PHILIPS CTT Hogeschool van Utrecht

Systeem en Regeltechniek

FMT / Mechatronica

Deel 3: Vervolg regeltechniek

Blok 7: De PD regelaar als veer-demper combinatie

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Cursus Systeem en Regeltechniek

Overzicht

Deel 1 *Wo. 14-04*	Blok 1.	Inleiding
	Blok 2.	Basisprincipes modelvorming massa-veersystemen
	Blok 3.	De regelaar als veer-demper combinatie
Deel 2 *Wo. 21-04*	Blok 4.	Frequentie-domein beschrijving
	Blok 5.	Basisconcepten in de regeltheorie
Deel 3 *Wo. 28-04*	Blok 6.	Verdere inleiding in de regeltheorie
	Blok 7.	De PD regelaar als veer-demper combinatie
Deel 4 *Wo. 12-05*	**Stabiliteit van regelsystemen**	
Deel 5 *Wo. 19-05*	**Toepassing: PID regelaarontwerp**	
Deel 6 *Wo. 26-05*	**Extra regeltechniek**	

The PD controller as spring-damper combination

What would be a good feedback controller for a moving mass?

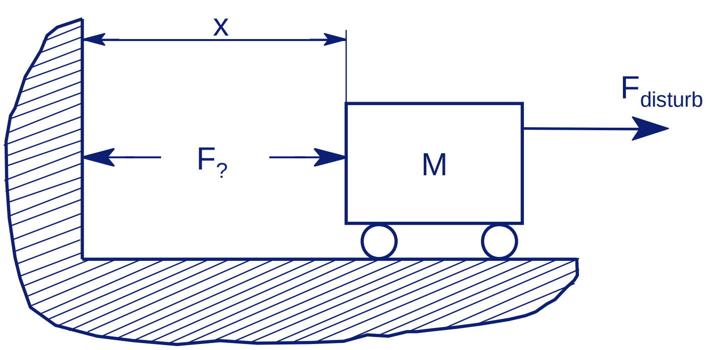
?

Recapitulation time domain interpretation

Introduction frequency domain interpretation

The PD controller as spring-damper combination

Controlling the position of a mass



Control Objectives:

- Getting there
- Staying there ←

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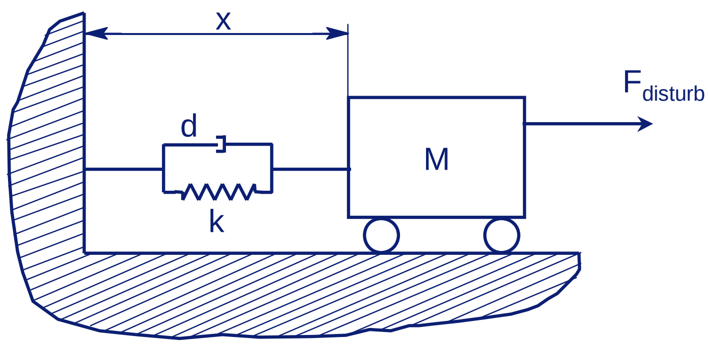
The PD controller as spring-damper combination

First control objective ‘staying there’:



Spring-damper force F: $F = -kx - d\dot{x}$

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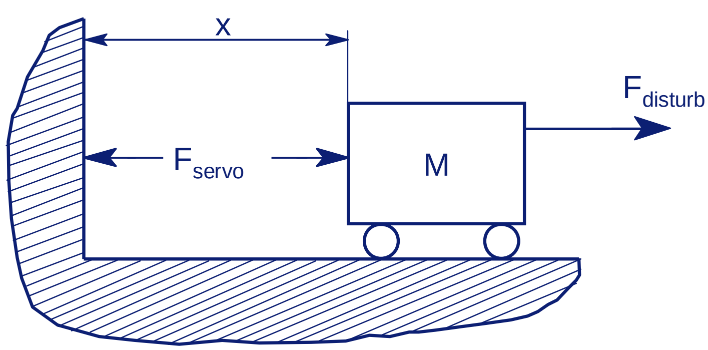
The PD controller as spring-damper combination

