general relativity – october 10

material discussed in class

Roughly 3.4, 3.6, 3.7 in the book.

exercises

- Show that
$$[\nabla_\mu, \nabla_\nu] V_\rho = -R^\sigma{}_{\rho\mu\nu} V_\sigma \tag{1}$$
given that
$$[\nabla_\mu, \nabla_\nu] V^\rho = R^\rho{}_{\sigma\mu\nu} V^\sigma. \tag{2}$$

- What would be the answer for $[\nabla_\mu, \nabla_\nu] A^\alpha{}_{\beta\gamma}$? Don't compute this explicitly, but use properties of the covariant derivative to figure out what the answer should be.

- Construct Riemann normal coordinates on the two-sphere around the north-pole, and express the metric in terms of these normal coordinates.

- Show that the parameter λ that appears in the geodesic equation is for timelike geodesics equal to $\lambda = a\tau + b$, with a, b constants and τ the proper time along the geodesic.