String Theory on AdS Backgrounds

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Abstract. Various exact two-dimensional conformal field theories with AdS_{2d+1} target space are constructed. These models can be solved using bosonization techniques and are examples of a more general novel type of coset construction. Some of them can be used to build perturbative superstring theories with AdS backgrounds, including AdS_5.

1. Introduction

In view of the relation between conformal field theories and string theory on AdS spaces (for a review see [1]), the latter have attracted considerable interest. The main problem in studying them is that they generically involve RR backgrounds, which makes quantization quite difficult.

Quantization in the NSR [3] and Green-Schwarz [2] formalism essentially leads to perturbation theory around flat space, and the calculations are as difficult as those that one would need to do in order to compute higher order corrections involving RR fields to the supergravity effective action. An alternative approach involves the hybrid formulation of string theory due to Berkovits (see the talk by N. Berkovits in these proceedings and references therein [4]). This formulation has been worked out for $AdS_2 \times S^2$ [5] and $AdS_3 \times S^3$ [6], and is a good starting point to do perturbation theory. The hybrid formulation involves theories that look like a WZW theory, but with an unconventional Wess-Zumino term. The only exception is the theory on $AdS_3 \times S^3$ with NS fluxes, which is described by a $PSU(2|2)$ WZW theory coupled to ghost-like fields. This theory can be quantized using exact conformal field theory techniques. Turning on RR fields corresponds to changing the coefficient in front of the Wess-Zumino term. Although the resulting sigma model is still exactly conformal, it is not known whether there is a corresponding exact conformal field theory description.

Having an exact conformal field theory description is clearly advantageous, especially if we are interested in questions that go beyond perturbation theory. Here we will summarize the construction of exact conformal field theories with AdS_{2d+1}

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backgrounds described in [7], study their symmetries and compute their central charges. All these theories are holographic, and we expect that they can be continuously deformed to AdS backgrounds with RR fields. Perturbation theory around these exact conformal field theories would presumably lead to a much better description of string theories with RR backgrounds than perturbation theory around flat spaces, since the starting point is already holographic and involves AdS spaces.

2. From AdS_3 to AdS_{2d+1}

The simplest way to describe the construction of the exact conformal field theories with AdS_{2d+1} backgrounds is to start with the well-known case of the $SL(2,R)$ WZW model. The conformal sigma model with AdS_3 target space is given by the WZW action for $SL(2,R)$ in the Gauss parametrization and reads [8]

$$S = \frac{k}{2\pi}\int d^2z\,(\partial\phi\bar{\partial}\phi + e^{2\phi}\partial\bar{\gamma}\bar{\partial}\gamma). \tag{1}$$

This theory has an sl_2 current algebra, and describes Lorentzian or Euclidean AdS_3 depending on whether we view γ and $\bar{\gamma}$ as independent and real, or as each others complex conjugate, respectively. By introducing auxiliary fields $\beta,\bar{\beta}$ and rescaling ϕ the action can be rewritten as

$$S = \frac{1}{4\pi}\int d^2z\,(\partial\phi\bar{\partial}\phi + \beta\bar{\partial}\gamma + \bar{\beta}\partial\bar{\gamma} - \beta\bar{\beta}e^{-2\phi/\alpha_+} - \frac{2}{\alpha_+}\phi\sqrt{g}R) \tag{2}$$

where $\alpha_+ = \sqrt{2(k-2)}$. This latter action describes a free field ϕ with some background charge and two free β,γ systems, perturbed by the exactly marginal operator $V = -\beta\bar{\beta}e^{-2\phi/\alpha_+}$. The operator V is of the form $S\bar{S}$, with $S = \beta e^{-2\phi/\alpha_+}$ a dimension $(1,0)$ operator known as the screening current. The contour integral of S is known as the screening charge of the theory. The only holomorphic operators in the theory that commute with $\oint S$ are those constructed out of the sl_2 currents. In addition, the correlation functions of the theory can, after an appropriate number of screening charges have been inserted, be computed in the free field approximation.

