general relativity – november 23, 2006

material discussed in class

Roughly 8.4,8.5, and a little bit of 8.7.

exercises

- Show that the Friedmann equations

$$\begin{aligned} \frac{\ddot{a}}{a} &= -\frac{4\pi G}{3}(\rho+3p) \\ \left(\frac{\dot{a}}{a}\right)^2 &= \frac{8\pi G}{3}\rho - \frac{k}{a^2}. \end{aligned} \tag{1}$$

  imply the equation we derived from the conservation of energy and momentum,

$$\nabla_\mu T^{\mu i} = \dot{\rho} + 3\frac{\dot{a}}{a}(p+\rho) = 0. \tag{2}$$

  why should they imply this?

- Verify for yourself once more that the redshift of the light emitted by a distant object, given in terms of the parameter z

$$z = \frac{\omega_{\text{emitted}}}{\omega_{\text{received}}} - 1, \tag{3}$$

  is equal to

$$z = \frac{a_{\text{receiver}}}{a_{\text{emitter}}} - 1 \tag{4}$$

  where a is the scale factor of the universe at the time of emission and reception of the signal respectively.

- The luminosity distance of an object was defined as

$$d_L = \sqrt{\frac{L}{4\pi F}} = ar(1+z) \tag{5}$$

  with L the absolute luminosity, F the flux received on earth, a the present size of the universe, r the coordinate distance to the object and z the redshift of the object. Verify once more for yourself that

$$d_L = H_0^{-1}(z + \frac{1}{2}(1-q_0)z^2 + \mathcal{O}(z^3)) \tag{6}$$

  with H_0, q_0 the present Hubble constant and deceleration parameter.