general relativity – october 10

material discussed in class

Roughly 3.4, 3.6, 3.7 in the book.

exercises

• Show that
  \[ [\nabla_\mu, \nabla_\nu]V_\rho = -R^\sigma_{\rho\mu\nu}V_\sigma \quad (1) \]
given that
  \[ [\nabla_\mu, \nabla_\nu]V^\rho = R^\rho_{\sigma\mu\nu}V^\sigma. \quad (2) \]

• What would be the answer for \([\nabla_\mu, \nabla_\nu]A^\alpha_{\beta\gamma}\)? Don’t compute this explicitly, but use properties of the covariant derivative to figure out what the answer should be.

• Construct Riemann normal coordinates on the two-sphere around the north-pole, and express the metric in terms of these normal coordinates.

• Show that the parameter \(\lambda\) that appears in the geodesic equation is for timelike geodesics equal to \(\lambda = a\tau + b\), with \(a, b\) constants and \(\tau\) the proper time along the geodesic.