Systeem en Regeltechniek
FMT / Mechatronica

Deel 4: Stabiliteit van regelsystemen
Blok 8: Stabiliteit van regelsystemen - Theorie

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Cursus Systeem en Regeltechniek
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	Blok 3.	De regelaar als veer-demper combinatie
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Stability of control systems

Why becomes a control system unstable?

?

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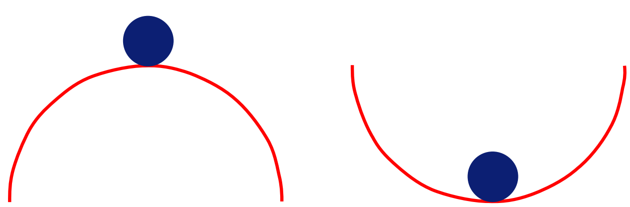
Stability of control systems

Introduction



Stable Instable

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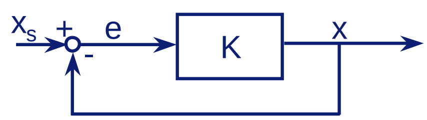
Stability of control systems

Introduction



$$\frac{x}{x_s} = \frac{K}{1+K}$$

K	0	1	-1
x/x_s	0	0.5	∞

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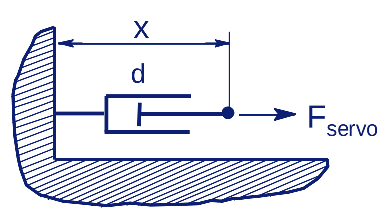
Stability of control systems

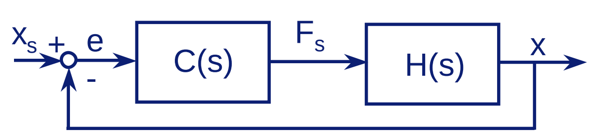
Introduction

Transfer functions:

$$H(s) = \frac{x(s)}{F(s)} = \frac{1}{ds}$$

$$C(s) = \frac{F(s)}{e(s)} = k_p$$





$$\frac{x}{x_s} = \frac{k_p/ds}{1+k_p/ds} = \frac{1}{d/k_p \cdot s + 1} = \frac{1}{\tau s + 1}$$

When stable?

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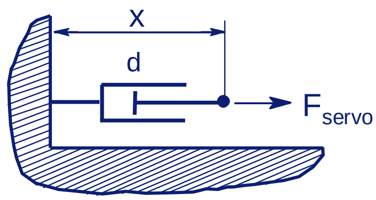
Stability of control systems

Stability in the time domain

First order system
(damper with P controller):

$$y = \frac{1}{\tau s + 1} u$$



Assume: u(t)=0, y(0)=y_0

Question: y(t) ?

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Stability of control systems

Stability in the time domain

Laplace domain:

$$\frac{y}{u} = \frac{1}{\tau s + 1}$$

$$(\tau s + 1) y = u$$

Time domain:

$$\tau \dot{y} + y = u$$

Initial condition:

u=0 y(0)=y_0

Solution:

$$\boxed{\begin{aligned} y(t) &= k e^{\lambda t} \\ \dot{y}(t) &= \lambda k e^{\lambda t} \end{aligned}}$$

Substitute in differential equation:

$$\tau \cdot \lambda k e^{\lambda t} + k e^{\lambda t} = 0 \qquad k \neq 0 \qquad e^{\lambda t} \neq 0$$

$$(\tau \lambda + 1) k e^{\lambda t} = 0 \qquad \boxed{\lambda = -\frac{1}{\tau}}$$

Initial condition: $y_0 = k \cdot e^{-\frac{1}{\tau} \cdot 0}$

$$y_0 = k$$

$$\boxed{y(t) = y_0 \cdot e^{-\frac{t}{\tau}}}$$

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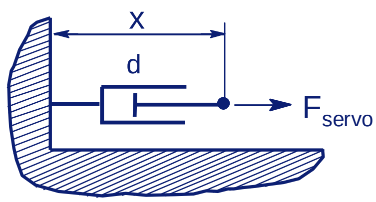
Stability of control systems

Stability in the time domain

First order system
(damper with P controller):



$$y = \frac{1}{\tau s + 1} u$$

Assume: $u(t)=0, \; y(0)=y_0$

Question: $y(t)$?

