general relativity – September 13, 2004

material discussed in class

Roughly equal to sections 1.4-1.7 in Carroll, plus variational principle (or page 8–19 in the lecture notes)

exercises

- Show that
$$\eta^{\mu'\nu'} = \Lambda^{\mu'}{}_{\mu}\Lambda^{\nu'}{}_{\nu}\eta^{\mu\nu}.$$
- Show that
$$\Lambda^{\mu}{}_{\rho}\Lambda_{\nu}{}^{\mu} = \delta^{\nu}_{\rho}.$$
- Define the charge and current as
$$\begin{aligned} Q &= \sum_n q_n \delta^{(3)}(x - x_n(t)) \\ J^i &= \sum_n q_n \delta^{(3)}(x - x_n(t)) \frac{dx_n^i(t)}{dt}. \end{aligned}$$
These combine nicely into a four-vector $J^\mu = (Q, J^i)$ which indeed transforms as a vector under Lorentz transformations. We can write
$$J^\mu = \sum_n q_n \delta^{(3)}(x - x_n(t)) \frac{dx_n^\mu(t)}{dt}. \tag{1}$$
Show that J^μ is a conserved current, $\partial_\mu J^\mu = 0$.
- Show that $Q = \int d^3x J^0(x)$ is conserved, i.e. time-independent, using only the fact that $\partial_\mu J^\mu = 0$, and not the detailed form of J^μ.
- Exercises 1.6-1.7 in Carroll.