Large extra dimensions

Anna Gimbrère*

August 27, 2010

Supervisor: Prof. Jan de Boer[1]

Abstract

The ADD scenario proposes that our world exists on a three-dimensional slice through a higher-dimensional universe. It tries to solve the hierarchyproblem (the question why gravity is so much weaker than the other forces in nature) by assuming that only gravity can propagate through the extra dimensions, thereby spreading its power over extra space. The model predicts potentially observable effects in astrophysical experiments and these can be used to derive bounds on the size of the extra dimensions and the higher-dimensional Planck scale. Supernova observations have provided some of the strongest bounds on the parameters. This report describes the ADD scenario and its physical implications for the purpose of considering the most important constraints on this model derived from supernova observations.

*Bachelor project for the bachelor physics & astrophysics (12 EC), student no. 0513652
[1]Institute for Theoretical Physics, University of Amsterdam

Layman summary

The concept of extra dimensions may sound like science fiction, but it has been explored in many fields of science and for many centuries, probably ever since the time Greek philosophers started thinking outside the box. At this moment the idea that extra dimensions may exist seems to be widely accepted in physics, even though real evidence has yet to be found.

We are used to describe our world in 4-dimensional space-time, using the space-time coordinates to specify 'where and when' a specific event took place. These space-time coordinates were introduced in Einstein's theory of relativity and they are denoted

$$(x, y, z, ct),$$

where x, y, z are the three spatial coordinates and ct is the coordinate of time multiplied by the speed of light, c, in order for all coordinates to have units of length. No other dimensions have ever been observed, but some physicists believe that more than three spatial dimensions exist. These extra dimensions would be too small to observe, but they can help to solve problems in physics, which can not be solved (easily) in 4-dimensional space-time.

One such problem is the so called *hierarchy problem*, which addresses the question: *"Why is the force of gravity so much weaker than the other forces in nature?"*. Many theories have tried to solve this problem by introducing extra spatial dimensions and in this paper I consider one of these theories: the *ADD scenario*.

The ADD scenario was proposed by three physicists named Arkani-Hamed, Dimopoulos and Dvali (ADD) in 1998. It is based on the assumption that our world is confined to a 4-dimensional subspace within a $(4+n)$-dimensional universe. The 4-dimensional subspace is referred to as a *3-brane*, because it can be thought of as a membrane with 3 spatial dimensions. The rest of the universe is referred to as the *bulk*. To solve the hierarchy problem, the ADD scenario suggests that only gravity can propagate through the bulk, while all the other forces are confined to the 3-brane. This way, gravity spreads its power over a larger space than the other forces, such that it seems weaker than the other forces when we measure it in our 3-dimensional world. The extra dimensions in this model are often referred to as "gravity only dimensions" (GOD's).

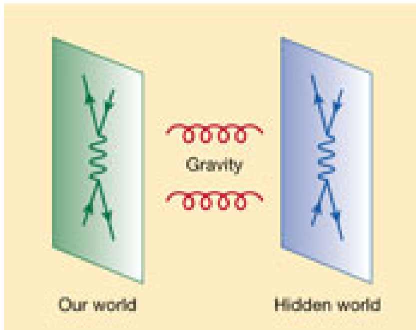

Figure 1: Our world on a brane, source: http://www.nature.com/nature/journal

The ADD scenario expects the extra dimensions to be curled up on very small circles of radius $R \lesssim 1mm$ and it suggests deviations from Newton's gravitational force law ($F = G\frac{m_1 m_2}{r^2}$) at distances smaller than R. The behavior of gravity has been tested in *short range gravity experiments*, but to date no deviations from Newton's law have been found for any distance larger than $\sim 40\mu m$. This means that, if GOD's exist, they have to be at least smaller than $40\mu m$.

Another prediction of the ADD scenario is that gravitational energy could escape into the extra dimensions when high amounts of energy are available. This could be observed as energyloss in high energy experiments, in particular in observations on stars and supernovae. Stars and supernovae have been observerd in astrophysical experiments and the measured energyfluxes have been used to put constraints on the size of the extra dimensions.

To date, the ADD scenario has not been proven right or wrong directly, but it has definatly led to some new ideas and excitement in the field of extra dimensional physics.

Contents

1 Introduction

Extra dimensions have been introduced in physical theories to solve problems that could not be solved with lower dimensional theories, especially in attempts to unify the forces in nature. In this report I will consider the so-called ADD-scenario which was proposed by the physicists named Arkani-Hamed, Dimopoulos and Dvali (ADD) in 1998. This theory uses the concept of extra dimensions to solve the *hierarchy problem,* which adresses the question:

"Why is gravity so much weaker than the other forces in nature?"

To properly introduce the ADD scenario I will give a short overview of the history and theoretical background of the model.

History and background

In 1919 one of the first and most important higher dimensional models was invented: the Kaluza-Klein model. It was originally invented by the German mathematician Theodor Kaluza in an attempt to unify general relativity and electromagnetism. He noticed that the generalization of Einstein's theory to 5 space-time dimensions (1 extra spatial dimension) could simultaneously describe the gravitational and electromagnetic fields in 4-dimensional space-time. To make this work, he had to assume that all fields are independent of the extra spatial dimension, but he could not explain this odd property.

The explanation was given by the Swedish mathematician Oscar Klein in 1926. He suggested that the extra dimension could be compactified on a circle of radius R, where R was expected to be of the size of the Planck length $L_{Pl} \sim 10^{-35}m$, too small to have ever been detected. By these means the extra dimension is in fact curled up such that it has limited length $L = 2\pi R$ and space is periodic with period L in this direction. In other words, the extra dimension has the geometry of a circle: S^1 and space is factorized into $M^4 \times S^1$, where M^4 is the geometrie of the 4-dimensional Minkowski space.

The idea of compactification and hidden dimensions can be illustrated by imagining a gardenhose. From a large distance a gardenhose looks like a line; a one-dimensional object. Taking a closer look however, one will see that the gardenhose in itself is actually two-dimensional: a curled up surface. Clearly this 'extra' dimension was already there, yet it was 'hidden' from us when looking from a greater distance.

The Kaluza Klein model failed as a complete theory in its original purpose due to several internal inconsistencies. Besides, no realistic applications for the model existed at that time. For these reasons, the Kaluza Klein model was essentially abondoned until the 1970/80's.

In this period supergravity and superstring theory were developed and in these theories, higher dimensions arise naturally. Moreover, it was demonstrated that a Kaluza Klein model with 6 extra dimensions, compactified on a 6 dimensional manifold (the Calabi-Yai manifold), could include features of the Standard model [2].

During the same period, new theories about the idea of localizing matter

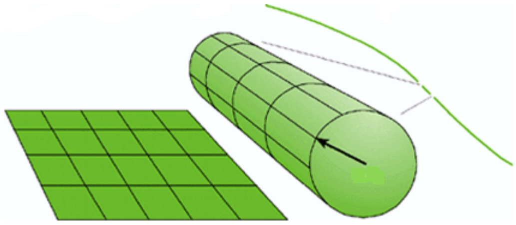

Figure 2: Hidden dimensions in Kaluza Klein compactification, source: www.physicsworld.com

on topological defects were developed. The essence of this idea is that our world could be confined to a 4-dimensional subspace within a higher-dimensional universe. All matter and all fields that we observe are trapped on the surface of a domain wall and we are therefore not able to observe the extra dimensions [3, 4].

While trying to describe our world on a domain wall, physicists found that there is an important distinction between the force of gravity and the gauge fields from the standard model (SM). It proved to be relatively easy to find field-theoretical mechanisms which ensured the localization of the SM fields on a domain wall, but for the gravitational field these mechanisms could not be found. The distinction can also be shown by the string-theoretical approach. In string theory the gauge fields from the SM are usually represented by open strings, of which he endpoints are attached to a brane: they are trapped on the brane. Gravity on the other hand is represented by a closed string which is not bound to any brane and is free to propagate through extra dimensions.

The ADD model In 1998 three scientists named Arkani-Hamed, Dimopoulos and Dvali (ADD) combined the Kaluza-Klein model with ideas from string theory, in particular the existence of branes. Their model [5, 6, 7] starts from the assumption that our universe consists of $D = (4+n)$ dimensions, where $n \geq 2$ dimensions are compactified on a n-dimensional torus as in the Kaluza Klein theory and space-time geometry is factorized:

$$R^{4+n} = M^4 \times S^n. \tag{1}$$

Within this framework, all matter and the SM fields are confined to a (3+1)-dimensional brane, called a 3-brane [2] and only gravity is able to propagate through the extra dimensions. In this way, gravity spreads its power over a higher dimensional space and that explains why gravity is so weak compared to the other forces in nature.

