holes can teach us. *arXiv:hep-ph/0412265*, December 2004.

[20] Panagiota Kanti. Black holes in theories with large extra dimensions: a review. *arXiv:hep-ph/0402168*, December 2004.

[21] G. Landsberg. Collider searches for extra dimensions. *arXiv:hep-ex/0412028*, December 2004.

[22] Daniel V. Schroeder. *An introduction to thermal physics.* Addison Wesley Longman Inc., first edition, 2000.

[23] J. Tanaka, T. Yamamura, S. Asai, and J. Kanzaki. Study of black holes with the atlas detector at het lhc. *arXiv:hep-ph/0411095*, November 2004.