Solution: $\boxed{y(t)=y_0 e^{-t/\tau}}$

stable if: $\tau>0$!!!

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Stability of control systems

Intermezzo: poles and zeros

Transfer function examples:

First order system: $H(s) = \frac{1}{\tau s + 1}$

Second order system: $H(s) = \frac{1}{ms^2 + ds + c}$

PD controller: $C(s) = (k_p + k_v s)$

Fourth order system w. zeros: $H(s) = \frac{\omega_o^2/\omega_n^2}{Ms^2} \frac{s^2 + 2\beta\omega_n s + \omega_n^2}{s^2 + 2\beta\omega_o s + \omega_o^2}$

Standard form: $$\boxed{H(s) = K \frac{(s - z_1)(s - z_2) \ldots (s - z_m)}{(s - p_1)(s - p_2) \ldots (s - p_n)}}$$

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Intermezzo: poles and zeros

$$H(s) = K \frac{(s - z_1)\,(s - z_2)\,\ldots\,(s - z_m)}{(s - p_1)\,(s - p_2)\,\ldots\,(s - p_n)}$$

⇐ zeros
⇐ poles

Zeros are roots of the numerator polynomial

(values of *s* for which numerator becomes zero)

Poles are roots of the denominator polynomial

(values of *s* for which denominator becomes zero)

n: number of poles = order of the system

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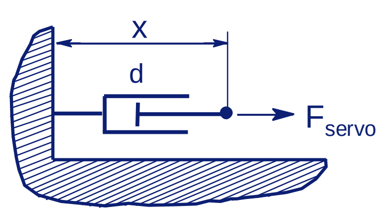
Stability in the frequency domain

First order system
(damper with P controller)



Transfer function in standard form:

$$H(s) = \frac{1}{\tau s + 1} = K \frac{1}{(s - p_1)}$$

⇐ zeros
⇐ poles

Zero: -

Pole: $\tau s+1=0 \Rightarrow s=-1/\tau$

$p_1=-1/\tau$ stable if $p_1<0$

Poles determine stability!!!

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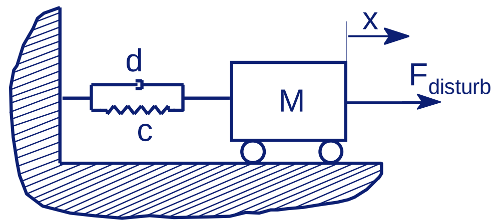
Stability of control systems

Stability in the frequency domain

Second order system (mass-spring-damper):

$$H(s) = \frac{1}{ms^2 + ds + c}$$



Transfer function in standard form:

$$H(s) = \frac{K}{(s - p_1)(s - p_2)}$$

⇐ zeros

⇐ poles

Zero: -

Poles: $p_{1,2} = -\frac{1}{2}\frac{d}{m} \pm \frac{1}{2}\sqrt{\frac{d^2}{m^2} - 4\frac{c}{m}}$

Stable if d>0, or: Re(p)<0

Poles determine stability!!!

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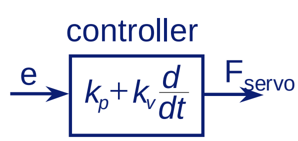
Stability of control systems

Stability in the frequency domain

PD controller

$$C(s) = (k_p + k_v s)$$



Transfer function in standard form:

$$C(s) = \frac{K\,(s - z_1)}{1}$$

⇐ zeros

⇐ poles

Zero: $z_1 = - k_p/k_v$

Pole: -

Always stable

Poles determine stability!!!