[2] in general we speak of a p-branes, which contain (p+1) space-time dimensions

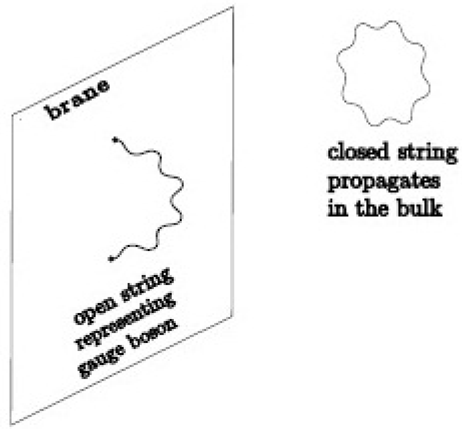

Figure 3: Illustration of string theory: the open string is confined to a brane and the closed string is free to propagate through bulk. Source: [4]

The ADD scenario suggests that a modification of Newton's gravitational force law will be observable at distances of the order of the compactification scale L, which in the ADD model is expected to be much larger than the Planck length. In fact, the original theory suggests that L could be in the range of 1 mm, for $n = 2$. This means that Newton's law would be modified at distances smaller than a millimetre. A funny coincidence at that time was, that Newton's law had only been verified for distances larger than 1 mm and it was expected to be measured at smaller distances in the near future. If the ADD scenario would be correct, these measurements could deliver the proof of the existence of extra dimensions by showing that Newton's law breaks down at these lengthscales.

Obviously, these predictions led to great excitement and popularity of the ADD scenario among experimental physicists. New experiments were designed to find signatures of large extra dimensions and existing data from observations in particle- and astrophysics were used to put constraints on the ADD theory.

In this report I consider the theory and phenomenology of extra dimensions to make clear how constraints on the ADD model can be derived from astrophysical experiments. I will start by considering the most important features of gravity and compactification used in extra dimensional theories. In the next section I will show how constraints on extra dimensions arise from astro-particle experiments and finally I will compare several approaches which have been used by different authors to derive constraints from supernova observations.

2 Theory

In the ADD model the gravitational field is the only field that propagates through the extra dimensions. It is therefore useful to understand how gravity can be described in higher dimenisonal spaces.

2.1 Gravitation

2.1.1 Gravitation in infinitely large extra dimensions

In 4-dimensional space-time the gravitational force between two point masses m_1 and m_2, separated by a distance r, is given by Newton's famous gravitational force law

$$F_{(4)}(r) = G_N \frac{m_1 m_2}{r^2}, \tag{2}$$

where G_N is Newton's gravitational constant. Accordingly the gravitational field at a distance r from a point mass M (in 4-dimensional space-time) is defined:

$$g_{(4)}(r) = -G_N \frac{M}{r^2}. \tag{3}$$

Comparing this field to the electric field (in the Heaviside-Lorentz system):

$$E(r) = \frac{\mathrm{Q}}{4\pi r^2} \tag{4}$$

shows that both fields fall of with $\frac{1}{r^2}$, and the two can be equated for $Q = -4\pi G_N M$. Now, from Gauss' law we know that the flux of the electric field through a closed surface is always equal to the enclosed charge Q:

$$\int \overrightarrow{E} \cdot d\overrightarrow{a} = Q \tag{5}$$

Equivalently, for the gravitational field it follows that:

$$\int \overrightarrow{g_{(4)}} \cdot d\overrightarrow{a} = -4\pi G_N M, \tag{6}$$

where M is now the enclosed mass. This equation holds for *any* closed surface, but since g(r) is spherically symmetric, it is most convenient to choose a spherical surface-integral. In that case the integral on the left side of the equation will always be equal to g(r) times the volume of the spherical shell around the point mass. In 4 space-time dimensions this tells us that:

$$vol([S(r)]) \cdot g(r) = -4\pi G_N M \tag{7}$$

where $vol[S(r)]$=$4\pi r^2$ is the volume of a 2 dimensional spherical shell with radius r (*see appendix A).* We can check the gravitional potential:

$$g(r) = \frac{-4\pi G_N M}{vol[S^2(r)]} = -\frac{G_N M}{r^2}. \tag{8}$$

Now, assuming that the gravitational potential around a point mass M is isotropic in all directions, we can generalize this formula to arbitrary dimensions. It is herefore important to note the difference between the number of space-time dimensions D and the number of spatial dimensions d. I define $D = d + 1$.

Assuming that the gravitational field is spherically symmetric in any number of spatial dimensions, the surface integral around a point mass M in D space-time dimensions is given by:

$$\int_{S^{d-1}} \overrightarrow{g_D} \cdot d\,\overrightarrow{a} = -4\pi G_N M \tag{9}$$

$$vol[S^{d-1}(r)] \cdot g(r) = -4\pi G_N M \tag{10}$$

where the factor 4π is just convention and $vol[S^n(r)] = r^n vol[S^n]$ is the volume of a n-dimensional spherical shell. The gravitational field in D dimensions is then given by:

$$g^{(D)}(r) = \frac{-4\pi G_N M}{r^{d-1} vol[S^{d-1}]} = G^{(D)} \frac{-M}{r^{d-1}} \tag{11}$$

where I have defined $G^{(D)}$, the gravitational constant in D dimensions, to be:

$$G^{(D)} = \frac{4\pi G_N}{vol[S^{d-1}]} \tag{12}$$

and Newton's law in arbitrary dimensions follows easily:

$$F^{(D)}(r) = \frac{G^{(D)} m_1 m_2}{r^{d-1}}. \tag{13}$$

This formula shows that in D space-time dimensions, the gravitational field should fall off with $\frac{1}{r^{d-1}} = \frac{1}{r^{D-2}}$. For instance, in the case of 1 extra dimension, gravity should fall off with $\frac{1}{r^{5-2}} = \frac{1}{r^3}$. However, experiments have shown that gravity obeys Newton's law (i.e. falls off with $\frac{1}{r^2}$) in the range of $10^{-4}m$ - $10^{20}m$.

Obviously, this tells us that if extra dimensions exist and gravity is able to propagate through these extra dimensions, they can not be infinitely large. If the extra dimensions are compactified however, the problem can be avoided.

2.1.2 Gravitation in spaces with compactified extra dimensions

When a dimension is compactified on a circle of radius R, all fields will have period $2\pi R$ in this direction. The gravitational field from a pointmass M will start out isotropically in all directions, but for distances $r > 2\pi R$, the spherical symmetry is broken and the fieldlines can only travel further along the ordinary (infinitely long) dimensions. From that point, the field does not depend on the extra dimension anymore and it behaves as if it propagates through only 3 spatial dimensions. Therefore one will measure the field falling of with approximately $\frac{1}{r^2}$ for any distance $r > 2\pi R$. Fig. 3 shows how fieldlines propagate through a compactified dimension.

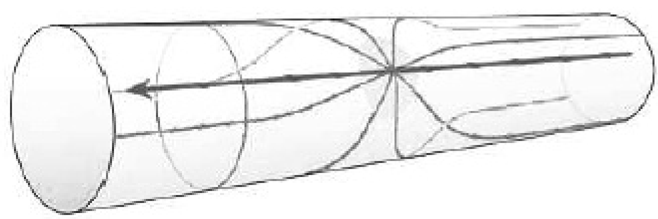

Figure 4: fieldlines in compactified dimension. source: [1]

From this picture we can see how the field behaves in the short- and long distance limits. For simplicity I will assume that all extra dimensions are compactified with the same size $L = 2\pi R$. For short distances ($r \ll L$), gravity propagates isotropically in all directions such that the gravitational force can be described by:

$$\overrightarrow{F}^{(D)}(r) = \frac{G^D m_1 m_2}{r^{d-1}} = \frac{G^{(4+n)} m_1 m_2}{r^{n+2}} \qquad r \ll L. \tag{14}$$

For distances $r \gg L$, when the fieldlines can only propagate along the ordinary spatial dimensions, gravity will behave like:

$$\overrightarrow{F}^{(D)}(r) = \frac{G^{(4+n)} m_1 m_2}{L^n r^2} \qquad r \gg L \tag{15}$$

Again, these formulas are just approximations of the short- and long distance limit of the general field. To find a general expression for the gravitational attraction between two pointmasses m_1 and m_2 in $D = 4 + n$ dimensions, we can use the *method of images*. This method approaches the compactified extra dimension, as if it were unrolled and has a point mass at every $2\pi Ra$ for all

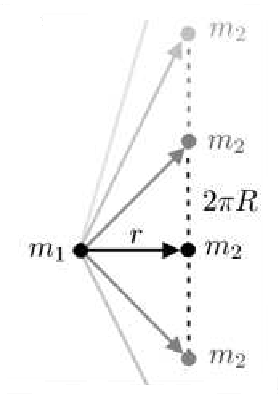

Figure 5: method of images applied on one compactified dimension: inifinite series of periodically spaced point masses.

integer $a \in \mathbb{Z}$. By these means, we automatically implement the periodicity of the field in the extra dimension: the extra dimension seems to contain an infinite number of pointmasses, which are periodically spread over the "length" of the extra dimension (see figure 4).

Adding the contributions of each mass to the field, we find that the vertical components cancel each other and what remains is a force in the $\vec{r}$-direction of the form:

$$F = \sum_{a_1=-\infty}^{\infty} \dots \sum_{a_n=-\infty}^{\infty} G^{(4+n)} \frac{m_1 m_2}{(r^2 + \sum_{i=1}^{n} (2\pi R a_i)^2)^{\frac{n+2}{2}}} \frac{r}{\sqrt{r^2 + \sum_{i=1}^{n} (2\pi R a_i)^2}}, \tag{16}$$

where r is the 'regular' distance from the pointmass $\sqrt{x^2 + y^2 + z^2}$ and the last term makes sure that only the horizontal components of the field contribute to the force. From this formula, one can describe the field at any point in space and also derive an expression for the short- and long distance limits. For the purposes of this paper, I will only use the limits: (14) & (15) .

