general relativity – September 12, 2005

material discussed in class

Roughly equal to sections 1.4-1.8 in Carroll.

exercises

- Show that
$$\eta^{\mu'\nu'} = \Lambda^{\mu'}{}_{\mu}\Lambda^{\nu'}{}_{\nu}\eta^{\mu\nu}.$$
- Verify that δ^{ρ}_{ν} is a tensor.
- Define the charge and current as
$$\begin{aligned} Q &= \sum_n q_n \delta^{(3)}(x - x_n(t)) \\ J^i &= \sum_n q_n \delta^{(3)}(x - x_n(t)) \frac{dx_n^i(t)}{dt}. \end{aligned}$$
These combine nicely into a four-vector $J^{\mu} = (Q, J^i)$ which indeed transforms as a vector under Lorentz transformations. We can write
$$J^{\mu} = \sum_n q_n \delta^{(3)}(x - x_n(t)) \frac{dx_n^{\mu}(t)}{dt}. \tag{1}$$
Show that J^{μ} is a conserved current, $\partial_{\mu} J^{\mu} = 0$.
- Show that $Q = \int d^3x J^0(x)$ is conserved, i.e. time-independent, using only the fact that $\partial_{\mu} J^{\mu} = 0$, and not the detailed form of J^{μ}.
- Exercises 1.6-1.7 in Carroll.