Stability in the frequency domain

PD controller in series with mass-spring-damper:

$$C(s)H(s) = \frac{(k_p + k_v s)}{ms^2 + ds + c}$$

Transfer function in standard form:

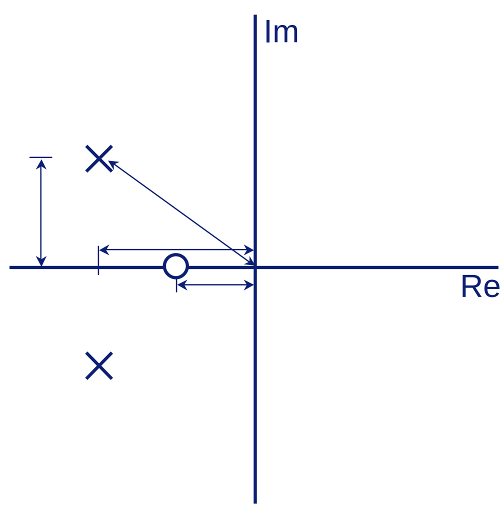
$$C(s)H(s) = \frac{K\,(s - z_1)}{(s - p_1)(s - p_2)}$$

⇦ zeros
⇦ poles

Zero: $z_1 = -\,k_p / k_v$

Pole: $p_{1,2} = -\tfrac{1}{2}\frac{d}{m} \pm \tfrac{1}{2}\sqrt{\frac{d^2}{m^2} - 4\frac{c}{m}}$

Pole-zero map in s-plane



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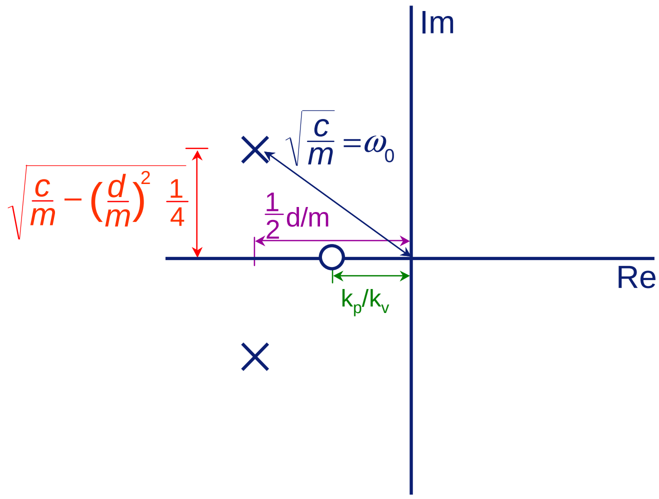
Stability of control systems

Pole-zero map in s-plane



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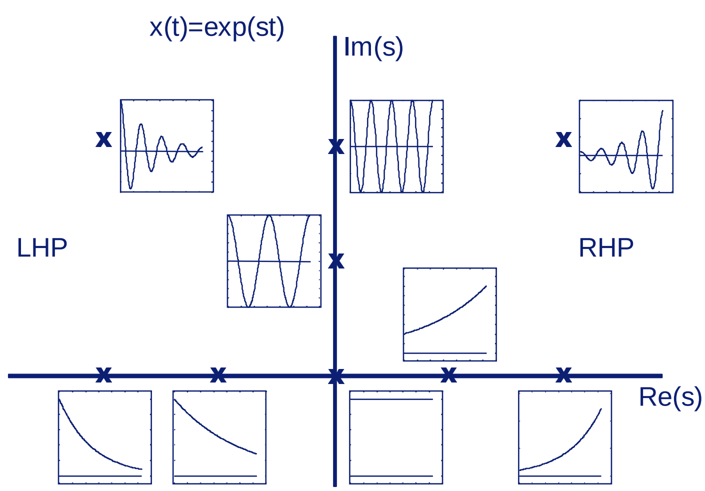
Stability of control systems