2.2 The Planck scale

In the study of gravitation it is convenient to use the Planckian system of units. In this system, the physical units of measurement are expressed in terms of the universal physical constants, such that the fundamental constants can be set to the numerical value of *one*.

2.2.1 Planck units and the Planck scale 4-D

One can derive the Planckian system of units starting from the values of the fundamental constants[3]:

$$G \backsimeq 6.67 \times 10^{-11} \frac{m^3}{kg \cdot s^2} \rightarrow [G^{(4)}] = \frac{L^3}{MT^2}$$

$$\hbar \backsimeq 1.06 \times 10^{-34} \frac{m^2 \cdot kg}{s} \rightarrow [\hbar] = \frac{L^2 M}{T}$$

$$c \backsimeq 3 \times 10^8 \frac{m}{s} \rightarrow [c] = \frac{L}{T}$$

where L, M and T stand for the units of length, mass and time, respectively. The Planck units are found by setting the numerical value of the fundamental constants to *one*. This leads to a set of three equations, which can be solved for the new units of length, time and mass. The corresponding units are called the Plancklength, -mass and -time and are denoted by: ℓ_{Pl}, m_{Pl} and t_{Pl} respectively.

$$G = 1 \times \frac{\ell_{Pl}^3}{m_{Pl} \cdot t_{Pl}^2}, \qquad \hbar = 1 \times \frac{\ell_{Pl}^2 m_{Pl}}{t_{Pl}}, \qquad and \quad c = 1 \times \frac{\ell_{pl}}{t_{Pl}} \tag{17}$$

an solving for ℓ_{Pl}, m_{Pl} and t_{Pl} gives:

$$\ell_{Pl} = \sqrt{\frac{G\hbar}{c^3}} \backsimeq 1.61 \times 10^{-35} m \tag{18}$$

$$m_{Pl} = \sqrt{\frac{\hbar c}{G}} \backsimeq 2.17 \times 10^{-8} kg \tag{19}$$

$$t_{Pl} = \sqrt{\frac{\hbar G}{c^5}} \backsimeq 5.4 \times 10^{-44} s. \tag{20}$$

Many other physical entities can be expressed in terms of these units. The most important one being the energy scale, or the so-called Planck scale [4]:

$$M_{Pl} = m_{Pl} c^2 \backsimeq 1.22 \times 10^{16} TeV. \tag{21}$$

Obviously this energy scale is very high. In fact, it is completely out of reach under terrestrial circumstances and therefore impossible to experiment with. Currently, the highest energy values have been reached at LHC and are about 7 TeV, which is high, but not even close to 10^{16}TeV.

---

[3]In the complete Planckian system of units also the coulomb constant $(4\pi\varepsilon_o)^{-1}$ and Boltzmann's constant k_B are taken into account, but these are not important for our purposes.

[4](Voetnoot: I use eV as the unit for energy because this is very common and convenient in particle physics. It can also be used as a unit for length, setting $c = \hbar = 1$.

2.3 The ADD scenario

Large extra dimensions as a solution to the hierarchy problem

The Planck scale $M_{Pl} \sim G^{-\frac{1}{2}} \sim 10^{16}TeV$ (setting $c = \hbar = 1$) is the fundamental energy scale for gravitational interactions, but it is not the only fundamental scale in physics. There is (at least) one other important energy scale, being the energy scale of the standard model, the electroweak scale: $M_{EW} \sim 1TeV$. Experiments with particle colliders have revealed that electromagnetism and the weak nuclear force are unified at this energy and the SM has been adjusted to fit this scale.

There is a large hierarchy between the Planck scale and the electroweak scale: $M_{Pl}/M_{EW} \sim 10^{16}$ and this is strange, because it is expected that the forces were all once united in the early beginning of the universe. To understand how this could have ever been possible, it would be helpful to understand where the large hierarchy comes from. In other words, we want to know why the force of gravity is so much weaker than the standard model interactions. This is called the *hierarchy problem*.

In 1998, the ADD scenario gave a new perspective on the hierarchy problem by suggesting that the higher-dimensional Planck scale could be equal to the electroweak scale M_{EW}. In fact, the ADD scenario is based on the philosophy that the electroweak scale should set the scale for all short distance interactions, including that of gravity. The idea behind this statement is that the electroweak scale is an experimental certainty: the SM interactions have been probed at distances of the order of M_{EW}^{-1}, whereas gravity on the other hand, has not remotely been probed at distances of the order of M_{Pl}^{-1} [5, 6, 7].

Within this framework, the 'observed' Planck scale M_{Pl} is just our 4-dimensional perception of the higher-dimensional (fundamental) Planck scale $M_{Pl(D)} = M_{EW}$. The fact that we observe the 4-dimensional Planck scale to be so big, is the effect of the existence of $n \geq 2$ compactified dimensions, which are large compared to the Planck length. By these means, the apparent weakness of gravity is explained by the idea that gravity spreads its power over more than 4 dimensions, while all the other forces are confined to a 3-brane.

2.3.1 ADD theory

The ADD scenario does not solve the hierarchy problem completely. It leaves us with a new parameter, being the compactification scale L. L becomes a new variable which is related to the higher dimensional Planck scale and the number of extra dimensions n.

The higher dimensional Planck scale We can find a general expression for the higher dimensional Planck scale, $M_{Pl(D)}$, in $D(= 4 + n)$ dimensions for which n dimensions are compactified on a circle. Again, for simplicity I will assume that all extra dimensions have the same size $L = 2\pi R$ such that they form a torus of volume $L^n = (2\pi R)^n$. Also I will use the convention $c = \hbar = 1$, such that the Planck scale can be expressed $M_{Pl} = \sqrt{G}$.

number of extra dimensions n	2	3	4	5	6	7
Compactification scale L in m	10^{-3}	10^{-8}	10^{-11}	10^{-13}	10^{-14}	10^{-14}

Table 1: compactification scale L for different numbers of n and $M_{Pl(D)} \backsimeq 1TeV$

I start from Newton's law in D dimensions, invoking formula (13) and using the relation between the Planck scale and the gravitational potential to find:

$$F^{(D)}(r) = G^D \frac{m_1 m_2}{r^{n+2}} \sim \frac{1}{M_{Pl(D)}^{n+2}} \frac{m_1 m_2}{r^{n+2}},$$

where $M_{Pl(D)}^{n+2}$ is defined to be the *fundamental Planck scale* in D dimension

s. From equation (14) and (15) it follows that:

$$F^{(D)}(r) = \begin{cases} \frac{m_1 m_2}{M_{Pl(D)}^{n+2} \cdot r^{n+2}} & for:\ r < L \\ \frac{m_1 m_2}{M_{Pl(D)}^{n+2} \cdot L^n r^2} & for:\ r > L \end{cases} \tag{22}$$

From this we can see the relation between the 4-dimensional planck scale and the D-dimensional Planck scale:

$$M_{Pl}^2 \sim M_{Pl(D)}^{n+2} L^n. \tag{23}$$

Now letting $M_{Pl(D)}^n$ $=M_{EW} \sim 1TeV$, and requiring L to be chosen such that it reproduces the observed 4-dimensional Planckscale $M_{Pl} \sim 10^{16}TeV$:

$$L \sim M_{Pl(D)} \left(\frac{M_{Pl}}{M_{Pl(D)}} \right)^{\frac{2}{n}} \sim 10^{32/n} TeV^{-1} \sim 10^{32/n - 19} m. \tag{24}$$

According to the ADD model only gravity propagates through the extra dimensions, so it should be possible to observe deviations from Newtonian gravity at distances of the order of the compactification scale L. Thus, using different values for n, we can compare theory with experiment. Letting $n = 1$, we find that $L \sim 10^{13}$m. This implies modifications of Newtonian gravity at distances of the scale of the solar system. Obviously this option is to be excluded. For other values of n, the results for L are shown in table 1.

In 1998, when the ADD model was first proposed, the shortest distance at which gravity had been measured was about 1 millimetre. Therefore, there was still a possibility that gravity propagates through higher dimensions at distances smaller than 1 millimetre; $n = 2$ was not yet excluded.

The SM fields on the other hand, had already been extensively tested up to distances of the order of $TeV^{-1} \sim 10^{-18}m$. This means that (if large extra dimensions exist) apparently the SM fields do not propagate through these extra dimensions for $L > TeV^{-1}$. They have to be confined to a 4-dimensional brane within the D-dimensional bulk.

2.4 Direct constraints on LED's

The ADD scenario predicts modifications of Newton's inverse square law at distances smaller than L, this means that the theory can be tested directly by probing gravity at short distances.

There are only few methods to measure the short range behavior of gravity, most of which use a so-called torsion pendulum. [8, 9] In general, these experiments search for deviations of the Newtonian potential at very short distances. Since the 1980's many groups over the world have been working on ever-more-sensitive tests of the inverse-square law at short distances and ever stronger bounds were derived on the compactification scale L.

