general relativity – october 27, 2009

material discussed in class

Roughly 3.4, 3.6, 3.7 in the book.

exercises

- Show that
  \[ [\nabla_\mu, \nabla_\nu] V_\rho = -R^\sigma_{\rho \mu \nu} V_\sigma \]  
  given that
  \[ [\nabla_\mu, \nabla_\nu] V^\rho = R_{\sigma \mu \nu} V^\sigma . \] (1)

- What would be the answer for \([\nabla_\mu, \nabla_\nu] A^\alpha_{\beta \gamma} \)? Don’t compute this explicitly, but use properties of the covariant derivative to figure out what the answer should be. In particular, first study what \([\nabla_\mu, \nabla_\nu] A^\alpha B_\beta C_\gamma \) is for arbitrary choices of tensors \(A, B, C\) using the Leibnitz rule, then infer from this what the answer for \([\nabla_\mu, \nabla_\nu] A^\alpha_{\beta \gamma} \) should be.

- Find a set of coordinates on the two-sphere which define a local inertial frame near the point \((\theta, \phi) = (\pi/2, 0)\) on the equator.

- Prove that any geodesic on the two-sphere always follows a ‘big circle’, i.e. a circle obtained by intersecting the two-sphere by a two-plane which passes through the origin. Here we view the two-sphere as the sphere \(x^2 + y^2 + z^2 = R^2\) in \(\mathbb{R}^3\). Hint: use the symmetries of the problem as much as possible!