general relativity – September 14, 2006

material discussed in class

Roughly equal to sections 1.4-1.8 in Carroll.

exercises

• Recall that $\Lambda^\mu \nu \mu$ and $\Lambda^\mu \nu \nu$ are not the same, but each others inverse, in other words
  $$\Lambda^\mu \nu \mu \Lambda^\mu \nu \nu = \delta^\mu \nu.$$ 
  Use this to show that $\eta^{\mu \nu}$ is a tensor, i.e.
  $$\eta^{\mu \nu} = \Lambda^\mu \nu \Lambda^\nu \mu \eta^{\mu \nu}.$$

• Define the charge and current of a collection of point particles which have charge $q_n$ and move along trajectories $x_n(t)$ as follows
  $$Q = \sum_n q_n \delta^{(3)}(x - x_n(t))$$
  $$J^i = \sum_n q_n \delta^{(3)}(x - x_n(t)) \frac{dx_n^i(t)}{dt}.$$ 
  These combine nicely into a four-vector $J^\mu = (Q, J^i)$ which indeed transforms as a vector under Lorentz transformations. Verify this.

• We can write
  $$J^\mu = \sum_n q_n \delta^{(3)}(x - x_n(t)) \frac{dx_n^\mu(t)}{dt}. \quad (1)$$
  Show that $J^\mu$ is a conserved current, $\partial_\mu J^\mu = 0$.

• Show that $Q = \int d^3x J^0(x)$ is conserved, i.e. time-independent, using only the fact that $\partial_\mu J^\mu = 0$, and not the detailed form of $J^\mu$.

• Exercises 1.6-1.7 in Carroll.