Short range gravity tests

As long as the inverse-square law holds, the gravitational potential for a pair of point masses can be written as:

$$V = -G\frac{m_1 m_2}{r}.$$

Generally researchers look for a new force that violates this law with a characteristic length scale λ. This is done by looking for a potential of the form:

$$V = -G_{(4)}\frac{m_1 m_2}{r}\left(1 + \alpha \cdot e^{-\frac{r}{\lambda}}\right) \tag{25}$$

where α is a dimensionless scaling factor which corresponds to the strength of the new interaction relative to Newtonian gravity and λ is a characteristic length scale. A potential of this form is called a 'Yukawa potential' and it generally describes a short-range force that will be carried by a particle with a mass of the order of $\sim \frac{\hbar}{c\lambda}$. In the case of the LED scenario the Yukawa potential can be used as an approximation to the effects of extra dimensions, by considering the effects of a higher-dimensional particle carrying the force of gravity.

In the next section I will show that such a particle is referred to as a 'Kaluza Klein' graviton and that it carries a mass inversely proportional to the radius of the extra dimension: $m_{KK} \sim 1/R$. When one such particle travels into the bulk, another one has to be emitted in opposite direction, to conserve momentum. From this it follows that the strength of the interaction can be estimated to be α ~2n (the number of particles leaving into one extra dimension) and λ is relative to the inverse mass of the graviton: $\lambda \simeq R$.

Figure 4 shows the progress made by several experiments in constraining the Yukawa potential parameter space. In the yellow region, new forces have been ruled out by experiments at the 95% confidence level (black lines).

The different experiments probe different length scales. The Stanford experiment explores the shortest lengths, while the Irvine experiment has set the lowest limits to $|\alpha|$, but probes larger distances. For a more detailed discussion of the various experiments see [9].

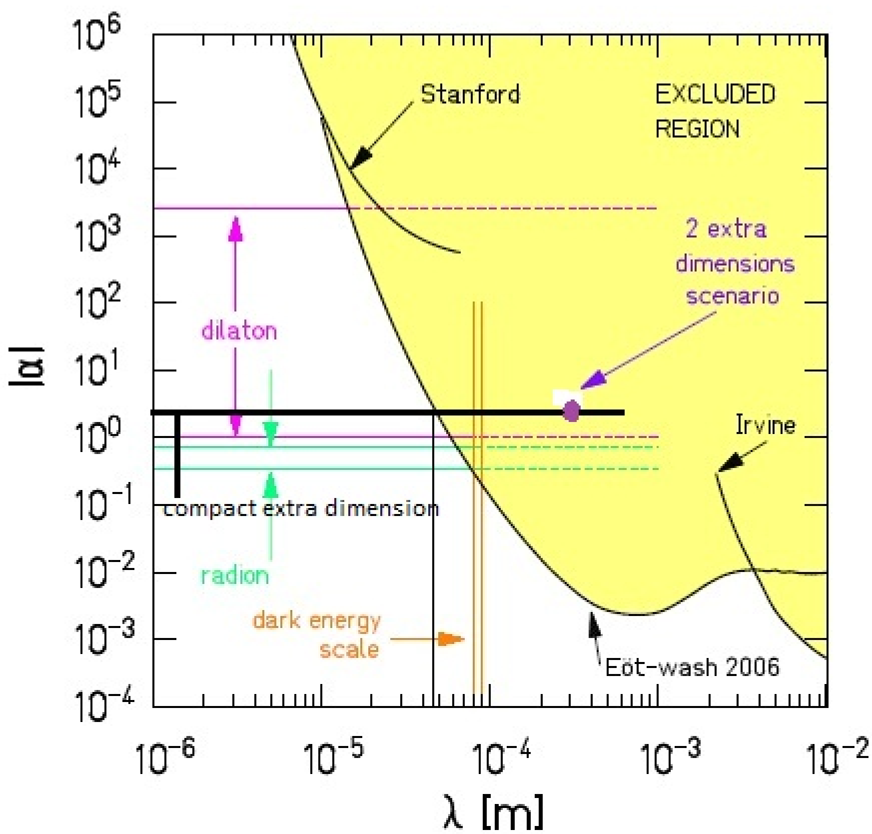

Figure 6: Results of short range gravity experiments, plotted in terms of the parameters λ and α. Within the excluded region (yellow) no deviations from Newton's law have been found, source: ref. [6].

For the purpose of this report, we are only interested in the case where ($\alpha \sim$2n). The two extra dimension scenario is shown by the black line, labeled "compact extra dimensions". Currently, the lowest bound for two extra dimensions of the same size is equal to $L < 37\mu m$ (see figure 5: $|\alpha| = log[4]$) This is much smaller than the predicted 1 millimetre from the ADD framework. It seems that the 6-dimensional scenario, for $M_{Pl(D)} \simeq M_{EW}$, is excluded by this result. If $M_{Pl(D)}$ is chosen to be slightly higher however, the 6-dimensional scenario may still resolve. In other words, we can interpretate short range gravity result as a bound on $M_{Pl(D)}$. We find this bound while using the relationship between the Planck-scales and plugging in the values n=2 and $L < 37\mu m$:

$$M^4_{Pl(6)} = \frac{M^2_{Pl}}{L^2}$$

$$M_{Pl(6)} = \sqrt{\frac{M_{Pl}}{L}} \gtrsim 3,1TeV. \tag{26}$$

This result tells us, that the ADD scenario with 2 extra dimensions is not capable of solving the hierarchy problem. Even stronger bounds on $M_{Pl(D)}$ have been derived from astrophysical observations. I will consider these bounds in chapter 3.

3 Constraints from Supernova observations

It is emphasized in the ADD papers [6, 7], that signatures of extra dimensions could be observed in astrophysical experiments. The idea is that large numbers of higher dimensional gravitons, called Kaluza Klein gravitons, could be produced in the hot cores of stars and particularly in supernovae. These Kaluza Klein (KK) gravitons can be very massive and they have the ability to escape into the extra dimensions, thereby taking away energy from the star. The maximum amount of energy the KK-gravitons can take away is constrained by astrophysical observations and these constraints have been used to calculate bounds on the Planck scale and the compactification scale.

To discuss these bounds I will start with a short theory on KK gravitons. In the next section I will consider the phenomenology of supernovae and in the last section I will compare some of the most important bounds derived from different experiments and by several authors.

3.1 Theory of Kaluza Klein gravitons

Kaluza Klein (KK) gravitons are hypothetical particles, which carry the force of gravity just like ordinary gravitons. The difference is that they have masses inversely proportional to the radius of compactification, so they can be very massive. Their most important properties follow from an aspect of compactification, called "Kaluza-Klein reduction". It shows how 4- and 5-dimensional physics are related and helps to understand Kaluza Klein gravitons.

3.1.1 Kaluza Klein reduction

Kaluza-Klein reduction shows how our 4-dimensional world can be interpreted as a projection of a higher-dimensional reality. To illustrate this, lets consider the 5-dimensional Kaluza Klein model.

In the 5-dimensional Kaluza Klein theory space is factorized into $M^4 \times S^1$, where $M^4$and S^1 are the geometries of the 4-dimensional Minkowski space and the compactified extra dimension, respectively. The factorization means that the 4-dimensional part of the metric does not depend on the extra dimension. The total set of coordinates is denoted by $x^M = (x^\mu, y)$, where x^μ, $\mu = 0, 1, 2, 3$ are the coordinates of the ordinary space-time dimensions and $y = x^5$ is the coordinate of the extra spatial dimension.

A field in the extra dimension is periodic with period $L = 2\pi R$. Thus, the points y and $y + 2\pi R$ are identified. I start by describing a massless scalar field:

$$\Phi\left(x^\mu, y\right) = \Phi\left(x^\mu, y + 2\pi R\right), \tag{27}$$

this can also be written with Fourier transformation:

$$\Phi(x^\mu, y) = \sum_{n=} \phi_n(x^\mu) \cdot e^{i\frac{n}{R}y}. \tag{28}$$

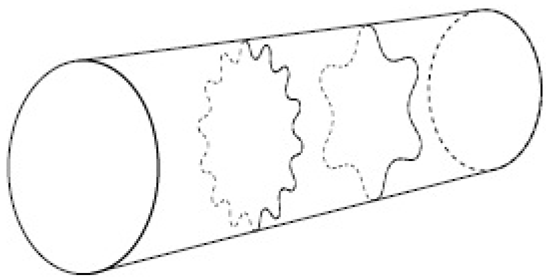

Figure 7: Kaluza Klein tower. source: [1]

The expansion coefficients ϕ_n are referred to as the 'modes' of the field and they only depend on the four 'ordinary' space-time coordinates x^μ. From the 4-dimensional point of view the modes form a series of 4-dimensional fields which is called the Kaluza Klein tower. (see figure 5)

To learn more about this Kaluza Klein tower, I will use the action to describe the field. In five dimensions the Lagrangian density is given by:

$$\mathcal{L} = -\frac{1}{2}\partial_{\mathrm{M}}\Phi\partial^{\mathrm{M}}\Phi \tag{29}$$

and plugging in the Fourier expansion gives:

$$\mathcal{L} = -\frac{1}{2}\sum_{n,m=-\infty}^{+\infty}\left(\partial_\mu\phi_n\partial^\mu\phi_m - \frac{nm}{R^2}\phi_n\phi_m\right)\cdot e^{i\frac{(n+m)}{R}y}. \tag{30}$$