To write down sigma models for AdS_{2d+1}, we simply take d copies of the single fields $\gamma,\bar{\gamma},\beta,\bar{\beta}$, and write down the same actions as above. The background charge in (2) remains unchanged in order for the perturbations to be exactly marginal. However, if we integrate out the fields $\beta,\bar{\beta}$ we now generate a nonzero background charge in (1), because each pair of $\beta,\bar{\beta}$ contributes $+2/\alpha_+$ to the background charge. Thus, the appropriate generalizations of (1) and (2) read

$$S = \frac{k}{2\pi}\int d^2z\,(\partial\phi\bar{\partial}\phi + e^{2\phi}\partial\bar{\gamma}_r\bar{\partial}\gamma_r) + \frac{1}{2\pi}\int d^2z\,(d-1)\phi\sqrt{g}R. \tag{3}$$

and

$$S = \frac{1}{4\pi}\int d^2z\,(\partial\phi\bar{\partial}\phi + \beta_r\bar{\partial}\gamma_r + \bar{\beta}_r\partial\bar{\gamma}_r - \beta_r\bar{\beta}_r e^{-2\phi/\alpha_+} - \frac{2}{\alpha_+}\phi\sqrt{g}R) \tag{4}$$

where $r = 1,\ldots d$ is summed over, and $\alpha_+ = \sqrt{2(k-2d)}$.

In (3) we recognize a standard sigma model on AdS_{2d+1} with nonzero B field, $B = e^{2\phi} d\gamma_r \wedge d\bar{\gamma}_r$, and linear dilaton $\Phi = (d-1)\phi$. One can verify that these background fields solve the (lowest order in α') supergravity equations of motion.

The exact central charge follows from the free field representation (4), and is given by

$$c = (2d+1) + \frac{6}{k-2d}. \tag{5}$$

3. Reality Conditions, Holography, and Symmetries

The string backgrounds described by (3) describe Euclidean AdS_{2d+1} geometries if γ_r and $\bar{\gamma}_r$ are each others complex conjugates, and Lorentzian AdS_{2d+1} geometries if we take one pair of $\gamma_r, \bar{\gamma}_r$ real and independent. The B field is real in the Lorentzian directions, and imaginary in the Euclidean directions. It is rather awkward to have imaginary B-fields from the space-time point of view. However, the world-sheet conformal field theory is perfectly well-defined for such B-fields, as the example of Euclidean AdS shows. Thus, we will simply view the string theory as defining a set of correlation functions of some boundary theory. It would be interesting to know whether this boundary theory is unitary or not. But even if it is not, the AdS_{2d+1} sigma model may still contain important and interesting information about the unitary deformations of the boundary theory.

In the Euclidean case the B-field defines a complex structure on the boundary of AdS_{2d+1}. From this it is clear that the Lorentz group $SO(2d)$ of the boundary is broken to at most $U(d)$. A more careful analysis shows that, ignoring the dilaton, the background preserves $(d+1)^2$ symmetries of the full $SO(2d+1,1)$ isometry group of AdS_{2d+1}. In addition, there are $d+1$ holomorphic and $d+1$ antiholomorphic currents. It would be interesting to analyze the complete chiral algebra of the theory.

If we approach the boundary of AdS_{2d+1}, the description in terms of free fields given by (4) becomes very useful. Both the string coupling and the perturbation become very small near the boundary. On the other hand, it seems that the space-time theory is strongly coupled near the boundary, because the string coupling goes to infinity. However, the growth of the strength of the string interactions has to compete against the rate at which points at fixed $\gamma, \bar{\gamma}$ separate near the boundary. The situation has some similarities to the one discussed in [9]. To analyze these competing effects, it is better to pass to the Einstein frame. In the Einstein frame, the metric on AdS_{2d+1} becomes

$$ds_E^2 = e^{2\phi/(2d-1)}(e^{-2\phi} d\phi^2 + d\gamma_r d\bar{\gamma}_r). \tag{6}$$

Because of the positive power of e^{ϕ} in front of this expression, distances on the boundary parametrized by $\gamma_r, \bar{\gamma}_r$ go to infinity as ϕ goes to infinity, and therefore we expect holographic behavior for large ϕ as in [9].

