general relativity – november 17, 2009

material discussed in class

Roughly 5.1, 5.2, 5.3 in the book.

exercises

1) (challenging! - Carroll problem 3.12) Show that any Killing vector obeys $\nabla_\mu \nabla_\sigma K^\rho = R^\rho{}_{\sigma\mu\nu} K^\nu$ and $K^\lambda \nabla_\lambda R = 0$.

2) Consider the following metric on $\mathbb{R}^2$

$$ds^2 = e^{\phi(x,y)}(dx^2 + dy^2). \tag{1}$$

Compute the Christoffel symbols and Riemann curvature tensor for this metric. Suppose that the function $\phi(x,y)$ is a function of $x^2 + y^2$ only. Show that in that case

$$K = x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}$$

is a Killing vector of (1). Which symmetry does this Killing vector generate?