general relativity – September 14, 2006

material discussed in class

Roughly equal to sections 1.4-1.8 in Carroll.

exercises

- Recall that ${\Lambda^{\mu'}}_{\mu}$ and ${\Lambda^{\mu}}_{\mu'}$ are not the same, but each others inverse, in other words
$$ {\Lambda^{\mu'}}_{\mu} {\Lambda^{\mu}}_{\nu'} = \delta^{\mu'}_{\nu'} . $$
Use this to show that $\eta^{\mu\nu}$ is a tensor, i.e.
$$ \eta^{\mu'\nu'} = {\Lambda^{\mu'}}_{\mu} {\Lambda^{\nu'}}_{\nu} \eta^{\mu\nu} . $$

- Define the charge and current of a collection of point particles which have charge q_n and move along trajectories $x_n(t)$ as follows
$$ \begin{aligned} Q &= \sum_n q_n \delta^{(3)}(x - x_n(t)) \\ J^i &= \sum_n q_n \delta^{(3)}(x - x_n(t)) \frac{dx_n^i(t)}{dt} . \end{aligned} $$
These combine nicely into a four-vector $J^\mu = (Q, J^i)$ which indeed transforms as a vector under Lorentz transformations. Verify this.

- We can write
$$ J^\mu = \sum_n q_n \delta^{(3)}(x - x_n(t)) \frac{dx_n^\mu(t)}{dt} . \tag{1} $$
Show that J^μ is a conserved current, $\partial_\mu J^\mu = 0$.

- Show that $Q = \int d^3x J^0(x)$ is conserved, i.e. time-independent, using only the fact that $\partial_\mu J^\mu = 0$, and not the detailed form of J^μ.

- Exercises 1.6-1.7 in Carroll.