With this we can define the action:

$$S = \int \mathcal{L}d^5x = \int_0^{2\pi R}\int \mathcal{L}d^4x \cdot dy$$

$$= -\frac{1}{2}\int_0^{2\pi R}\int \sum_{n,m=-\infty}^{+\infty}\left(\partial_\mu\phi_n\partial^\mu\phi_m - \frac{n^2}{R^2}\phi_n\phi_m\right)\cdot e^{i\frac{(n+m)}{R}y}d^4x \cdot dy$$

$$= -\frac{1}{2}\cdot 2\pi R\int \sum_{n=-\infty}^{+\infty}\left(\partial_\mu\phi_n\partial^\mu\phi_{-n} - \frac{n^2}{R^2}\phi_n\phi_{-n}\right)d^4x,$$

where I integrated with respect to y, since the 4-dimensional metric does not depend on y. I define a new more convenient coefficient:

$$\phi_j = \sqrt{2\pi R}\phi_n$$

and note that reality of Φ implies that $\phi_{-n} = \phi_n^\dagger$. Using these simplifications the action becomes:

$$S = -\int \left\{ \frac{1}{2}\partial_\mu \phi_0 \partial^\mu \phi_0^\dagger + \sum_{j=1}^{\infty} \left(\partial_j \phi_j \partial^\mu \phi_j^\dagger - \frac{j^2}{R^2}\phi_j \phi_j^\dagger \right) \right\} d^4x \tag{31}$$

It shows that from the 4-dimensional point of view, the spectrum of the 5-dimensional compactified field exists of:

- One real massless scalar field ϕ_0 called the zero mode.
- An infinite series of complex massive scalar fields, the Kaluza Klein tower. These fields have masses inversely proportional to the radius of compactification:
$$m_j^2 = j^2/R^2 \rightarrow m_j = |j|/R. \tag{32}$$

One can generalize this to the case of a massive field with mass m_0. If we do this, the masses of the Kaluza Klein states are given by $m_j^2 = m_0^2 + j^2/R^2$ and for the case of more than *one* extra dimension, in which the extra dimensions are compactified on a n-dimensional torus with radii $R_1, R_2, ...$*etc.*, the masses become:

$$m_j^2 = m_0^2 + \frac{j_1^2}{R_1^2} + \frac{j_2^2}{R_2^2} + ... \tag{33}$$

Our 4-dimensional world consists only of the massless zero-modes and we can not observe the higher modes directly. Obviously the energy needed to produce these massive particles has to be very high, since the masses are inversely proportional to the compactification radius R and R is very small.

3.1.2 Real emission of Kaluza-Klein gravitons

Processes with Kaluza-Klein (KK) graviton production may be an important signature of extra dimensions [5, 6]. They appear as missing-energy events when a particle is produced and no observable particle balances its transverse momentum in the center of mass frame. To estimate how much energy escapes into the extra dimensions due to KK graviton production, one mainly needs to estimate two things: the number of particles that is produced and the amount of energy they carry away.

Theoretically, there are many processes in which KK-gravitons can be produced and every process has different production rates. In addition, one needs to take in consideration the specific physical conditions in which the production takes place, because this also influences the production rate of the particles.

There is a simple method which is often used in particle physics to give a rough estimate of the production rate in a specific process. It starts by estimating the probability that an interaction takes place. In particle- and nuclear physics the probability is given by the so-called *cross section* σ. The cross section is a hypothetical measure of area which helps estimate the number of succesful interactions between particles. Since the dimension of a cross-section is $[\sigma]$ =Length^2, it can be approximated by a typical area corresponding to the relevant interaction.

KK-gravitons are expected to interact with the same strenght as ordinary gravitons and the typical energy scale for a graviton interaction is the Planck energy, M_{Pl}. Moreover, the typical length scale for a similar interaction would be the Planck length $l_{Pl} \sim M_{Pl}^{-1}$. By these means, the production cross-section of a KK-graviton can be estimated at:

$$\sigma(G_{KK}) \sim l_{Pl}^2 \sim M_{Pl}^{-2} \tag{34}$$

where G_{KK} denotes one single graviton. Actually, it is not possible to produce just one KK-Graviton, with momentum in the extra dimension. Conservation of momentum tells us, that a second graviton should leave in opposite direction, therefore I should include a factor 2 to be more precise. However, I just give a very rough estimate of the real cross-section here, so I might just as well forget about this extra factor.

This method of dimensional analysis might seem somewhat coarse, yet it appears to work quite well as a first estimate and we will see later that specific calculations of the cross section of KK-graviton production processes also contain this factor of M_{Pl}^{-2}.

Obviously, the probability that a KK-graviton is produced is extremely small. One may wonder whether or not it is possible to observe any effect of KK-graviton production. It turns out however, that in high energy processes, the small likelihood of a graviton being produced is compensated by the large mass degeneracy of the KK modes. In fact, it fully compensates for the tiny cross section.

The total cross-section is simply the sum over all states:

$$\sigma_{tot} = \sum_{\vec{j}} \sigma(G_{KK}). \tag{35}$$

Now since $m_j \sim (j/R)$ and R is considerably large in the ADD scenario, the mass separations between the adjacent KK modes is much smaller than typical energies in a physical process. One can therefore approximate the sum over the adjacent $\overrightarrow{j}$-states with an integral.

$$\sum_{\overrightarrow{j}} \rightarrow \int dj. \tag{36}$$

I estimate the number of KK modes with quantumnumber j in the extra dimensions, between j and$j + dj$ in j-space:

$$dN = S_{n-1} j^{n-1} dj$$

$$= S_{n-1} m_j^{n-1} R^{n-1} R \cdot dm_j$$

$$= \frac{S_{n-1}}{(2\pi)^n} \frac{M_{Pl}^2}{M_{Pl(D)}^{n+2}} m_j^{n-1} dm_j \tag{37}$$

where I used the relation (23) from the section 2.2:

$$L^n = (2\pi R)^n = \left(\frac{M_{Pl}^2}{M_{Pl(D)}^{n+2}} \right)$$

and S_{n-1} is again the volume of a sphere with unit radius in a n-dimensional space. It follows that the number of KK-modes accessible at energy $E \sim m$ is:

$$N(E) \sim \left(\frac{M_{Pl}}{M_{Pl(D)}} \right)^2 \cdot \left(\frac{E}{M_{Pl(D)}} \right)^n . \tag{38}$$

From this equation it follows that:

- More KK modes are accessible at higher energies, thus, stronger constraints on the LED's will follow from processes with higher energies;
- For $E < M_{Pl(D)}$, there are fewer KK modes for higher numbers of n; for these processes we will find weaker constraints for large n;
- The large degeneracy of the KK states compensates for the low probability of one interaction.

Finally we can estimate the total cross-section for given energy E:

$$\sigma_{tot} \sim \left(\frac{M_{Pl}}{M_{Pl(D)}} \right)^2 \left(\frac{E}{M_{Pl(D)}} \right)^n M_{Pl}^{-2} = \frac{E^n}{M_{Pl(D)}^{2+n}} \tag{39}$$

where we can see how the factor M_{Pl}^{-2} is factored out of the cross-section as a result of the combined effect of all gravitons together. According to this total cross-section it should statistically be possible to observe effects of KK-Graviton production in high energy processes - like stars [7].

3.2 phenomenology of supernovae

Physically there are two types of SNe, based on what mechanism powers them. Type I, the *thermonuclear* SNe and Type II, the *core-collapse* SNe. Type II will be our main interest.

Core collapse SNe mark the evolutionary end of a massive star, with mass $M > 8M_{\odot}$. During their life, these stars accumulate iron in their inner core as a result of nuclear fusion at the edge. Once the mass of the core reaches the so-called Chandrasekhar limit (about 1.38 solar masses), the star starts to collapse. The collapse continues until degeneracy pressure within the stellar core abruptly stoppes it, at a nuclear density of about $3 \times 10^{4} g \cdot cm^{-3}$. This bounce causes a shockwave moving outward and is observed as a huge explosion. It is important to note that this explosion is in fact a reversed implosion, so the power derives from gravitational energy, not from nuclear energy.

Typically, the released energy of this explosion is about $3 \times 10^{53} erg$ [5], of which 1% is carried by kinetic energy and the other 99% goes into neutrino's. This highly energetic neutrino-burst can be (and has been) detected by astrophysical experiments, however quantitative differences between the predicted- and the measured neutrinoflux have led to the idea that higher dimensional KK-gravitons might take away energy from the SN core.

During the first few seconds after the collapse, the temperatures in the SN core are estimated at $T \sim 30 - 70 MeV$ and the densities at $\rho \sim (3-10) \times 10^{14} g \cdot cm^{-3}$. At this time the core contains neutrons, protons, electrons, neutrino's and photons. There are three relevant processes that can possibly produce KK-gravitons:

- nucleon nucleon bremmsstrahlung: $N + N \mapsto N + N + G_{KK}$;
- photon fusion: $\gamma + \gamma \mapsto G_{KK}$;
- electron-positron annihilation: $e^- + e^+ \mapsto G_{KK}$.