First control objective ‘staying there’:



Controller: $F_{servo} = -k_p x - k_v \dot{x}$

k_p servo stiffness
k_v servo damping

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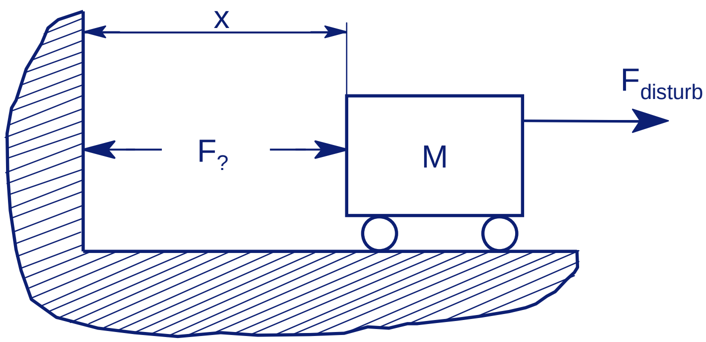
The PD controller as spring-damper combination

Controlling the position of a mass



Control Objectives:

- Getting there ←
- Staying there

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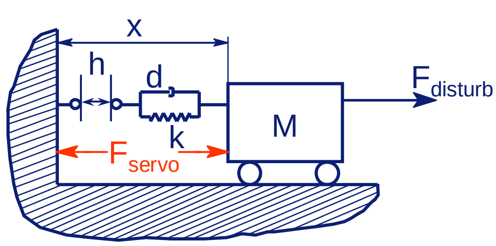
The PD controller as spring-damper combination

Second control objective ‘getting there’:



Spring-damper force F: $F = -k(h - x) - d(\dot{h} - \dot{x})$

Controller: $F_{servo} = k_p(x_s - x) + k_v(\dot{x}_s - \dot{x})$

(x_s: setpoint)

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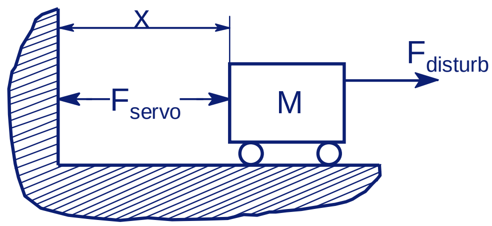
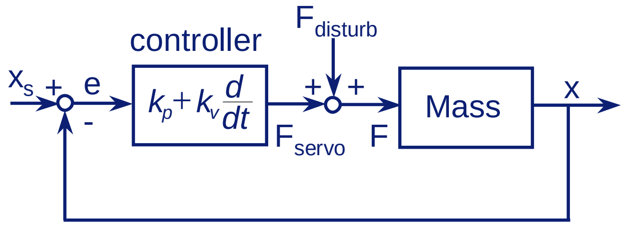
The PD controller as spring-damper combination

The controlled system



Controller: $F_{servo} = k_p(x_s - x) + k_v(\dot{x}_s - \dot{x})$



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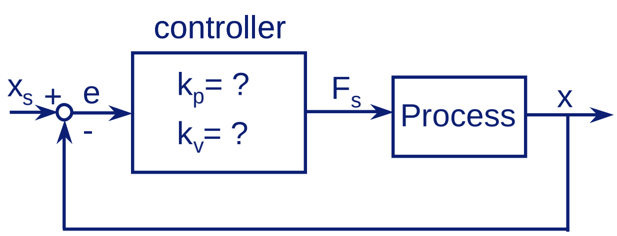
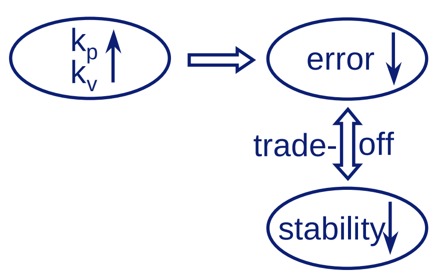
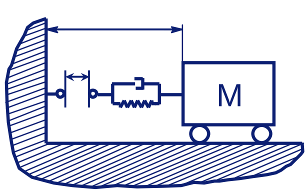
The PD controller as spring-damper combination

Tuning the k_p-k_v controller







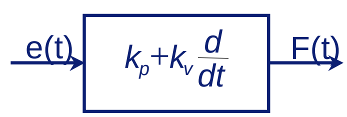
How does the k_p k_v controller look like in the frequency domain?