Stability of poles in s-plane



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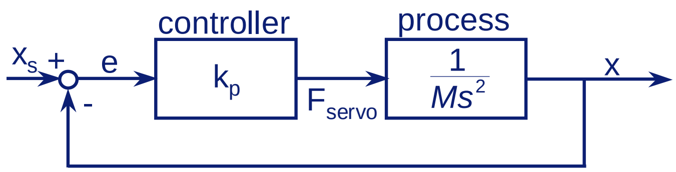
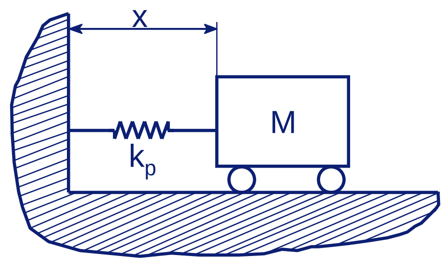
Stability of control systems

Stability of closed loop systems

Closed loop system
(simple mass with P controller):



Mechanical equivalent:



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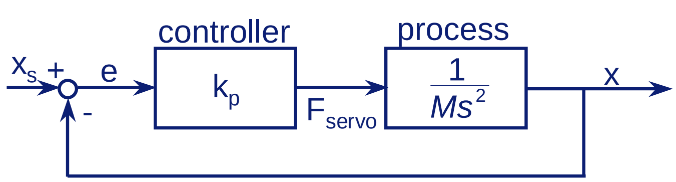
Stability of control systems

Stability of closed loop systems



Transfer function: $\frac{x}{x_s} = \frac{k_p}{Ms^2 + k_p}$

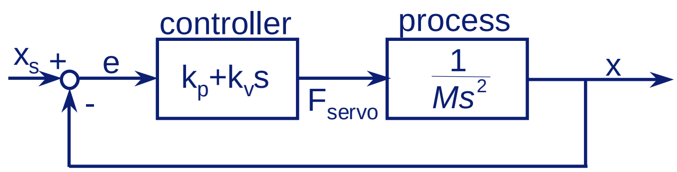
Poles: $Ms^2 + k_p = 0$, or

$$s^2 = -\frac{k_p}{M} \Rightarrow s = \pm j\sqrt{\frac{k_p}{M}}$$

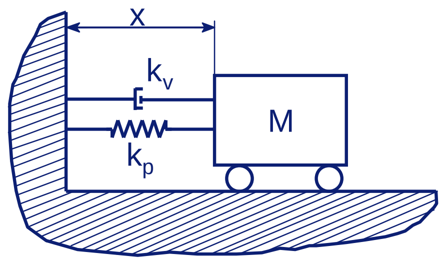
Marginally stable!!!

Stability of closed loop systems

Closed loop system
(simple mass with PD controller):



Mechanical equivalent:



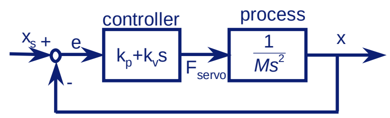
Exercise Poles

For the controlled mechanical system of the previous slide, take M=1 Kg, k_p=2 N/m and k_v=2 Ns/m.

1. Calculate the poles of the open-loop system.
2. Calculate the poles of the closed-loop system.
3. Is the system stable?

Stability of closed loop systems

Open-loop: $\frac{x}{e} = \frac{k_v s + k_p}{Ms^2} = H(s)$



Closed-loop: $\frac{x}{x_s} = \frac{H(s)}{1+H(s)}$

$$\frac{x}{x_s} = \frac{H}{1+H} = \frac{\frac{k_v s + k_P}{Ms^2}}{1+\frac{k_v s + k_p}{Ms^2}} = \frac{k_v s + k_p}{Ms^2 + k_v s + k_p}$$

Closed-loop poles: 1+H(s)=0, or;

