general relativity – october 27, 2009

material discussed in class

Roughly 3.4, 3.6, 3.7 in the book.

exercises

- Show that
$$[\nabla_\mu, \nabla_\nu] V_\rho = -R^\sigma{}_{\rho\mu\nu} V_\sigma \tag{1}$$
given that
$$[\nabla_\mu, \nabla_\nu] V^\rho = R^\rho{}_{\sigma\mu\nu} V^\sigma . \tag{2}$$
- What would be the answer for $[\nabla_\mu, \nabla_\nu] A^\alpha{}_{\beta\gamma}$? Don't compute this explicitly, but use properties of the covariant derivative to figure out what the answer should be. In particular, first study what $[\nabla_\mu, \nabla_\nu] A^\alpha B_\beta C_\gamma$ is for arbitrary choices of tensors A, B, C using the Leibnitz rule, then infer from this what the answer for $[\nabla_\mu, \nabla_\nu] A^\alpha{}_{\beta\gamma}$ should be.
- Find a set of coordinates on the two-sphere which define a local inertial frame near the point $(\theta, \phi) = (\pi/2, 0)$ on the equator.
- Prove that any geodesic on the two-sphere always follows a 'big circle', i.e. a circle obtained by intersecting the two-sphere by a two-plane which passes throught the origin. Here we view the two-sphere as the sphere $x^2 + y^2 + z^2 = R^2$ in $\mathbb{R}^3$. Hint: use the symmetries of the problem as much as possible!