?

PD controller: Time / Laplace / Frequency

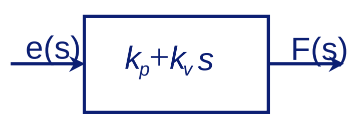
Time domain:

$$F = k_p e + k_v \dot{e}$$



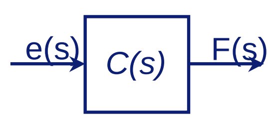
Laplace domain:

$$F(s) = (k_p + k_v s) e(s)$$



Transfer function:

$$C(s) = \frac{F}{e}(s) = (k_p + k_v s)$$



Frequency response:

$$C = k_p + j k_v \omega$$

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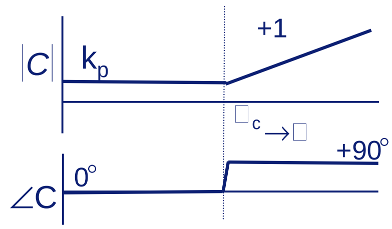
The PD controller as spring-damper combination

The Bode plot of a PD controller

Transfer Function:

$$C(s) = \frac{F}{e}(s) = (k_p + k_v s)$$



Asymptotes in Bode plot:

Amplitude: $|C| = \sqrt{k_p^2 + k_v^2 \omega^2}$

$s \to 0 \quad \Rightarrow \quad C \to k_p \quad \Rightarrow \quad |C| \to k_p$, $\angle C \to 0^o$

$s \to \infty \quad \Rightarrow \quad C \to k_v s \quad \Rightarrow \quad |C| \to k_v \omega$, $\angle C \to 90^o$

Break point: $\log k_p = \log k_v + \log \omega \quad \Rightarrow \quad \omega_c = \frac{k_p}{k_v}$

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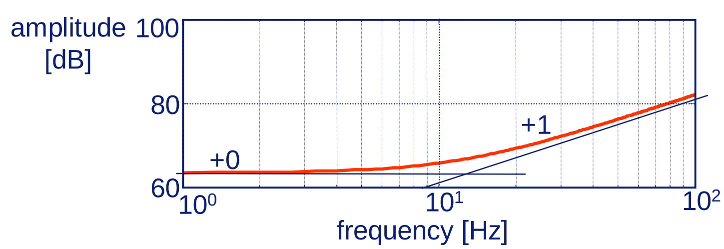
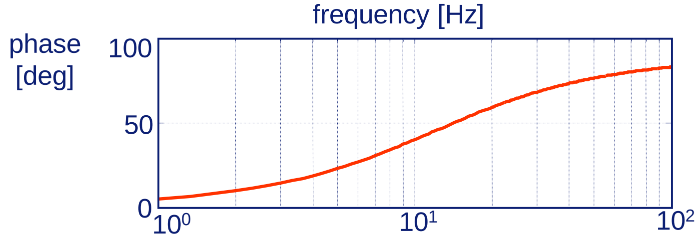
The PD controller as spring-damper combination

The Bode plot of a PD controller

k_p = 1500 N/m; k_v = 20 Ns/m





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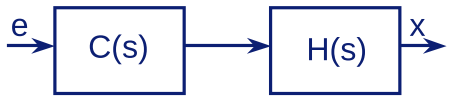
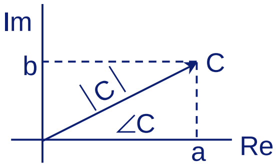
The PD controller as spring-damper combination

