1 Carroll 1.2

Immagine that space (not spacetime) is actually a finite box, or in more sophisticated terms, a three-torus, of size L. By this we mean that there is a coordinate system $x^\mu = (t, x, y, z)$ such that every point with coordinates (t, x, y, z) is identified with every point with coordinates $(t, x + L, y, z)$,$(t, x, y + L, z)$ and $(t, x, y, z + L)$. Note that the time coordinate is the same. Now consider two observers; observer A is at rest in this coordinate system (constant spatial coordinates), while observer B moves in the x-direction with constant velicity v. A and B begin at the same event, and while A remains atill, B moves once around the univers and comes back to intersect the worldline of A without ever having to accelerate (since the univers is periodic). What are the relative proper times experienced in this interval by A and B? Is this consistent with your understanding of Lorentz invariance?

2 Carroll 1.3

Three events A, B, C, are seen by observer $\mathcal{O}$ to occur in the order A B C. Another observer, $\bar{\mathcal{O}}$, sees the events to occur in the order C B A. Is it possible that a third observer sees the events in the order A C B? Support your conclusion by drawing a spacetime diagram.

3 Carroll 1.4

Projection effects can trick you into thinking that an astrophysical object is moving "superluminally". Consider a quasar that ejects gas with speed v at angle θ with respect to the line-of-sight of the observer. Projected onto the sky, the gas appears to travel perpondicular to the line-of-sight with angular speed v_{app}/D, where D is the distance to to the quasar and v_{app} is the apparent speed. Derive an expression for v_{app} in terms of v and θ. Show that, for appropriate values of v and θ, v_{app} can be greater than 1.

4 Carroll 1.5

Particle physicists are used to setting $c = 1$ that they measure mass in units of energy. In particular, they tend to use electron volts (1 eV $= 1.6 \times 10^{-12}$ erg $= 1.8 \times 10^{-33}$ g), or, more commonly, keV, MeV, and GeV (10^3 eV, 10^6 eV, and 10^9 eV, respectively). THe muon has been measured to have a mass of 0.106 GeV and a rest frame lifetime of 2.19×10^{-6} seconds. Imagine that such a muon is moving in the cicular storage ring of a particle accelerator, 1 kilometer in diameter, such that the muon's total energy is 1000 GeV. How long would it appear to live from the experementer's point of view? How many radians would it travel around the ring?