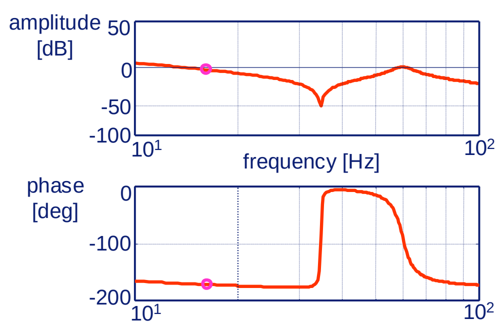
$Ms^2 + k_v s + k_p = 0$ Stable if $k_v > 0$

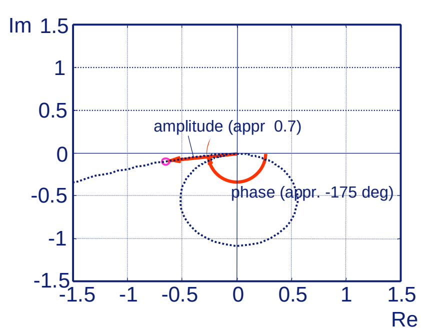
Intermezzo: Nyquist plot

H(s)

Bode plot:
- log(|**H**|) vs. log(**f**)
- ∠**H** vs. log(**f**)

Nyquist plot:
- Re(**H**) vs. Im(**H**) in complex plane





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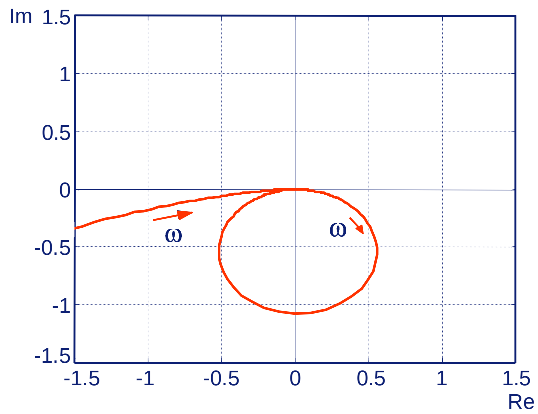
Stability of control systems

Intermezzo: Nyquist plot



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Stability of control systems

Exercise Nyquist plots

Make Bode and Nyquist diagrams for the following systems:

1. A mass (double integrator)
2. A double integrator in series with K_p/K_v controller
3. A first order system

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Stability of control systems

Towards the Nyquist stability criterion

Stability:

- Poles of closed loop system in LHP
- Poles of closed loop are solutions of 1 + H(s) = 0
- Nyquist plot of open loop H(s) can be used to evaluate stability...

Yet another exercise...

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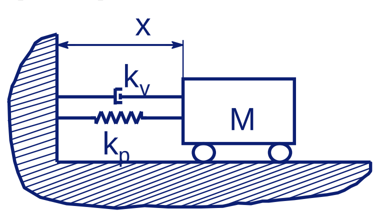
Stability of control systems

Exercise s→H(s) map

For the controlled mechanical system of earlier pages, take M=1 Kg, k_p=2 N/m and k_v=2 Ns/m.



Open-loop: $$\frac{x}{e} = \frac{k_v s + k_p}{Ms^2} = H(s)$$

Closed loop: $$\frac{x}{x_s} = \frac{H}{1+H} = \frac{k_v s + k_p}{Ms^2 + k_v s + k_p}$$

Check if s=2 is a solution to 1+H(s)=0. Same for s=-2+3j, s=-1+j, s=-1-j. Plot these s values in the complex plane (s-plane). Plot the corresponding value of H(s) in another complex plane plot (H(s) plane).