Intermezzo: Multiplication of Bode plots

The amplitude story





$C = a + bj$ $\quad$ $H = c + dj$ $\quad$ $CH = ac + bcj + adj - bd$

$Re(C) = a$ $\quad$ $Re(H) = c$ $\quad$ $Re(CH) = ac - bd$

$Im(C) = b$ $\quad$ $Im(H) = d$ $\quad$ $Im(CH) = bc + ad$

$|C| = \sqrt{a^2 + b^2}$ $\quad$ $|H| = \sqrt{c^2 + d^2}$ $\quad$ $|CH| = \sqrt{(a^2 + b^2)\ (c^2 + d^2)}$

$$|C \cdot H| = |C| \cdot |H|$$

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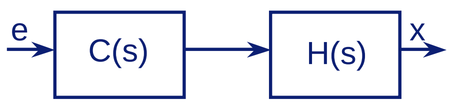
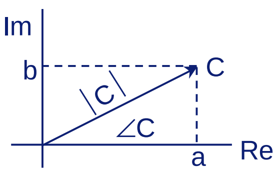
The PD controller as spring-damper combination

Intermezzo: Multiplication of Bode plots

The phase story





$C = |C| \cdot [\cos(\varphi) + j\sin(\varphi)]$ $\quad$ $H = |H| \cdot [\cos(\psi) + j\sin(\psi)]$

$$C \cdot H = |C| \cdot |H| \cdot [\cos(\varphi) + j\sin(\varphi)] \cdot [\cos(\psi) + j\sin(\psi)]$$

$$= |C \cdot H| \cdot [(\cos(\varphi)\cos(\psi) - \sin(\varphi)\sin(\psi)) + \ldots$$

$$\ldots\ j\,(\cos(\varphi)\sin(\psi) + \sin(\varphi)\cos(\psi))]$$

$$= |C \cdot H| \cdot [\cos(\varphi + \psi) + j\sin(\varphi + \psi)]$$

$\left.\begin{matrix}\varphi = \angle C \\ \psi = \angle H\end{matrix}\right\} \longrightarrow$ $\boxed{\angle(C \cdot H) = \angle C + \angle H}$

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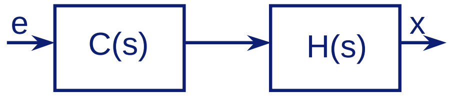
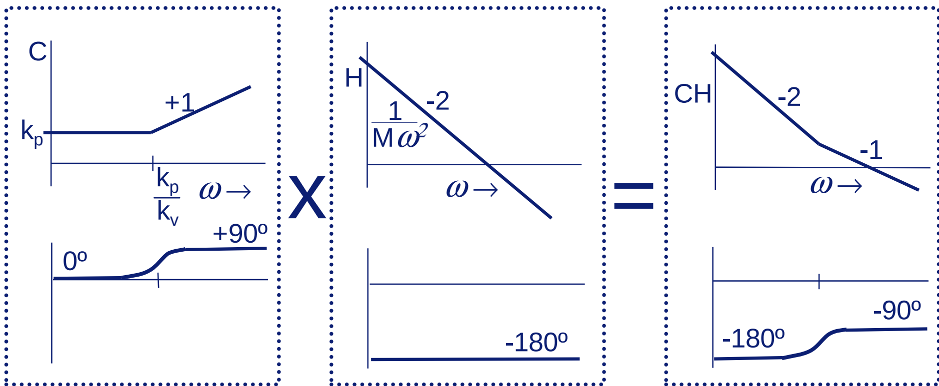
The PD controller as spring-damper combination

Intermezzo: Multiplication of Bode plots



$$\frac{x(s)}{e(s)} = C(s)H(s)$$



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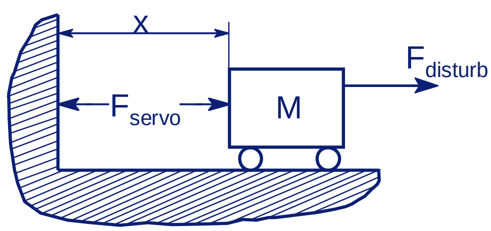
The PD controller as spring-damper combination

