PHILIPS CTT Hogeschool van Utrecht

Systeem en Regeltechniek
FMT / Mechatronica

Deel 2: Basisbegrippen regeltechniek
Blok 5: Basisconcepten in de regeltheorie

Gert van Schothorst

Philips Centre for Technical Training (CTT)
Philips Centre for Industrial Technology (CFT)
Hogeschool van Utrecht - PTGroep

Cursus Systeem en Regeltechniek
Overzicht

Deel 1 *Wo. 14-04*
- Blok 1. Inleiding
- Blok 2. Basisprincipes modelvorming massa-veersystemen
- Blok 3. De regelaar als veer-demper combinatie

Deel 2 *Wo. 21-04*
- Blok 4. Frequentie-domein beschrijving
- Blok 5. Basisconcepten in de regeltheorie

Deel 3 *Wo. 28-04* **Vervolg regeltechniek**

Deel 4 *Wo. 12-05* **Stabiliteit van regelsystemen**

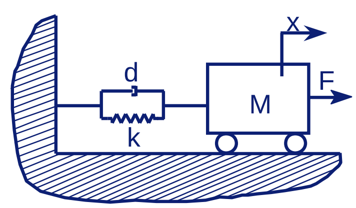
Deel 5 *Wo. 19-05* **Toepassing: PID regelaarontwerp**

Deel 6 *Wo. 26-05* **Extra regeltechniek**

What is the Bode plot of a system?

?

Time ⇒ Frequency ⇒ Laplace

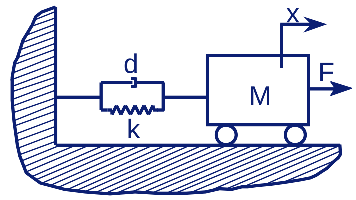


Time domain representation
in block scheme:

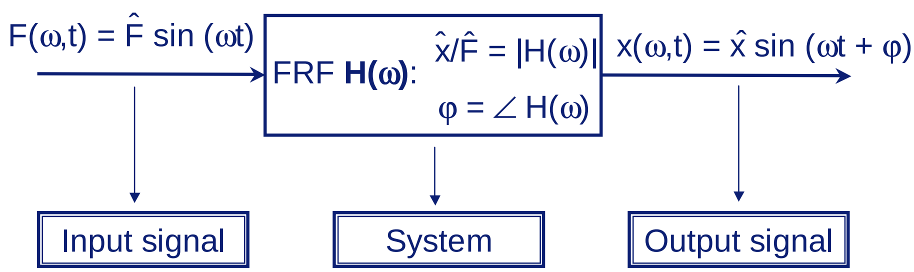
F(t) → **Equation of motion:** $M\ddot{x}(t)+d\dot{x}+kx=F(t)$ → x(t)

Input signal | System | Output signal

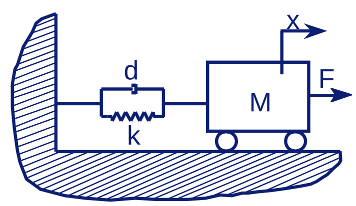
Time ⇒ Frequency ⇒ Laplace



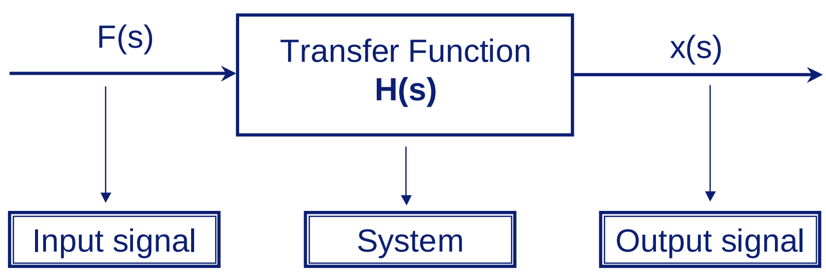
Frequency domain
representation in block scheme:



Time ⇒ Frequency ⇒ Laplace



Laplace domain representation
in block scheme:



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Basic concepts in control theory

What does H(s) look like?

