exercise

1. (The so-called fourth test of general relativity) Consider the sun, mercury and earth, and assume that the sun is at $r = 0$, the metric around it is the Schwarzschild metric, and assume that mercury is at $(r_1, \theta, \phi)$ for all $t$, and that earth is at $(r_2, \theta, \phi)$ for all $t$. In other words, we assume both mercury and earth to be static. Assume that we send a radar signal from earth to mercury, where it is reflected back to earth. After some proper time on earth we measure the reflected signal. The total proper time as measured on earth it takes for the signal to go back to mercury and back is denoted $\Delta \tau$. Compute this quantity, and show that it is not equal to twice the proper distance between earth and mercury divided by the speed of light (which we take to be $c = 1$ throughout.)

2. Consider a massive particle, initially at rest at some fixed $\theta, \phi$, at some fixed value $r = r_0$. The particle will disappear into the black hole described by the Schwarzschild metric. At some point the particle will reach the horizon. Show that it takes infinite coordinate time, but only finite proper time for the massive particle to reach the horizon.

3. Compute some of the periods of the orbits of massive particles described in the lecture.