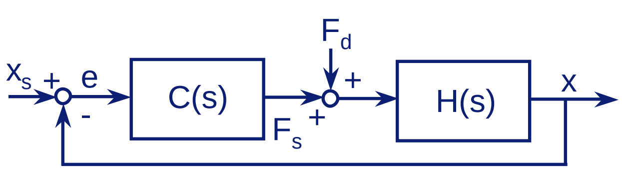
Closed loop with PD controller

Transfer functions:

$$H(s) = \frac{x(s)}{F(s)} = \frac{1}{Ms^2}$$

$$C(s) = \frac{F(s)}{e(s)} = (k_p + k_v s)$$





Recall: $x \approx x_s$ if: $C(s) \cdot H(s) >> 1$

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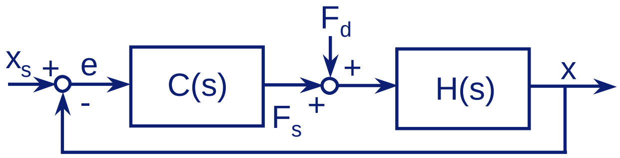
The PD controller as spring-damper combination

Four important transfer functions



Open Loop: $H_o(s) = \frac{x}{e} = C(s)H(s)$

Closed Loop: $H_c(s) = \frac{x}{x_s}(s) = \frac{C(s)H(s)}{1 + C(s)H(s)}$

Sensitivity: $S(s) = \frac{e}{x_s}(s) = \frac{1}{1 + C(s)H(s)}$

Process Sensitivity: $H_{ps}(s) = \frac{x}{F_d}(s) = \frac{H(s)}{1 + C(s)H(s)}$

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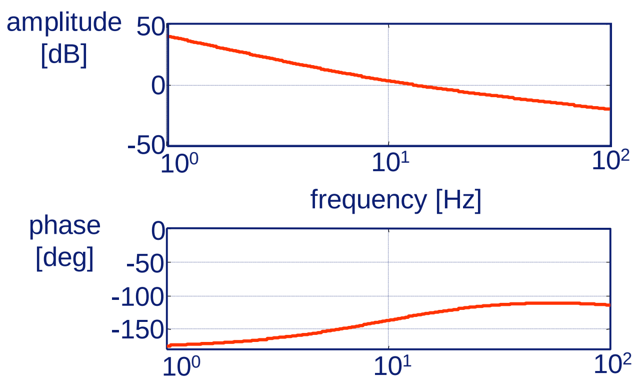
The PD controller as spring-damper combination

Open-Loop

$$H_o(s) = \frac{x}{e} = C(s)H(s)$$



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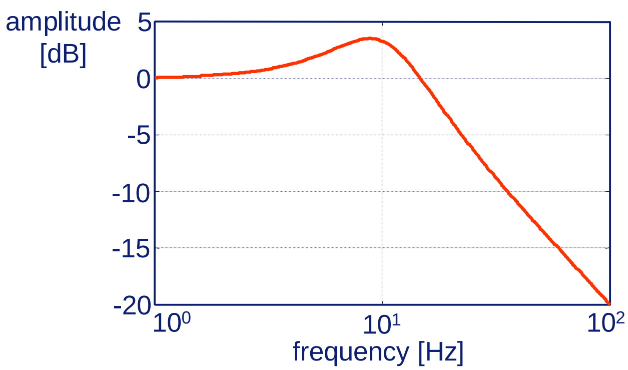
The PD controller as spring-damper combination

Closed-Loop

$$H_c(s) = \frac{x}{x_s}(s) = \frac{C(s)H(s)}{1 + C(s)H(s)}$$



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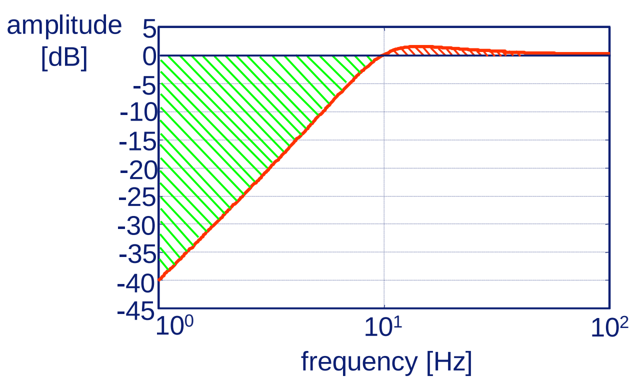
The PD controller as spring-damper combination