The first process is the most dominant one, because nucleons are the most abundant in SNe and because the core temperature is so high that interactions between nucleons is unsuppressed. However, large uncertainties are involved in the theory around this process, mainly concerning the temperature of the SN core. Photons are less abundant than nucleons, so the photon fusion process leads to less-restrictive bounds (about three orders of magnitude), but it can still be useful. The process of electron-positron annihilation is practically insignificant for the derivation of bounds on $M_{Pl(D)}$. I will not consider this process.

For any process that could produce KK-gravitons, the emission rate should always be low enough to preserve the current understanding of neutrino observations. Most authors use a criterion, suggested by G. Raffelt [13]. Raffelt used detailed supernova simulations to show that the emission rate of any mechanism taking away energy from the supernova core should be below $10^{19} ergs \cdot g^{-1} \cdot s^{-1}$

[5] $1\, erg = 10^{-7} J = 624.15\, GeV$

to be compatible with the current understanding of the neutrino signal of a supernova type II.

The KK-gravitons that are produced in the SN-core tend to have small kinetic energies and they can be trapped in the gravitational potential of the SN core. The result is a neutron star (NS), surrounded by a halo of KK-gravitons. These gravitons can be decaying and annihilating, causing a flux of neutrino's, electrons, positrons and γ-rays. Special experiments, designed for the observation of γ-ray fluxes from localized sources, will be able to detect the possible effects of the halo. The strongest bounds arise from nearby neutronstars.

3.3 Constraints

3.3.1 SN 1987A

SN 1987A was a type II supernova and observations were done by two collaborations named Kamiokande [10] and IMB [11]. The observations essentially confirmed the predicted neutrinosignal, but the measured energy fluxes were much lower than expected. Numerical simulations predicted that the neutrino fluxes should yield the (time-intergrated) value of $\langle E_{\bar{\nu}_e} \rangle \simeq 16\, MeV$ [12], while the data implied an average neutrino-energy of $\langle E_{\bar{\nu}_e} \rangle \backsimeq 7.5\ MeV$ from Kamiokande (or at least below 12.5 MeV, taking the 95% confidence range into account) and $\langle E_{\bar{\nu}_e} \rangle \backsimeq 11.1\ MeV$ from IMB.

This difference between predictions and observation is the so-called *energy-loss argument,* that has led to the idea that KK-gravitons might take away energy from the supernova core. To derive constraints on the ADD model from this idea, one has to calculate the cross section for the relevant production processes. I will consider bounds derived from the photon-fusion process and nucleon-nucleon bremmstrahlung by different authors.

Photon fusion

The first process I consider is photon-fusion and I will follow the calculation done by Satheeshkumar and Suresh [14]. The authors start from the idea that photon fusion is an initial two particle reaction, $a + b \to c$ [6]and they use the general formula for the energyloss rate of an process [15]:

$$\dot{\epsilon}_{(a+b\to c)} = \frac{\left\langle n_a n_b \sigma_{(a+b\to c)} v_{rel} E_c \right\rangle}{\rho} \tag{40}$$

where the brackets indicate thermal averaging, $n_{a,b}$ are the number densities for the initial particles, ρ is the total mass density, v_{rel} is the relative velocity between the particles a and b, E_c is the energy of the particle c and $\sigma_{(a+b\to c)}$ is the scattering cross-section of an initial two particle interaction. The precise calculation of the cross-section goes beyond the purpose of this paper, but the

[6]generally two gravitons are produced, each with opposite momenta, this is taken into account in the calculation of the cross section.

used method and parameter values can be found in [16] and even more details are described in [14]. The obtained cross-section is given by

$$\sigma(m_{\vec{j}}) = \frac{\pi \kappa^2 \sqrt{s}}{16} \delta \left(m_{\vec{j}} - \sqrt{s} \right) \tag{41}$$

where $m_{\vec{j}}$ is the mass of the KK Graviton in the j^{th} state, $\sqrt{s} = E_a + E_b$ is the total initial energy in the center of mass frame and $\kappa^2 = 16\pi G_{(4)} \sim \frac{16\pi}{M_{Pl}^2}$ is the coupling constant of the gravitational field, which determines the strength of an interaction between particles. Obviously in this case the coupling is very weak ($\kappa^2 \sim \frac{1}{M_{Pl}^2}$) and therefore the probability that a graviton is produced is very small. However, as we already saw in the previous chapter, the large mass-degeneracy of the KK states compensates for this small cross-section and we can write the total cross-section

$$\sigma_{total} = \kappa^2 \sum \sigma(m_{\vec{j}}) \tag{42}$$

where the coupling strength is factored out and summation over the KK states can again be approximated by an integral. Filling in the parameters $T = 30MeV$, $\rho = 3 \times 10^{14} g \cdot cm^{-3}$ (the other parameters can be found in the literature) and doing the integration then leads to the following bounds (see table 2):

$$M_{Pl(D)} \gtrsim 14.72\, TeV \qquad n = 2 \tag{43}$$

$$M_{Pl(D)} \gtrsim 1.62\, TeV \qquad n = 3 \tag{44}$$

Nucleon nucleon bremsstrahlung

The nucleon-nucleon bremsstrahlung (NNB) process is denoted by

$$N + N \mapsto N + N + G_{KK}$$

where N can be either a proton or a neutron. Again, to calculate limits on $M_{Pl(D)}$, the main task is to compute the cross-section and the rate of the energy-loss for this process. There are many processes that contribute to the emission rate of KK-gravitons in NNB however, and various authors use different approaches.

Generally the emission rate from this process is of the form:

$$\dot{\epsilon} = a \cdot erg \cdot g^{-1} \cdot s^{-1} (X_n^2 + X_p^2 + b \cdot X_n X_p) \rho T^c M_{Pl(D)}^{-(D-2)}$$

where $a, b \,\&\, c$ are numerical factors, X_p and $X_n = 1 - X_p$ are the proton and neutron fractions in the core, T is the temperature and ρ is the total mass density within the core. The values of these parameters vary per author, but to give an idea of the ranges of the numerical factors:

$$a \sim 10^{18} - 10^{12} (for\, n = 2 - n = 3) \qquad b \sim 8.0, \qquad c \sim 5 - 8$$

Generally, the temperature depence causes the biggest uncertainty, because the temperature of SNe is hard to estimate (it ranges between 30-70 MeV). The dependence on the proton- and neutron fractions, X_p and X_n, on the other hand is not very important. It is therefore common to set the value of X_p to *zero*, so that $X_n = 1$. For comparison of the results of different authors, I only used the results for the parameters: $T = 30\, MeV$, $\rho = 3 \times 10^{14}\, g \cdot cm^{-3}$ and $X_p = 0$ (other parameters depend on the models chosen by the author). The results are listed below.

Table 2: Energyloss rates $\dot{\epsilon}$ ($erg \cdot g^{-1} \cdot s^{-1} M_{Pl(D)}^{-(n+2)}$) from photon fusion (first row) and NNB in SN 1987A. Parameters: $T = 30\, MeV$, $\rho = 3 \times 10^{14}\, g \cdot cm^{-3}$

n	2	3
Satheeshkumar & Suresh [14]:	$\dot{\epsilon} = 4.7 \times 10^{23}$	$\dot{\epsilon} = 1.1 \times 10^{20}$
Cullen & Perelstein [19]:	$\dot{\epsilon} = 6.79 \times 10^{25}$	$\dot{\epsilon} = 1.12 \times 10^{22}$
Barger, Han, Kao, Zhang [20]:	$\dot{\epsilon} = 6.7 \times 10^{25}$	$\dot{\epsilon} = 6.3 \times 10^{21}$
Hanhart *et. al.* [21]:	$\dot{\epsilon} = 9.24 \times 10^{24}$	$\dot{\epsilon} = 1.57 \times 10^{21}$

Table 3: Bounds on the fundamental Planck scale $M_{Pl(D)}$(TeV) derived from SN 1987A neutrino observations. Parameters: $T = 30 MeV$, $\rho = 3 \times 10^{14} g \cdot cm^{-3}$.

n	2	3
Satheeshkumar & Suresh [14]:	$M_{(D)} \gtrsim 14.72$	$M_{(D)} \gtrsim 1.62$
Cullen & Perelstein [19]:	$M_{Pl(D)} \gtrsim 50$	$M_{Pl(D)} \gtrsim 4$
Barger, Han, Kao, Zhang [20]:	$M_{Pl(D)} \gtrsim 51$	$M_{Pl(D)} \gtrsim 3.6$
Hanhart *et. al.* [21]:	$M_{Pl(D)} \gtrsim 31$	$M_{Pl(D)} \gtrsim 2.75$

3.3.2 Bounds from KK-decay

Even more stringent constraints can be obtained by considering the subsequent decays of the KK modes. Depending on their mass, the KK-modes typically have a lifetime comparable to the Hubble time or longer (4.35×10^{17}s or 13.8 billion years) [17] and their decay would contribute to the cosmic gamma-ray background. [22, 23, 24]. Generally, the decay rate of gravitons to photons can be estimated by:

$$\Gamma_{decay}(m_{\vec{k}}) = \frac{m_{\vec{k}}^3}{80\pi M_{Pl}^2}. \tag{45}$$

Hannestad & Raffelt [22] have used previously obtained data from several experiments to derive bounds on the compactification scale L and the Planck-scale $M_{Pl(D)}$ based on the theory of KK-decay.

