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Systemen en Regeltechniek FMT / Mechatronica

Deel 2: **Basisbegrippen regeltechniek**

Blok 4: Frequentie-domein beschrijving

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Cursus Systemen en Regeltechniek

Overzicht

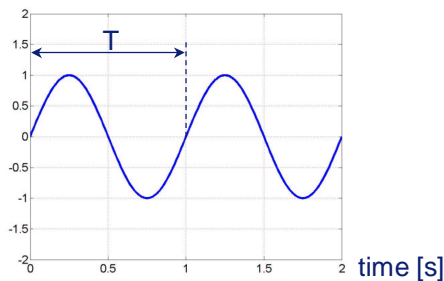
- Deel 1 Blok 1. Inleiding
Wo. 14-04 Blok 2. Basisprincipes modelvorming massa-veersystemen
 Blok 3. De regelaar als veer-demper combinatie
- Deel 2 Blok 4. [Frequentie-domein beschrijving](#)
Wo. 21-04 Blok 5. Basisconcepten in de regeltheorie
- Deel 3 **Vervolg regeltechniek**
Wo. 28-04
- Deel 4 **Stabiliteit van regelsystemen**
Wo. 12-05
- Deel 5 **Toepassing: PID regelaarontwerp**
Wo. 19-05
- Deel 6 **Extra regeltechniek**
Wo. 26-05

What is a frequency domain description?



Intro Frequency Domain

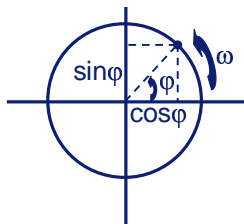
What is frequency ?



$$f = \frac{1}{T} \quad [\text{Hz}]$$

smaller period →
higher frequency

$$\omega = 2\pi f \quad [\text{rad/s}]$$

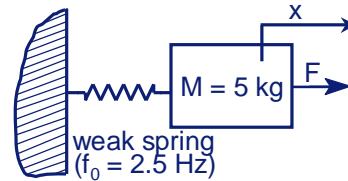


φ: phase angle

one period

- ⇔ one revolution
- ⇔ 360° degrees
- ⇔ 2π radians

From Time Domain to Frequency Domain



For sinusoidal excitation:

$$F(t) = 400 \sin(2\pi 7t) \quad (f = 7 \text{ Hz})$$

A linear system gives a sinusoidal response:

$$x(t) = \hat{x} \sin(2\pi 7t + \varphi) \quad \hat{x}=?; \varphi=? \quad @ f = 7 \text{ Hz}$$

Frequency Domain Response: **H(f)**

$$\longrightarrow \hat{x}/\hat{F} = |H(7\text{Hz})|$$

$$\longrightarrow \varphi = \angle H(7\text{Hz})$$

Intermezzo: complex numbers

$$s = a + bj$$

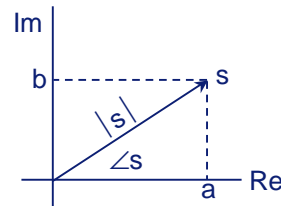
$$j^2 = -1$$

$$\text{Re}(s) = a$$

$$|s| = \sqrt{a^2 + b^2}$$

$$\text{Im}(s) = b$$

$$\angle s = \text{atan}\left(\frac{b}{a}\right)$$



Define: $\varphi = \angle s$

Then: $s = |s| [\cos(\varphi) + j \sin(\varphi)]$

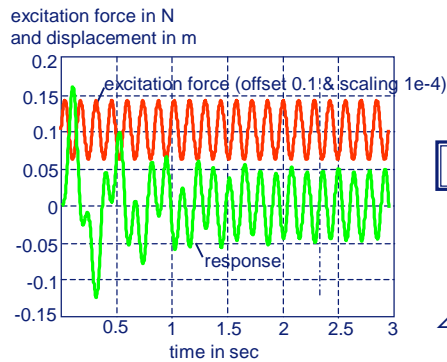
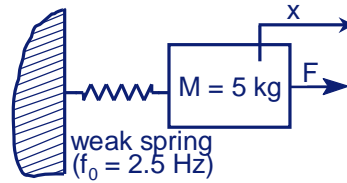
So: $a = |s| \cos(\varphi)$

$b = |s| \sin(\varphi)$

From Time Domain to Frequency Domain

$$F(t) = 400 \sin(2\pi 7t)$$

$$x(t) = \hat{x} \sin(2\pi 7t + \varphi)$$



Frequency Domain Response: $H(7\text{Hz})$

$$|H(7\text{Hz})| \approx 0.045 / 400 = 1e-4 \text{ m/N}$$

$$\angle H(7\text{Hz}) \approx -180^\circ$$

Frequency Domain solution for equation of motion

$$F = M\ddot{x}$$

1) Choose a sinusoidal input:

$$F = \hat{F} \sin(\omega t)$$

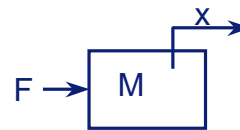
2) Then for a linear system:

$$x = \hat{x} \sin(\omega t + \varphi)$$

$$\dot{x} = \omega \hat{x} \cos(\omega t + \varphi)$$

$$\ddot{x} = -\omega^2 \hat{x} \sin(\omega t + \varphi)$$

$$\hat{x} = ?; \varphi = ?$$



Frequency Domain solution for equation of motion

$$F = M\ddot{x}$$

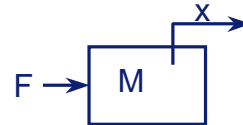
3) Solution:

$$\hat{F}\sin(\omega t) = -M\omega^2 \hat{x}\sin(\omega t + \varphi)$$

$$\varphi = \angle H = -180^\circ$$

$$\hat{x} = |H| \hat{F} = \frac{\hat{F}}{M\omega^2}$$

$$H = \frac{x}{F} = -\frac{1}{M\omega^2} \leftarrow \text{Frequency Response Function !}$$



Summary

- Frequency
- Frequency response H(f)
 - Amplitude
 - Phase
- Simple mass as example:

$$H = \frac{x}{F} = -\frac{1}{M\omega^2}$$