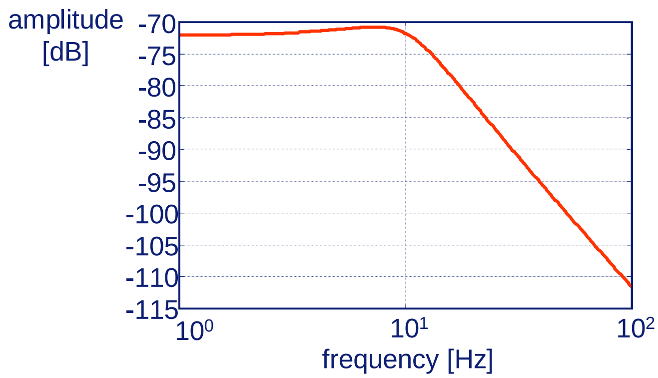
Sensitivity

$$S(s) = \frac{e}{x_s}(s) = \frac{1}{1 + C(s)H(s)}$$



Process Sensitivity

$$H_{ps}(s)=\frac{x}{F_d}(s)=\frac{H(s)}{1+C(s)H(s)}$$

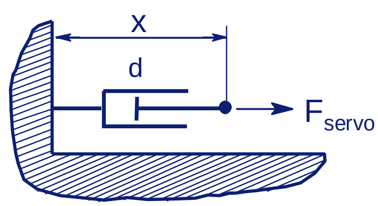


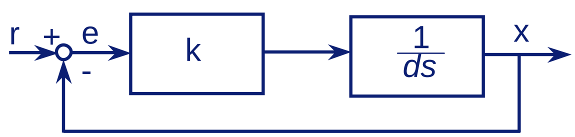
The Bandwidth Concept - First order system

Transfer functions:

$$H(s)=\frac{x(s)}{F(s)}=\frac{1}{ds}$$

$$C(s)=\frac{F(s)}{e(s)}=k_p$$





Closed loop: $H_c(s)=\frac{1}{\tau s+1}$ $(\tau=\frac{d}{k_p})$

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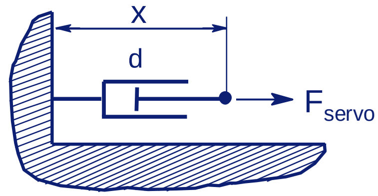
The PD controller as spring-damper combination

The Bandwidth Concept - First order system

Closed loop:

$$H_c(s) = \frac{1}{\tau s + 1} \quad (\tau = \frac{d}{k_p})$$



Bandwidth:

- -3 dB closed loop
- -45º closed loop
- 0 dB open loop

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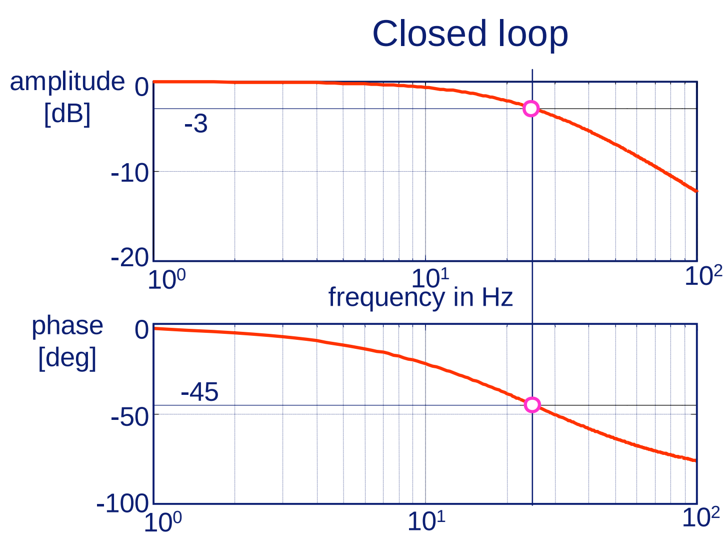
The PD controller as spring-damper combination