?

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Basic concepts in control theory

Laplace transform

Laplace transform of a signal: $\mathfrak{L}\{x(t)\} = X(s)$

$\mathfrak{L}\{F(t)\} = F(s)$

Example: $x(t) = \sin(t) \xrightarrow{\mathfrak{L}} X(s) = \frac{1}{s^2+1}$

Laplace transform of a system:

Differential equation (equation of motion) $\xrightarrow{\mathfrak{L}}$ Transfer Function H(s)

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Laplace transform

Property of the Laplace transform:

$$\mathfrak{L}\{\dot{x}(t)\} = s \cdot X(s) - x(0)$$
$$\mathfrak{L}\{\ddot{x}(t)\} = s^2 \cdot X(s) - \dot{x}(0) - s \cdot x(0)$$

As x(0) and $\dot{x}(0)$ are zero most of the time, this simplifies to:

$$\boxed{\frac{d}{dt} \rightarrow s}$$

Application to mechanical system (equation of motion):

$$M\ddot{x}(t) + d\dot{x} + kx = F(t) \quad \overset{\mathfrak{L}}{\rightarrow} \quad Ms^2 \cdot x(s) + ds \cdot x(s) + k \cdot x(s) = F(s)$$

Or: $(Ms^2 + ds + k) \cdot x(s) = F(s)$

Transfer Function

So we have: $(Ms^2 + ds + k)x(s) = F(s)$

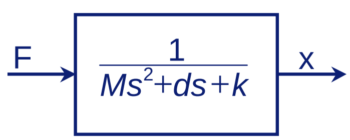
This leads to the Transfer Function H(s):

$$\boxed{H(s) = \frac{x(s)}{F(s)} = \frac{1}{Ms^2 + ds + k}}$$

Note that in Laplace domain:

x(s) = H(s)·F(s) So: *output* = *system* x *input* !!!

In block representation:



Frequency Response Function (FRF)

Consider sinusoidal signals (using "Euler's notation"):

$$x(t) = \mathrm{Re}\{\hat{x}(\cos\omega t + j\sin\omega t)\} = \mathrm{Re}\{\hat{x}e^{j\omega t}\}$$

$$\dot{x}(t) = \mathrm{Re}\{\omega\hat{x}(-\sin\omega t + j\cos\omega t)\} = \mathrm{Re}\{j\,\omega\hat{x}e^{j\omega t}\}$$

Apparently: s = jω for sinusoidal signals

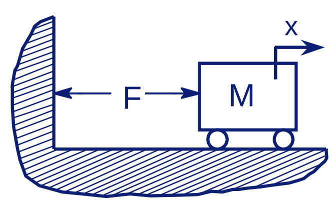
By replacing all 's' in the Transfer Function, s → jω

We find the Frequency Response Function:

$$H(j\omega) = \frac{1}{-M\omega^2 + jd\,\omega + k}$$

Bode plot

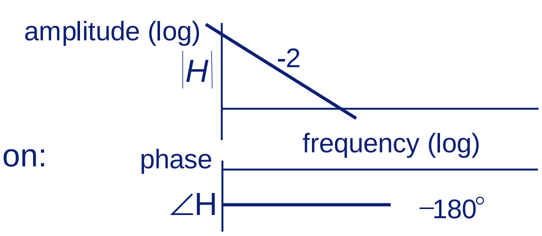
Recall Frequency Response Function (FRF):



$$H = \frac{x}{F} = -\frac{1}{M\omega^2}$$

$$\frac{\hat{x}}{\hat{F}} = |H| = \frac{1}{M\omega^2}$$

$$\varphi = \angle H = -180^\circ$$



Graphical representation:
Bode plot

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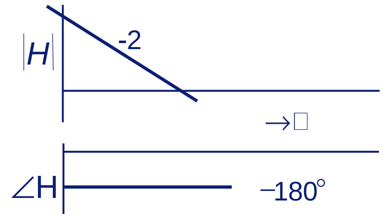
Logarithmic representation (in dB)

