1 Exercise 5.1

Consider an inertial frame $S$ with coordinates $x^\mu = (t, x, y, z)$ and a frame $S'$ with coordinates $x'^\mu$ related to $S$ by a boost with velcity parameter $v$ along the $y$-axis. Imagine we have a wall at rest in $S'$, lying along the line $x' = -y'$. From the point of view of $S$, what is the relationship between the incident angle of a ball hitting the wall (travelling in the $x$-$y$ plane) and the reflected angle? What about the velocity before and after?

2 Exercise 5.2

Imagine that space (not spacetime) is actually a finite box, or in more sophisticated terms, a three-torus, of size $L$. By this we mean that there is a coordinate system $x^\mu = (t, x, y, z)$ such that every point with coordinates $(t, x, y, z)$ is identified with every point with coordinates $(t, x + L, y, z), (t, x, y + L, z)$ and $(t, x, y, z + L)$. Note that the time coordinate is the same. Now consider two observers; observer $A$ is at rest in this coordinate system (constant spatial coordinates), while observer $B$ moves in the $x$-direction with constant velocity $v$. $A$ and $B$ begin at the same event, and while $A$ remains still, $B$ moves once around the universe and comes back to intersect the worldline of $A$ without ever having to accelerate (since the universe is periodic). What are the relative proper times experienced in this interval by $A$ and $B$? Is this consistent with your understanding of Lorentz invariance?

3 Exercise 5.3

Three events $A, B, C$, are seen by observer $\mathcal{O}$ to occur in the order $A B C$. Another observer, $\mathcal{O}^*$, sees the events to occur in the order $C B A$. Is it possible that a third observer sees the events in the order $A C B$? Support your conclusion by drawing a spacetime diagram.

4 Exercise 5.4

Projection effects can trick you into thinking that an astrophysical object is moving "superluminally". Consider a quasar that ejects gas with speed $v$ at angle $\theta$ with respect to the line-of-sight of the observer. Projected onto the sky, the gas appears to travel perpendicular to the line-of-sight with angular speed $v_{\text{app}}/D$, where $D$ is the distance to to the quasar and $v_{\text{app}}$ is the apparent speed. Derive an expression for $v_{\text{app}}$ in terms of $v$ and $\theta$. Show that, for appropriate values of $v$ and $\theta$, $v_{\text{app}}$ can be greater than 1.
5 Exercise 5.5

Particle physicists are used to setting $c = 1$ that they measure mass in units of energy. In particular, they tend to use electron volts ($1 \text{ eV} = 1.6 \times 10^{-12} \text{ erg} = 1.8 \times 10^{-33} \text{ g}$), or, more commonly, keV, MeV, and GeV ($10^{3} \text{ eV}$, $10^{6} \text{ eV}$, and $10^{9} \text{ eV}$, respectively). The muon has been measured to have a mass of $0.106 \text{ GeV}$ and a rest frame lifetime of $2.19 \times 10^{-6}$ seconds. Imagine that such a muon is moving in the circular storage ring of a particle accelerator, 1 kilometer in diameter, such that the muon’s total energy is $1000 \text{ GeV}$. How long would it appear to live from the experimenter’s point of view? How many radians would it travel around the ring?