The Bandwidth Concept - First order system



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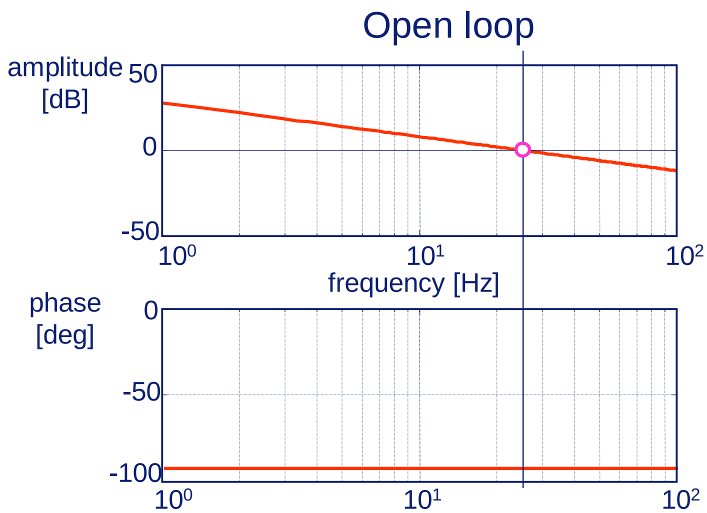
The PD controller as spring-damper combination

The Bandwidth Concept - First order system



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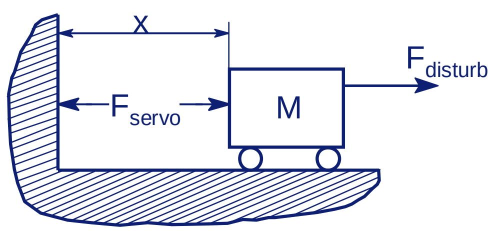
The PD controller as spring-damper combination

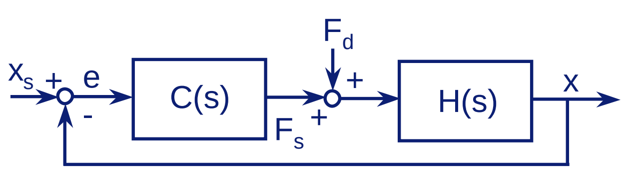
The Bandwidth Concept - Second order system

Transfer functions:

$$H(s) = \frac{x(s)}{F(s)} = \frac{1}{Ms^2}$$

$$C(s) = \frac{F(s)}{e(s)} = (k_p + k_v s)$$





Bandwidth???

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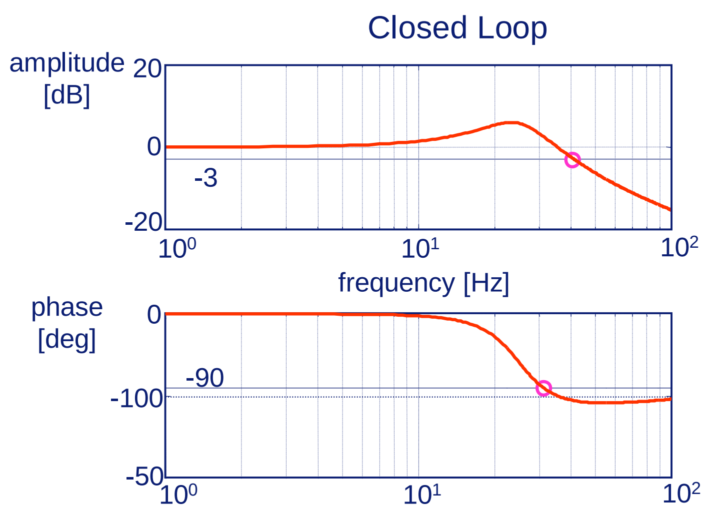
The PD controller as spring-damper combination

