general relativity – november 20, 2008

material discussed in class

Roughly 8.4,8.5, and a little bit of 8.7.

exercises

• Show that the Friedmann equations

\[
\frac{\dot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)
\]

\[
\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}.
\] (1)

imply the equation we derived from the conservation of energy and momentum,

\[
\nabla \mu T^\mu = \dot{\rho} + 3\frac{\dot{a}}{a}(p + \rho) = 0.
\] (2)

why should they imply this?

• Verify for yourself once more that the redshift of the light emitted by a distant object, given in terms of the parameter \(z\)

\[
z = \frac{\omega_{\text{emitted}}}{\omega_{\text{received}}} - 1,
\] (3)

is equal to

\[
z = \frac{a_{\text{receiver}}}{a_{\text{emitter}}} - 1
\] (4)

where \(a\) is the scale factor of the universe at the time of emission and reception of the signal respectively.

• The luminosity distance of an object was defined as

\[
d_L = \sqrt{\frac{L}{4\pi F}} = a r (1+z)
\] (5)

with \(L\) the absolute luminosity, \(F\) the flux received on earth, \(a\) the present size of the universe, \(r\) the coordinate distance to the object and \(z\) the redshift of the object. Verify once more for yourself that

\[
d_L = H_0^{-1}(z + \frac{1}{2}(1 - q_0)z^2 + \mathcal{O}(z^3))
\] (6)

with \(H_0,q_0\) the present Hubble constant and deceleration parameter.