general relativity – September 11, 2008

material discussed in class

Roughly equal to sections 1.4-1.8 in Carroll.

exercises

• Recall that $\Lambda^\nu_\mu$ and $\Lambda^{\mu}_\nu$ are not the same, but each others inverse, in other words

$$\Lambda^\nu_\mu \Lambda^{\mu}_\nu = \delta^\nu_\nu.$$ 

Use this to show that $\eta^{\mu\nu}$ is a tensor, i.e.

$$\eta^{\mu\nu} = \Lambda^\mu_\mu \Lambda^{\mu}_\nu \eta^{\mu\nu}.$$ 

• Define the charge and current of a collection of point particles which have charge $q_n$ and move along trajectories $x_n(t)$ as follows

$$Q = \sum_n q_n \delta^{(3)}(x - x_n(t))$$

$$J^i = \sum_n q_n \delta^{(3)}(x - x_n(t)) \frac{dx^i_n(t)}{dt}.$$ 

These combine nicely into a four-vector $J^\mu = (Q, J^i)$ which indeed transforms as a vector under Lorentz transformations. Verify this.

• We can write

$$J^\mu = \sum_n q_n \delta^{(3)}(x - x_n(t)) \frac{dx^\mu_n(t)}{dt}. \quad (1)$$ 

Show that $J^\mu$ is a conserved current, $\partial_\mu J^\mu = 0$.

• Show that $Q = \int d^3x J^0(x)$ is conserved, i.e. time-independent, using only the fact that $\partial_\mu J^\mu = 0$, and not the detailed form of $J^\mu$.

• Exercises 1.6-1.7 in Carroll.