The Bandwidth Concept - Second order system



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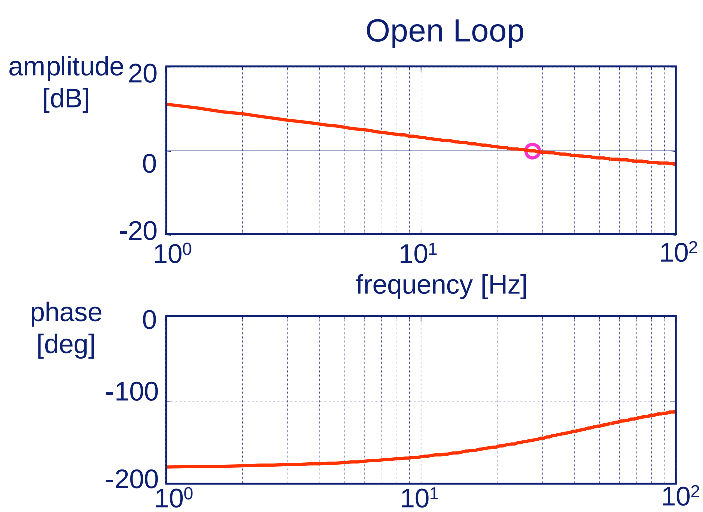
The PD controller as spring-damper combination

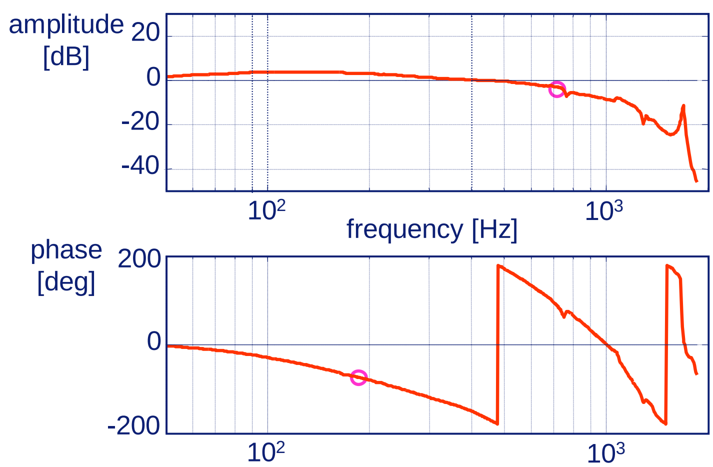
The Bandwidth Concept - Second order system



The PD controller as spring-damper combination

The Bandwidth Concept - Experimental result

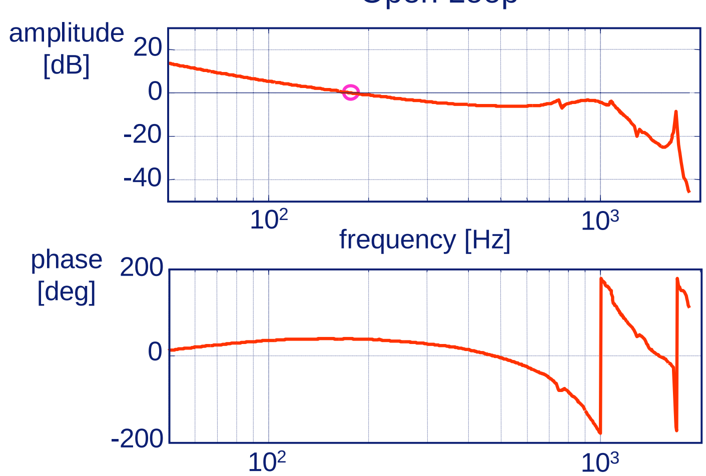
Closed Loop



The PD controller as spring-damper combination

The Bandwidth Concept - Experimental result

Open Loop



The Bandwidth Concept - ‘Definition’

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Bandwidth: 0 dB crossing open loop

Summary

- PD controller:
  - Time domain interpretation
  - Frequency domain description
  - Bode plot
    - Intermezzo: Multiplication of Bode plots
- Closed loop with PD controller
- The Bandwidth concept

Exercise with 20-sim