Why -2 slope?



$$\log(|H|) = \log\left|\frac{1}{M\omega^2}\right| = \log\frac{1}{M} + \log\frac{1}{\omega^2}$$

$$= \log\left(\frac{1}{M}\right) - 2\log(\omega)$$

Towards amplitude in dB...

|H| in dB = 20 log |H|

-2 slope ⇔ -40 dB/decade

1 ⇔ 0 dB
10 ⇔ 20 dB
100 ⇔ 40 dB
1000 ⇔ 60 dB
...

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What is 0.01 in dB?

?

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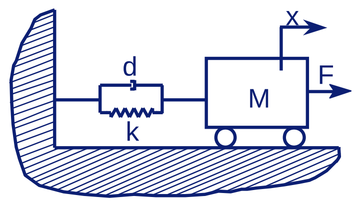
The Bode plot of a mass-spring-system

Transfer Function:

$$H(s) = \frac{x(s)}{F(s)} = \frac{1}{Ms^2 + ds + k}$$

$$H(s) = \frac{1/M}{s^2 + 2\beta\omega_o s + \omega_o^2}$$



Recall: $\omega_o = \sqrt{\frac{k}{M}}$ Eigenfrequency

$\beta = \frac{d}{2\sqrt{Mk}}$ Relative damping

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The Bode plot of a mass-spring-system

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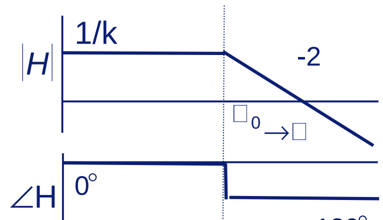

Asymptotes in Bode plot:

$s \to 0 \quad \Rightarrow \quad H \to 1/k \quad \Rightarrow \quad |H| \to 1/k$, $\angle H \to 0^o$

$s \to \infty \quad \Rightarrow \quad H \to 1/Ms^2 \quad \Rightarrow \quad |H| \to 1/M\omega^2$, $\angle H \to 180^o$

Break point:

$\log(1/k) = \log(1/M\omega^2) = \log(1/M) - 2\log(\omega)$

$$\omega = \omega_o = \sqrt{\frac{k}{M}}$$

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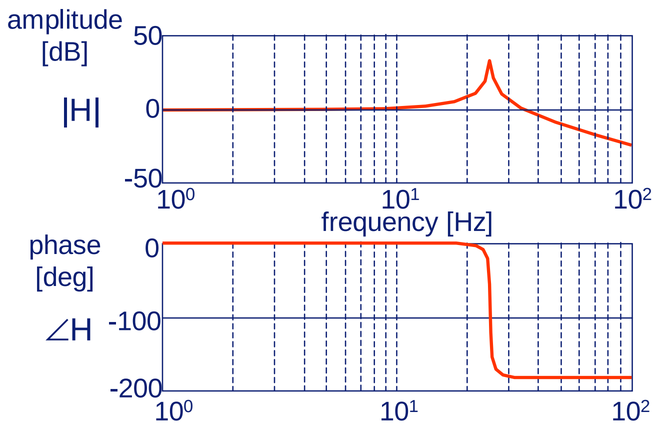
The Bode plot of a mass-spring-system

