Quantisation of Open String with Dirichlet Boundary Conditions

Demanding the action to be stationary, yields the equations of motion $(\partial_\tau^2 - \partial_\sigma^2)X^\mu(\tau,\sigma) = 0$ as well as equations which determine boundary conditions:

$$X'^\mu(\tau,\sigma)\delta X^\mu(\tau,\sigma)|_{\sigma=0,\pi} = 0$$

1. What is the physical interpretation of the boundary condition $\delta X = 0$ and what consequences does it have for the symmetries, both on the worldsheet and in spacetime?

2. Choose $X(\tau,\sigma)|_{\sigma=0,\pi} =$ constant as a particular realisation of $\delta X = 0$. Write down the most general solution to the string equations of motion obeying these boundary conditions.

3. Using the results obtained in excercises 2.9 and 2.10 for the open string with *Neumann* conditions (i.e. $X'^\mu(\tau,\sigma)|_{\sigma=0,\pi} = 0$), show that the massless physical states of the open string with *Neumann* conditions have one Lorentz index and have D-2 linearly independent polarisations, as is expected for a massless gauge vector particle.

4. Repeat excercise 2.9 for an open string with *Dirichlet* boundary conditions (i.e. $\delta X(\tau,\sigma)|_{\sigma=0,\pi} = 0$). Pay attention to the operators α_0^r. Do they appear in the constraints this time?

5. In a Lorentz covariant theory in D dimensions the mass is defined as

   $$m^2 = -\sum_{\mu=0}^{D-1}\sum_{\nu=0}^{D-1}\eta^{\mu\nu}p_\mu p_\nu$$

   Recalling the first question item above, what would the (classical) definition of the mass be for an open string which obeys Neumann conditions in the directions $X^\mu, \mu = 0,\cdots,k-1$ and Dirichlet conditions in the directions $X^r, r = k,\cdots,D-1$ (Use the solutions to the string equations of motion: eq. (1.18) for Neumann conditions and the result you obtained above for Dirichlet conditions)

6. As in equation (2.11), in the quantummechanical case, there can be massless states of the string with Dirichlet boundary conditions, which are of the form $\alpha_{-1}^r|0,k^\nu\rangle$. Now what is the number of *physical* massless states, when now the oscillator

index labels $D-k$ directions with Dirichlet boundary conditions?