PHILIPS CTT Hogeschool van Utrecht

Systeem en Regeltechniek

FMT / Mechatronica

Deel 1: Inleidende verkenning

Blok 2: Basisprincipes modelvorming massa-veersystemen

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Cursus Systeem en Regeltechniek

Overzicht

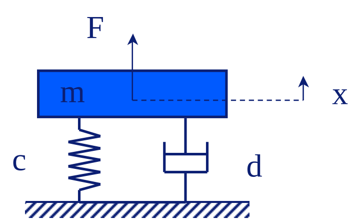
What is a Mass-Spring-System?

?

Basic principles modelling mass-spring-systems

Basic equations

Linear		Rotary	
F	[N]	M	[Nm]
$\ddot{x}$	[ms^{-2}]	$\ddot{\varphi}$	[s^{-2}]
c	[Nm^{-1}]	k	[Nm]
m	[kg]	J	[kgm^2]
d	[N/(m/s)]	d_φ	[Nms]



$F = m \cdot \ddot{x}$	$M = J \cdot \ddot{\varphi}$	Acceleration
$F_c = -c \cdot x$	$M_k = -k \cdot \phi$	Spring
$F_d = -d \cdot \dot{x}$	$M_d = -d_\varphi \cdot \dot{\varphi}$	Damper

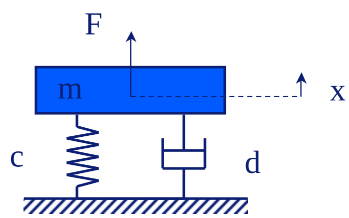
Equation of motion

Translation:

$$m \cdot \ddot{x} + d \cdot \dot{x} + c \cdot x = F$$

Rotation:

$$J \cdot \ddot{\varphi} + d_{\varphi} \cdot \dot{\varphi} + k \cdot \varphi = M$$



Free motion (translation)

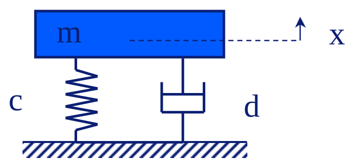
$$m \cdot \ddot{x} + d \cdot \dot{x} + c \cdot x = 0$$

$$\Updownarrow$$

$$\ddot{x} + \frac{d}{m} \cdot \dot{x} + \frac{c}{m} \cdot x = 0 \quad \Leftrightarrow \quad \frac{m}{c} \cdot \ddot{x} + \frac{d}{c} \cdot \dot{x} + x = 0$$

$$\Updownarrow \qquad\qquad \Updownarrow$$

$$\ddot{x} + 2\beta\omega_e \cdot \dot{x} + \omega_e^2 \cdot x = 0 \quad \Leftrightarrow \quad \frac{1}{\omega_e^2} \cdot \ddot{x} + \frac{2\beta}{\omega_e} \cdot \dot{x} + x = 0$$



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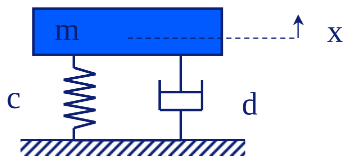
Basic principles modelling mass-spring-systems

Resonance frequency & damping

$$\frac{m}{c}\cdot\ddot{x}+\frac{d}{c}\cdot\dot{x}+x=0$$

$$\frac{1}{\omega_e^2}\cdot\ddot{x}+\frac{2\beta}{\omega_e}\cdot\dot{x}+x=0$$



Resonance frequency: $\omega_e=\sqrt{\frac{c}{m}},\quad f_e=\frac{1}{2\pi}\sqrt{\frac{c}{m}}$

Relative damping: $\beta=\frac{d}{2\sqrt{m\cdot c}}=\frac{d}{2\cdot m\cdot\omega_e}\leq 1$

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Intermezzo

$$\omega_e=2\pi f_e$$

$$\omega_e^2=4\pi^2 f_e^2$$

$$\omega_e^2\approx 40 f_e^2$$

Approximation???

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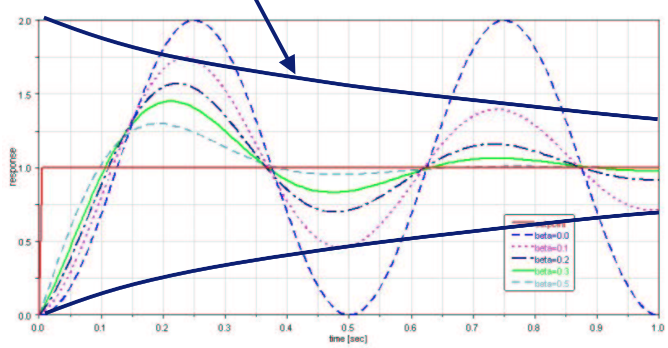
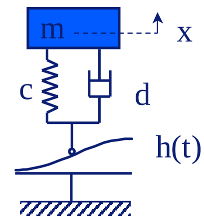
Step response of mass-spring-system

$$x = 1 - e^{-\beta\omega_e t} \cdot \cos(\omega_e \sqrt{1-\beta^2} \cdot t)$$





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Summary

- Equation of motion
- Resonance frequency
- Relative damping
- Step response