4. Superstrings with AdS_{2d+1} Backgrounds

Any bosonic sigma model can classically be promoted to a NSR type sigma model with $N = (1,1)$ supersymmetry, by rewriting it in terms of superfields. If we apply this to (4) and work out the component formulation of the result, we obtain

$$\begin{aligned} S(AdS_{2d+1},\alpha_+) = \frac{1}{4\pi}\int (& - \lambda^L\bar\partial\lambda^L + \partial\phi\bar\partial\phi + \partial\lambda^R\lambda^R \\ & - \bar\sigma_r\partial\bar\psi^R_r + \bar\beta_r\partial\bar\gamma_r - \sigma_r\bar\partial\psi^L_r + \beta_r\bar\partial\gamma_r \\ & - e^{-2\phi/\alpha_+}(\bar\beta_r - \frac{2}{\alpha_+}\lambda^R\bar\sigma_r)(\beta_r - \frac{2}{\alpha_+}\lambda^L\sigma_r) \\ & - e^{-4\phi/\alpha_+}\sigma_r\bar\sigma_r\sigma_s\bar\sigma_s - \frac{2}{\alpha_+}\phi\sqrt{g}R). \end{aligned} \tag{7}$$

Here, λ, ψ, σ are fermionic degrees of freedom.

If we want to construct a critical string background, we also need to correct central charge. The central charge of

$$S(AdS_{2d_1+1},\alpha_+) + S(AdS_{2d_2+1}, i\alpha_+) \tag{8}$$

equals $c = 3(d_1 + d_2 + 1)$, so a critical string background is obtained e.g. by taking $d_1 = d_2 = 2$. Notice that the second term in (8) has an imaginary background charge $i\alpha_+$. In the case of $S(AdS_3, i\alpha_+)$, it is precisely this theory combined with an analytic continuation $\phi \to i\phi$ that has been used to compute the correlation functions of the $SU(2)$ WZW theory, although the action does not resemble that of the $SU(2)$ WZW theory at all. This is all very suggestive, and we expect that the analytic continuation of the $S(AdS_{2d+1}, i\alpha_+)$ theory is closely related to the theory of strings propagating on the sphere S^{2d+1}.

For the usual NSR strings, the spacetime supersymmetry generators are constructed by bosonizing the fermions, and involve a factor $e^{\varphi/2}$ where φ comes from the bosonized superghosts. Similar operators can be constructed for (8), but there is a new ingredient, namely the zero modes of γ_r and $\bar\gamma_r$ are well-defined and we can multiply any of the usual supersymmetry generators by an arbitrary function of these zero modes and still get a candidate supersymmetry generator with the right conformal weight. However, not all of these are good operators in the theory defined by (8). Good operators have to satisfy additional requirements, namely the supersymmetry operators need to have the right operator product expansions with the generators of the $N = 1$ superconformal symmetry of (7), and in addition they need to commute with the screening charges. These conditions were analyzed in [7] and it was found that there are $2(d_1 + 1)(d_2 + 1)$ supersymmetry generators that survive. In particular, for "$AdS_5 \times S^5$" we find that the theory has 18 supersymmetries. If we denote the RR ground states as usual with five $\pm$ signs, then the nine left-moving ground states correspond to those combinations $(\pm,\pm,\pm,\pm,\pm)$ for which the first two signs are not both minus, the last two signs are not both minus, and for which the product of all signs is $+1$.

One may wonder how it is possible to have a four dimensional theory with 18 supersymmetries. Usually, the number has to be a multiple of four, but this argument assumes unbroken $SO(4)$ Lorentz invariance. In our case Lorentz invariance is broken to $U(2)$, and the number no longer needs to be a multiple of four.

