general relativity – October 31

material discussed in class

Roughly 7.1,7.4,7.5,7.6 in the book.

exercises

• Consider a massive particle, initially at rest at some fixed \( \theta, \phi \), at some fixed value \( r = r_0 \). The particle will disappear into the black hole described by the Schwarzschild metric. At some point the particle will reach the horizon. Show that it takes infinite coordinate time, but only finite proper time for the massive particle to reach the horizon.

• Consider a system of particles moving in a plane with common angular frequency \( \omega \). Assume the \( i^{th} \) particle is located at time zero at position \((r_i \cos \phi_i, r_i \sin \phi_i)\) in the plane. Show that the average total emitted power is given by

\[
\left\langle \frac{\partial E}{\partial t} \right\rangle = -\frac{32G\omega^6}{5} \sum_{i,j} m_i m_j r_i^2 r_j^2 \cos 2(\phi_i - \phi_j).
\]

• Consider a bar with length one meter, with a mass of one kilogram at each endpoint. Assume this bar rotates once a second. Compute the power emitted due to gravitational radiation. Remember \( c = 1 \) in our units.