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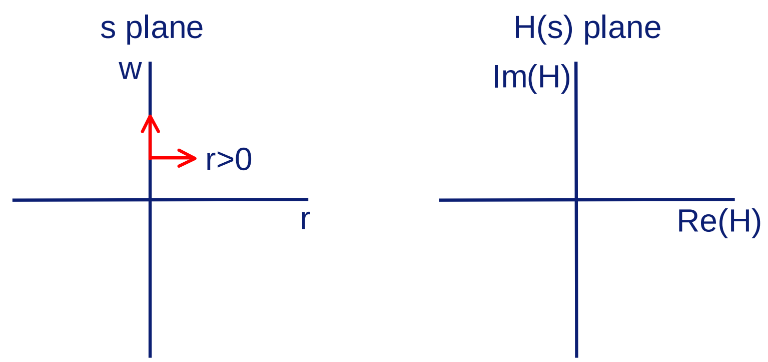
Stability of control systems

Exercise s→H(s) map



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Stability of control systems

Exercise s→H(s) map

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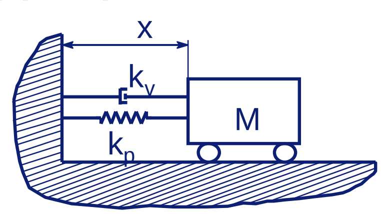
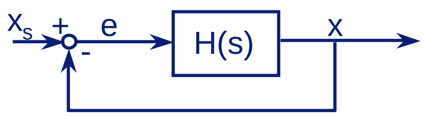


Open-loop: $$\frac{x}{e} = \frac{k_v s + k_p}{Ms^2} = H(s)$$

Closed loop: $$\frac{x}{x_s} = \frac{H}{1+H} = \frac{k_v s + k_p}{Ms^2 + k_v s + k_p}$$

1. Assume that we consider sinus-types of signals, i.e. s=jω. Make the H(s) plot for ω=0….∞.
2. Is the system stable?

Nyquist stability criterion



$$H_c(s) = \frac{H(s)}{1+H(s)}$$

Stability: check RHP poles of $H_c(s)$

$1+H(s)=0$ for $Re(s)>0$

Question:
Given a frequency response $H(j\omega)$, how to check stability?

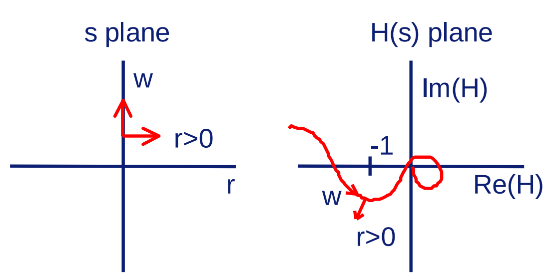
Answer:
Use Nyquist plot of $H(j\omega)$ in the complex plane, and evaluate with respect to the (-1,0)

Nyquist stability criterion

Graphical evaluation of stability

> For increasing frequency along the curve of $H(j\omega)$ in the complex plane, the point (-1,0) should stay at the left hand side of the curve

H(s) with $s=r+j\omega$



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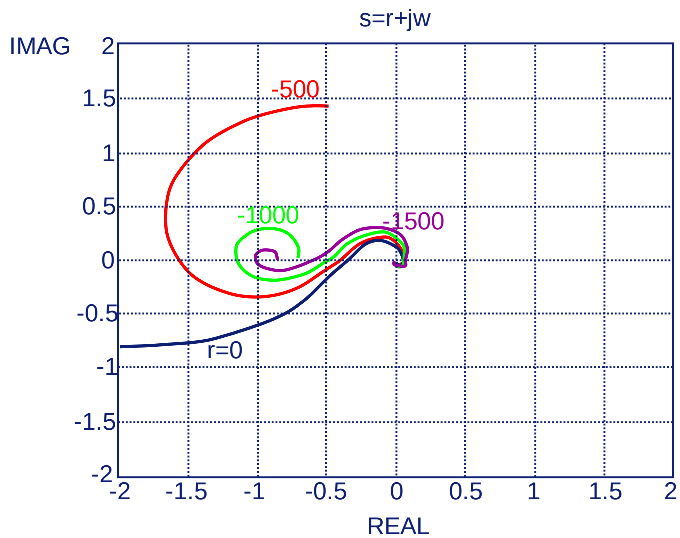
Stability of control systems