5. A Novel Coset Construction

The AdS_{2d+1} sigma model given by (3) is an example of the sort of sigma models that can be obtained by means of a novel type of coset construction. Anti-de Sitter spaces are homogeneous spaces of the form G/H, and it seems natural to construct conformal field theories for such spaces using a coset construction. However, in the usual coset construction we identify $g \sim hgh^{-1}$ rather than $g \sim hg$, and the resulting space is usually singular. Furthermore, its isometries form the group $Z_H \times Z_H$, where Z_H is the centralizer of H in G. On the other hand, homogeneous spaces have a much larger isometry group, namely $Z_H \times G$. Thus usual cosets never describe homogeneous spaces. The novel cosets also generically do not describe homogeneous spaces, but resemble them more than usual cosets, and in particular they have more isometries than usual cosets.

Hints for the existence of these novel cosets appeared in [10], where it was noticed that given a group G and a subgroup $H = H_1 \times H_2 \times ...$, such that the levels of the corresponding current algebras satisfy $k^G + c_V^G = k^{H_1} + c_V^{H_1} = ...$, one can attempt to define some sort of "coset" theory, by getting rid of the degrees of freedom residing in H. The basic idea of this construction is to do a partial bosonization of the G currents, by expressing them in terms of the H currents and extra β, γ, ϕ systems, and to subsequently set the H currents to zero. The partial bosonization is based on on a Gauss-like decomposition $G = G_+ H G_0 G_-$ of G, explicit expressions for the currents and screening operators are given in [11].

In order to remove the H degrees of freedom, it would be nice to do some sort of gauging of the theory. However, we have not been able to find a gauging that reproduces the novel coset construction (see also the comments in the final section). Alternatively, one could attempt to impose $H = 1$ by means of some BRST procedure. This is problematic, because the conformal weight of H is nonzero in the quantum theory, so that H and 1 do not have the same conformal weight. The only way we found to get rid of H is to put $H = 1$ by brute force, by subsequently bosonizing the theory and by adjusting the background charges so that the screening charges have precisely conformal weight one. It turns out that there is always a unique answer for the background charges that works. Let us illustrate this with the example of $SL(3)/SL(2)$. We first write

$$G = \begin{pmatrix} 1 & \bar\gamma_1 & \bar\gamma_2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} H \begin{pmatrix} e^{-2\phi} & 0 & 0 \\ 0 & e^{\phi} & 0 \\ 0 & 0 & e^{\phi} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ \gamma_1 & 1 & 0 \\ \gamma_2 & 0 & 1 \end{pmatrix} \tag{9}$$

where H is an $SL(2)$ element embedded in the bottom right 2×2 block. Next we drop H and evaluate $S_{WZW}[G_+ G_0 G_-]$, and obtain

$$S = \frac{1}{2\pi} \int (3\partial\phi\bar\partial\phi + e^{3\phi}(\partial\bar\gamma_1\bar\partial\gamma_1 + \partial\bar\gamma_2\bar\partial\gamma_2)). \tag{10}$$

Bosonization of this theory leads to the same action as the one given in (4), but with the background charge $-2/\alpha_+$ replaced by $-4/\alpha_+$. In the final step we replace $-4/\alpha_+$ by $-2/\alpha_+$, so that the screening charges have conformal weight one, and we recover the action given in (4) with $d = 2$.

More generally, the novel cosets based on $SL(d+1)/SL(d)$ yield the sigma models with AdS_{2d+1} backgrounds given in (3). General G/H models have a holomorphic current algebra corresponding to the group G_+G_0, and an anti-holomorphic current algebra corresponding to G_0G_-, and $\dim G + \dim G_0$ isometries. The group of isometries does not contain a subgroup isomorphic to G (and therefore the target spaces are not the homogeneous spaces G/H, but they could be a different homogeneous space), but does contain a subgroup isomorphic to H.