1. **Measurements from the EGRET** instrument (Energetic Gamma Ray Experiment Telescope) [26] constrain the number of KK-gravitons that may have been emitted by all cosmic SNe. The data imply that a typical SN-core must not lose more than about 0,5% of its energy in KK-gravitons. Observations of SN 1987A were used to derive constraints from this restriction.

2. **Supernova Remnant Cas A** is a young SN remnant, probably corresponding to a SN in 1680. Based on its age, a cloud of emitted KK-gravitons should still appear as a point source to EGRET. However, EGRET does not observe a photon flux at the expected point in space. The difference between the theoretically expected flux and observation restricts the emissivity of KK gravitons and the fundamental Planck scale.

3. **Gamma radiation from nearby neutron stars** should be visible to EGRET as a result of the decaying KK-gravitons which are trapped in the halo around a NS after the SN. The expected $\gamma - ray$ flux was calculated such that only the trapped gravitons are counted, and the results were compared to EGRET observations in the same way as was done for the SN remnant Cas A.

4. **Neutron-star excess heat** gives the most stringent constraints. There are two mechanisms to be considered.

   - KK-decay: When KK-gravitons in the halo around the NS decay, a fraction of the decay-photons hits the surface and heat the star. The total energy absorbed by the star in this way can not be too high, otherwise the star would not be able to cool down as it does. The derived bounds are very strong compared to bounds derived from SN observations and temperature dependence is about the same as for NNB.
   - Reabsorption in the neutron-star: while the trapped KK-gravitons can be inside the NS during part of their trajectory, they can be reabsorbed by the nuclear medium. The authors estimate the reabsorbtion rate for average gravitons and compare their results to observations obtained from different NSs. This process does not provide significant bounds.

Finally Hannestad and Raffelt have calculated a bound from NNB and the neutrino signal from SN 1987A. These results are shown in the first row of table 4 and 5. Comparing this result to the results in table 2 and 3, one can see that this NNB bound is about half the size of the bounds derived by the other authors. Hannestad en Raffelt claim that this difference is mainly due to the different approach on calculating the emission rate for gravitational bremsstrahlung in the SN core.

The authors have explicitly calculated constraints for $2 \leq n \leq 7$. Lower bounds on the Planck scale are given in table 4 and upper bounds on the compactification scale L are given in table 5.

n	2	3	4	5	6	7
Neutrino signal SN 1987A	26.522	2.284	0.45	0.144	6×10^{-3}	10^{-4}
EGRET $\gamma -$ray limits:						
SN 1987A	83.44	5.071	0.97	0.28	0.11	10^{-2}
SN remnant Cas A	43.21	4.29	0.89	0.29	0.12	10^{-2}
Nearby stars	115	9.169	1.3	0.48	0.19	10^{-2}
Neutron star Excess heat	2.1×10^{3}	88.2	10.6	2.33	0.73	10^{-2}

Table 4: lower bounds on $M_{Pl(D)}$ (TeV) as derived by Hannestad & Raffelt [23] from $\gamma - ray$ observations.

n	2	3	4	5	6	7
Neutrino's SN1987A	10^{-6}	1.1×10^{-9}	3.8×10^{-11}	2×10^{-12}	1.2×10^{-12}	4.4×10^{-13}
EGRET γ ray:						
SN 1987A	10^{-7}	2.5×10^{-10}	1.2×10^{-11}	2×10^{-12}	5.6×10^{-13}	2.3×10^{-13}
SN remnant CasA	3.6×10^{-7}	4×10^{-10}	1.4×10^{-11}	2×10^{-12}	5.0×10^{-13}	1.9×10^{-13}
Nearby stars	2.6×10^{-8}	7.2×10^{-11}	3.9×10^{-12}	7×10^{-13}	2.2×10^{-13}	10^{-13}
NS Excess heat	10^{-10}	2.6×10^{-12}	3.4×10^{-13}	10^{-13}	4.4×10^{-14}	2.5×10^{-14}

Table 5: Upper bounds on the compactifiaction scale L(m), derived by Hannestad & Raffelt [23] from $\gamma - ray$ observations.

Note that I express bounds on the Planck scale in terms of the parameter $M_{Pl(D)}$, while Hannestad and Raffelt use a different parameter in the original paper:

$$\overline{M}_{4+n}^{2+n} = \overline{M}_{Pl}^{2}/L^{n} \tag{46}$$

where $\overline{M}_{Pl} = M_{Pl}/\sqrt{8\pi}$ is the reduced Planck scale, and L^n is the volume of the extra dimensions. This parameter is related to $M_{Pl(D)}$ by:

$$M_{Pl(D)} = 2^{1/(n+2)} \, (2\pi)^{n/(n+2)} \, \overline{M}_{4+n}. \tag{47}$$

To get the limits on $M_{Pl(D)}$ as shown in table 4, the results from [22] were multiplied by the factor $M_{Pl(D)}/\overline{M}_{4+n}$.

4 Discussion

We have seen how the ADD gives a different perspective on the hierarchy problem by putting the higher dimensional Planck scale at the electroweak scale. I have considered some physical implications of the theory and the most restrictive constraints from short range gravity experiments and supernova observations.

Short range gravity experiments exclude the 6 dimensional scenario ($n = 2$) directly for $M_{Pl(D)} = M_{EW}$, meaning that 2 extra dimensions can not solve the hierarchy problem in the ADD framework. Neutrino observations of SN 1987A have been used to put bounds on the Planck scale for low values of n. The strongest bounds derived from these experiments imply $M_{Pl(D)} \gtrsim 50$ TeV ($n = 2$) and $M_{Pl(D)} \gtrsim 4$ TeV ($n = 3$). Stronger bounds on the Planck scale and bounds for larger numbers of n, have been derived from γ-ray observations by EGRET. The most stringest constraints follow from the consideration of NS excess heat and the results are summarized in the table below. According to these results, the hierarchy problem can not be solved for $n \leq 5$.

n	2	3	4	5	6	7
$M_{Pl(D)}(TeV) \gtrsim$	2.1×10^3	88.2	10.6	2.3	0.7	10^{-2}
$L(m) \lesssim$	10^{-10}	10^{-12}	10^{-13}	10^{-13}	10^{-14}	10^{-14}

I used the assumption that all extra dimensions are compactified with the same size L. Theoretically though, the extra dimensions can have different sizes and this would lead to different results.

Besides the constraints derived from short range gravity experiments and SN observations, several other experiments have also put strong constraints on $M_{Pl(D)}$ and L. In the field of astrophysics, observations on black holes, the sun and red giants have been used to derive constraints in similar ways as SN observations have been [20], but none of these are as strong as the constraints I discussed in this paper. Particle collider experiments search for signatures of gravitons in the form of missing energy in particle collisions. The results do not depend on the number of extra dimensions, n, as much as astrophysical experiments. For example, experiments done at (LEP II) have led to the following upperbounds on the Planck scale: $M_{Pl(D)} \gtrsim 2$ TeV (for n = 2) and 0.6 TeV (for n = 6) [27].

In cosmology, constraints arise from various models, like inflation models and Big-Bang Nucleosynthesis [28]. All approaches lead to particularly high bounds on the $M_{Pl(D)}$ for small n, but for higher numbers of n the results are not as significant. For more details on these bounds see for example [29].

All in all, the ADD scenario is strongly constrained by several experiments, but no hard proof of extra dimensions has been found so far. The theory has not been proven wrong either, but even if we were to find extra dimensions, we can ask ourselves: "does the ADD solve then really solve the hierarchy problem?". It seems to solve the hierarchy between the electroweak scale and the Planck scale (for the right choice of L), but it also leaves us with a new hierarchy problem: the hierarchy between the electroweak scale and the compactification scale L. In the highest dimensional scenario (according to string theory), n=7:

$L \sim 10^{-14}m$, while $M_{EW}^{-1} \sim 10^{-19}m$. For L to reach the order of 10^{-19}, the universe would have to contain infinitly many extra dimensions, since (24):

$$L \sim 10^{32/n-19}m$$

and obviously this is not considered a realistic scenario.

Several other models have been based on the concept of large extra dimensions. The most important ones are the 'Universal extra dimension' (UED) model and the Randall-Sundrum (RS) models.

The UED model is similar to the ADD scenario, only in this model all fields can propagate through the extra dimensional bulk [30]. The Randall-Sundrum models (RS I and RS II), are based on the assumption that our universe contains two 3-branes, with equal but opposite tensions and space-time is curved within the extra dimensions. This curving is called 'warping' of the extra dimensions and the models are often referred to as 'warped extra dimension' models. For more details see [31]. Many more models extist, all giving different perspectives on different problems in physics.

At this point it seems that we will just have to wait for proof to know whether extra dimensions exist and if so, which model describes the physics within these extra dimensions best. If direct evidence of extra dimensions is to be found, this would have major consequences for our understanding of the universe and it could possibly be an important step towards the unification of the forces into one overarching theory.

Acknowledgements

I would like to thank my supervisor Jan de Boer for his useful advice and finding time to read my paper and answer my questions. Further I want to thank Willem for helping me out when I got stuck with theory. Finally I want to thank my sister, Minou, for giving me advice on writing a paper in English. [7]

[7] In case of any mistakes or questions, I will probably send you through to one of these people.