$$H(s) = \frac{x(s)}{F(s)} = \frac{1}{Ms^2 + ds + k}$$

$$H(\omega) = \frac{1}{-M\omega^2 + jd\,\omega + k}$$



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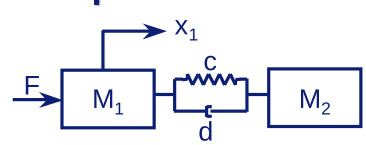
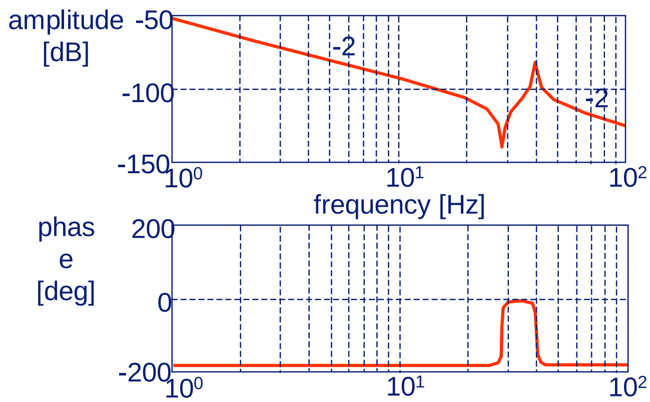
Other Bode plot examples

$$H(s) = \frac{\omega_o^2/\omega_n^2}{Ms^2}\;\frac{s^2 + 2\beta\omega_n s + \omega_n^2}{s^2 + 2\beta\omega_o s + \omega_o^2}$$





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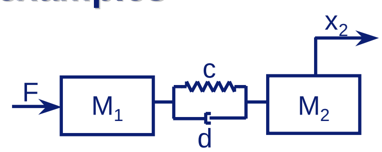
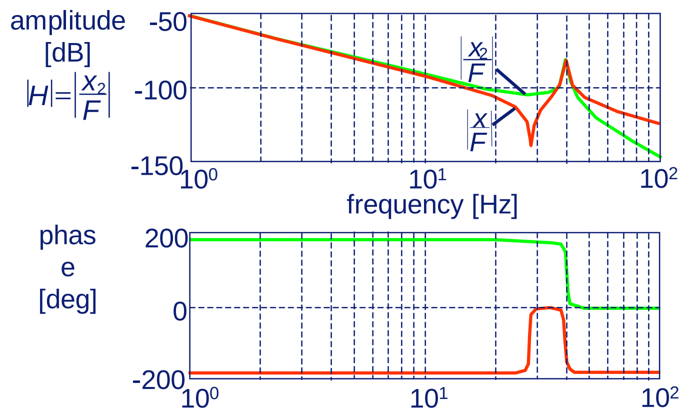
Other Bode plot examples

$$H(s) = \frac{1}{Ms^2}\,\frac{\omega_o^2}{s^2 + 2\beta\omega_o s + \omega_o^2}$$





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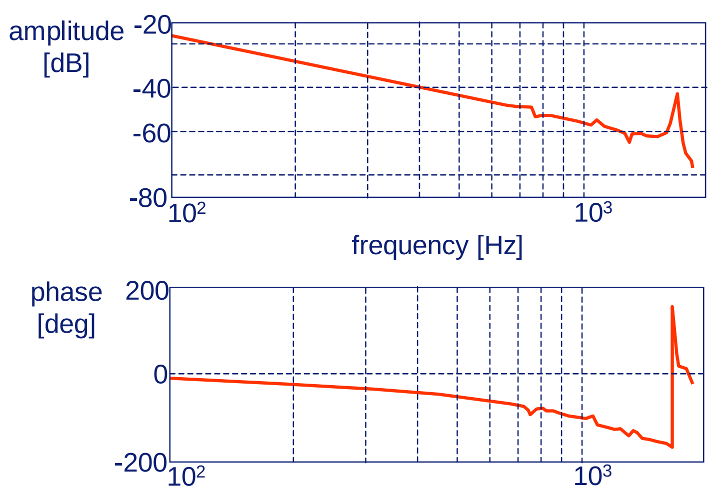
Other Bode plot examples

Measurement mechanics of reticle stage



Summary

- Laplace transform: $t \rightarrow \omega \rightarrow s$
- Transfer function: $x(s) = H(s) \cdot F(s)$
- Frequency Response Function (FRF): $s = j\omega \rightarrow H(\omega)$
- Bode plot:
  - $\log(|H|)$ vs. $\log(\omega)$
  - $\angle H$ vs. $\log(\omega)$

Exercise with 20-sim

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