6. Conclusions

One interesting set of novel cosets is obtained by applying the general construction to cosets of supergroups, especially in view of the results of [5, 6]. For example, $SL(3|3)/SL(2)XSL(2)$ yields a sigma model with target space $AdS_5 \times S^5$, 18 anti-commuting scalars, central charge $c = -8$, and with both bosonic and fermionic B fields turned on. The number 18 is similar to the number of space-time supersymmetries that appeared in the NSR formulation of "$AdS_5 \times S^5$", and it is tempting to think that one is some kind of Green-Schwarz reformulation of the other (although the supercoset does not have any κ-symmetry). Another example is $SL(4|4)/SP(2)^2$, which also has $AdS_5 \times S^5$ target space, 32 anti-commuting scalars (again suggesting a relation to the GS string) and $c = -22$. At this stage, these supercosets are just conformal field theories, and it is not known how to promote them to full-fledged string theories and in particular what the space-time interpretation of these theories is. Once their space-time interpretation is understood, one could analyze whether they do in fact describe theories with some RR background turned on (we suspect that RR backgrounds are related to fermionic screening charges) or whether one needs to perturb the theories away from the "WZW" point in order to turn on RR backgrounds.

We already mentioned that it would be desirable to understand the holographic properties and unitarity of the AdS sigma models better, and to what extend they are really exactly solvable conformal field theories. It would also be helpful to know whether they arise as the near-horizon limit of some brane configuration. The only brane configurations with only NS charges are configurations of fundamental strings and NS fivebranes, and a suitable delocalized superposition of them corresponds to the AdS sigma models. The meaning of these configurations is, however, rather obscure.

Recently, there was considerable interest in noncommutative Yang-Mills theories that arise on branes in the presence of B-fields, and one naturally wonders whether there is a relation between our AdS sigma models and noncommutative gauge theories. The B-field that appears in (3) is proportional to $e^{2\phi}$, whereas the supergravity solution for D3 branes in the presence of a background B field [12] involves B fields of the form

$B = ae^{4\phi}/(1 + be^{4\phi})$, which behaves differently from $e^{2\phi}$ for both large and small ϕ. Therefore the AdS sigma models are at most relevant for some intermediate regime.

So far we only described AdS_p spaces with odd p. It should also be possible to construct sigma models for theories involving AdS_p with p even. For example, $AdS_2 \times S^2 \times S^1$ with B-field given by $dB = d\theta \wedge (dvol(S^2) + dvol(AdS_2))$, with θ the angular coordinate along S^1 and $dvol$ indicating the volume form, is a solution of the supergravity equations of motion and should be described by some exact CFT.

The analysis of the AdS sigma models would be simplified considerably if we would know how to realize them in terms of e.g. gauged WZW models. In fact, many exact solutions of string theory can be obtained from various gauged WZW models (see e.g. [13] and references therein). An interesting set of gauged WZW theories are the following. Write $H = H_+ H_0 H_-$. One can now easily construct a gauged WZW theory where one gauges H_+ from the left, H_- from the right, and one gauges H_0 either axially or vectorially. As far as we know, these gauged WZW models, which we could also denote by G/H, have not been considered in the literature, and are much closer to our novel coset than the usual coset construction. However, by looking at a simple example one quickly sees they do not give rise to AdS sigma models. A derivation of the AdS sigma models directly from WZW theory remains an interesting open problem‖.

Finally, what about string theories on $AdS_5 \times S^5$ with RR flux¶? One way to construct such a string theory could be as follows. First one rewrites the $AdS_5 \times S^5$ sigma model with NS flux in terms of the ten-dimensional $U(5)$ covariant hybrid variables that were introduced in [14]. Subsequently one deforms the theory by turning on a RR background and by turning off the NS background. Since the theory already has the right target space geometry, it should be considerably easier to do this than to deform the theory starting from flat space. The $U(5)$ covariant hybrid formulation has five anticommuting scalars, and an interesting candidate for the final theory is that it is a deformation of the novel coset $GL(3|3)/GL(2|2)$, but more work remains to be done to find out whether this can be made more precise.

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‖ One way to see why the AdS sigma models do not correspond to ordinary WZW cosets is by looking at their central charge (5), which can not in any obvious way be written as the difference of the central charges of two WZW theories.

¶ Another interesting string theory to think about is the description of string theory on $AdS_3 \times S^3 \times S^3$ with RR flux, for which one suspects a close relation to the $D(2, 1, \alpha)$ WZW theory.

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