A Appendix A: Spheres in higher dimensions

It is important to be exact in the definition of spheres, because they are often confused with balls in daily language. When we are speaking of a sphere, at least in mathematical terms, we mean the spherical shell of radius R (a surface) that surrounds a ball of t Radius 0 < r < R.

In mathematics a ball of radius R in 3-dimensional space $\mathbb{R}^3$ with coordinates (x^1, x^2, x^3) is called a three-ball B^3 (where the superscript denotes the dimensionality) and it is enclosed by a two dimensional spherical shell S^2, called a two-sphere . The three-ball is then the region mathematically defined by:

$$B^3(R) = x_1^2 + x_2^2 + x_3^2 \leq R^2 \tag{48}$$

and the two-sphere:

$$S^2(R) = x_1^2 + x_2^2 + x_3^2 = R^2. \tag{49}$$

In general a d-ball B^d is a ball in d-dimensional space $\mathbb{R}^d$ with coordinates $(x_1 + x_2 + ... + x_d)$. It is mathematically defined by the region:

$$B^d(R) = x_1^2 + x_2^2 + ... + x_d^2 \leq R^2 \tag{50}$$

and enclosed by the (d-1)-sphere:

$$S^{(d-1)}(R) = x_1^2 + x_2^2 + ... + x_d^2 = R^2 \tag{51}$$

Now, to calculate the flux of a field through a sphere in d-dimensional space $\mathbb{R}^d$, one needs to know how to calculate the volume of the sperical shell $S^{(d-1)}(R)$. Note that I call this a volume, even though I am speaking of a sperical shell. Mathematicians always speak of volumes, no matter what the dimensionality is of the space they are talking about.

As an example I will use the volume of a ball and the corresponding sphere in 2- and 3-dimensions. The volume of a 2-ball in $\mathbb{R}^2$ is

$$vol[B^2(R)] = \pi R^2,$$

it is the area of the circle surrounded enclosed by a line, the 1-sphere $S^1$of volume:

$$vol[S^1(R)] = 2\pi R. \tag{52}$$

In the 3-dimensional case, the volume of the 3-ball is obvious,

$$vol[B^3(R)] = \frac{4}{3}\pi R^3$$

and the volume of the 2 dimensional spherical shell is the surface area that encloses the 3-ball:

$$vol[S^2(R)] = 4\pi R^2$$

Now in arbitrary dimensions the volume of a sphere of radius R is related to the volume of a sphere of unit radius by:

$$vol[S^{(d-1)}(R)] = R^{(d-1)} vol[S^{(d-1)}] \tag{53}$$

where the volume of a sphere with unit radius is given by:

$$vol[S^{(d-1)}] = \frac{2\pi^{\frac{d}{2}}}{\Gamma(\frac{d}{2})} \tag{54}$$

and $\Gamma(\frac{d}{2})$ is the Gamma function:

$$\Gamma(x) = \int_0^\infty dt \cdot e^{-t} t^{(x-1)}, \qquad x > 0. \tag{55}$$

For a more detailed derivation see reference [32]. For d=1 and d=2 the gamma function is easy:

$$\Gamma(\frac{1}{2}) = \int_0^\infty dt \cdot e^{-t} t^{(\frac{1}{2})} = \sqrt{\pi} \tag{56}$$

$$\Gamma(1) = \int_0^\infty dt \cdot e^{-t} t^{(0)} = 1 \tag{57}$$

and for other values of d, the gamma function follows from the relation:

$$\Gamma(x) = (x-1)\Gamma(x-1). \tag{58}$$

All in all, the volume of a (d-1)-sphere in d-space is given by

$$vol[S^{(d-1)}(R)] = R^{(d-1)} \frac{2\pi^{\frac{d}{2}}}{\Gamma(\frac{d}{2})} \tag{59}$$

and the volume of a d-ball B^d is

$$vol[B^d(R)] = R \frac{\pi^{\frac{d}{2}}}{\Gamma(1+\frac{d}{2})} \tag{60}$$

References

[1] For a more complete overview on large extra dimensions see for example: TASI Lecture - introduction to extra dimensions, Hsin-Chia CHeng, (2009)

[2] P. Candelas, G. T. Horowitz, A. Strominger and E. Witten, Nucl. Phys. B 258, 46 (1985).

[3] V. A. Rubakov and M. E. Shaposhnikov, *do we live inside a domain wall?* Phys. Lett. B 125, 136 (1983).

[4] M. Shifman: LARGE EXTRA DIMENSIONS, Becoming acquainted with an alternative paradigm. arXiv:0907.3074v2 [hep-ph] Nov 2009.

[5] N. Arkani-Hamed, S. Dimopoulos and G.R. Dvali, Phys.Lett.B 429, 263 (1998) [arXiv:hep-ph/9803315]

[6] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G.R. Dvali, New dimensions at a millimeter to a fermi and superstrings at a TeV, Phys. Lett. B 436 (1998) 257 [hep-ph/9804398].

[7] N. Arkani-Hamed, S. Dimopoulos and G.R. Dvali, Phys.Rev.D 59, 086004 (1999) [arXiv:hep-ph/9807344]

[8] For a general introduction to short range gravity experiments and information about the mechanism of torsion pendula, see for example: *http://www.npl.washington.edu/eotwash/experiments/shortRange/sr.html,* or read: *Sub-millimeter tests of the gravitational inverse-square law: A search for "large" extra dimensions* C. D. Hoyle, U. Schmidt, B. R. Heckel, E. G. Adelberger, J. H. Gundlach, D. J. Kapner, H. E. Swanson arXiv:hep-ph/0011014,

[9] D.J. Kapner et al., "*Tests of the gravitational inverse-square law below the dark-energy length scale*," Phys. Rev. Lett. 98 021101 (2007).

[10] K. Hirata et al., Phys. Rev. Lett. 58, 1490 (1987).

[11] R. M. Bionta et al., Phys. Rev. Lett. 58, 1494 (1987).

[12] Future detection of SN neutrino burst and explosion mechanism -T. TOTANI, the astrophysical journal, 496:216È225, (1998)

[13] G.G. Raffelt, *Stars as laboratories for fundamental physics,* Chicago University Press, 1996.

[14] V.H. Satheeshkumar and P.K. Kuresh, *Bounds on large extra dimensions from photon fusion process in SN1987A*, arXiv:0805.3429v1 [astro-ph] (2008)

[15] E.W. Kolb and M. S. Turner, *The early universe.*

[16] S Weinberg, The Quantum Theory of Fields, Vol 1 Cambridge University Press (1995)

[17] T. Han, J.D. Lykken and R.-J. Zhang, *On Kaluza-Klein states from large extra dimensions*, Phys. Rev. D 59 (1999) 105006 [hep-ph/9811350];

[18] G.F. Giudice, R. Rattazzi and J.D. Wells, *Quantum gravity and extra dimensions at high-energy colliders,* Nucl. Phys. B 544 (1999) 3 [hep-ph/9811291].

[19] S. Cullen and M. Perelstein, SN1987A constraints on large compact dimensions, Phys. Rev. Lett. 83 (1999) 268 [hep-ph/9903422]

[20] V.D. Barger, T. Han, C. Kao and R.-J. Zhang, Astrophysical constraints on large extra dimensions, Phys. Lett. B 461 (1999) 34 [hep-ph/9905474].

[21] C. Hanhart, D.R. Phillips, S. Reddy and M.J. Savage, Extra dimensions, SN1987A and nucleon nucleon scattering data, Nucl. Phys. B 595 (2001) 335 [nucl-th/0007016].

[22] S. Hannestad and G.G. Raffelt, *New supernova limit on large extra dimensions*, Phys. Rev. Lett. 87 (2001) 051301 [hep-ph/0103201];

[23] Hannestad and Raffelt, *Stringent neutron-star limits on large extra dimensions*, Phys. Rev. Lett. 88 (2002) 071301 [hep-ph/0110067].

[24] [9] S. Hannestad, Strong constraint on large extra dimensions from cosmology, Phys. Rev. D 64 (2001) 023515 [hep-ph/0102290].

[25] S. Weinberg, "Gravitation and Cosmology" (Wiley, New York, 1972).

[26] D. A. Kniffen et al., "EGRET Observations of the high latitude diffuse radiation," Astron. Astrophys. Suppl. Ser. 120, 615 (1996).

[27] E. A. Mirabelli, M. Perelstein and M. E. Peskin, Phys. Rev. Lett. 82 (1999) 2236 [arXiv:hep-ph/9811337].

[28] A.R. Liddle, *An Introduction to Modern Cosmology* (Chichester 1999).

[29] M. Fairbairn [arXiv:hep-ph/0101131], D. Kazanas and A. Nicolaidis [arXiv:hep-ph/0109247]

[30] Appelquist, Thomas; Cheng, Hsin-Chia; Dobrescu, Bogdan A. (2001). "Bounds on universal extra dimensions". Physical Review D 64: 035002. doi:10.1103/PhysRevD.64.035002.

[31] Randall, Lisa; Sundrum, Raman (1999). "Large Mass Hierarchy from a Small Extra Dimension". Physical Review Letters 83 (17): 3370–3373. doi:10.1103/PhysRevLett.83.3370.

[32] Barton Zwiebach, *A First course to string theory*, Cambridge University Press 2004 (third edition 2005).