Nyquist stability criterion



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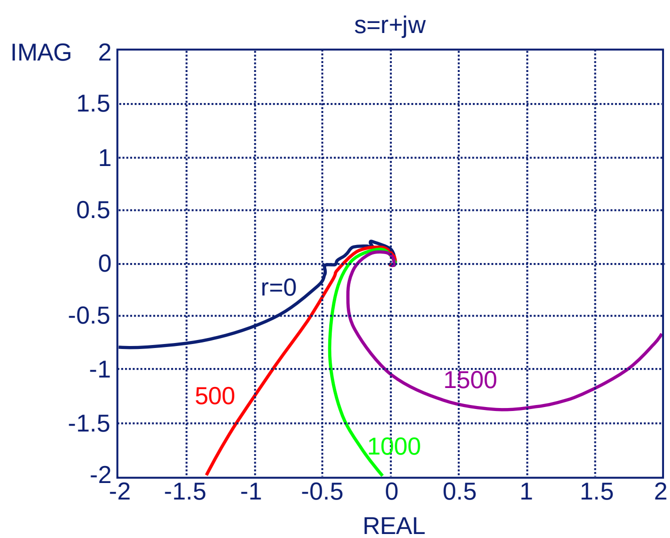
Stability of control systems

Nyquist stability criterion



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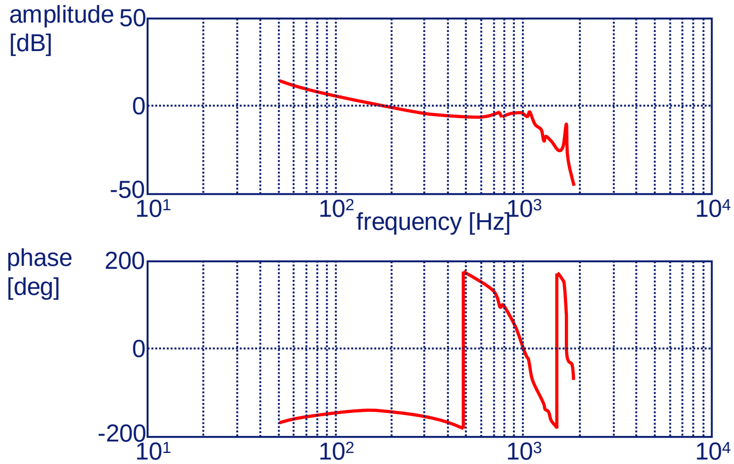
Stability of control systems

Example of experimental system

Measurement of open loop reticle stage



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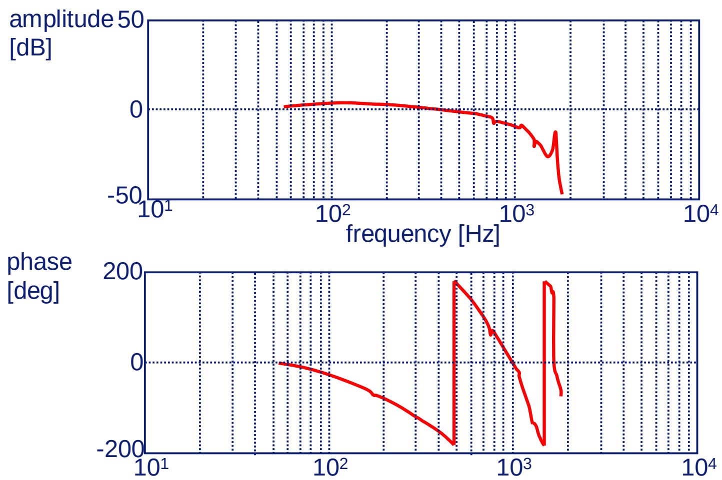
Stability of control systems

Example of experimental system

Measurement of closed loop reticle stage



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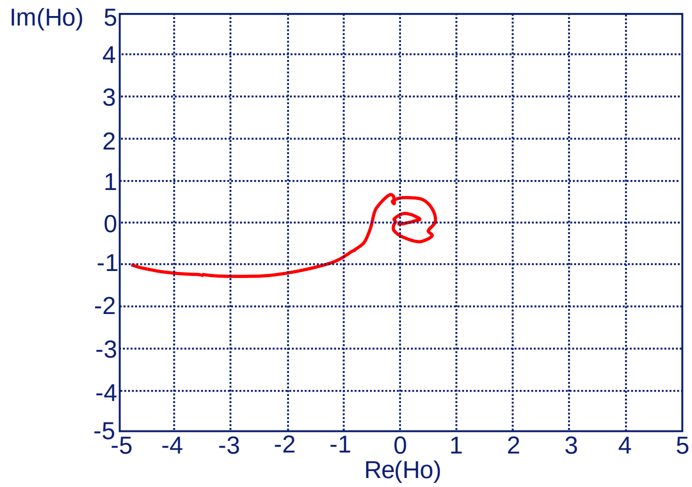
Stability of control systems

Example of experimental system

Nyquist plot of open loop reticle stage



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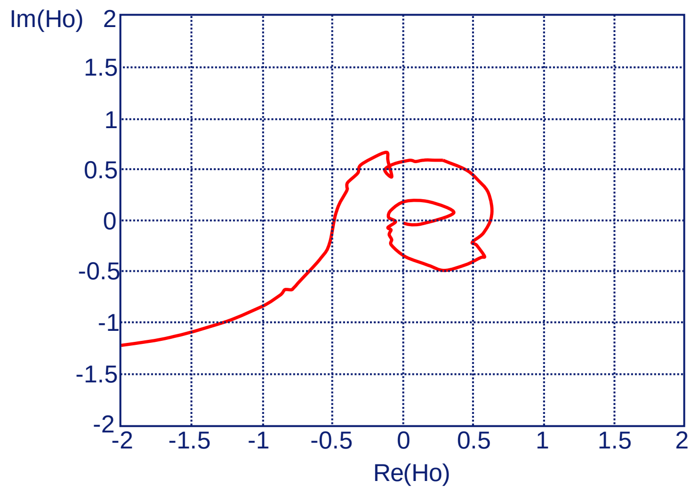
Stability of control systems

Example of experimental system

Nyquist plot of open loop reticle stage (zoomed)



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Stability of control systems

Stability margins

Margin = Distance of H(jω) to the critical point

Phase Margin (PM) = allowable phase shift up to instability (rotation of H(jω) diagram)

Or: check phase of H(jω) where $|H(j\omega)| = 1$

Gain Margin (GM) = allowable gain increase up to instability (enlargement of H(jω) diagram)

Or: check gain of H(jω) where $\angle H(j\omega) = -180^o$

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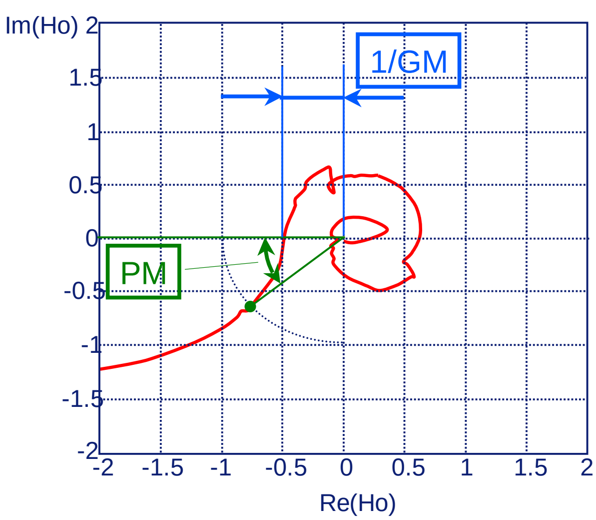
Stability of control systems

Stability margins

Nyquist plot of open loop reticle stage



Summary

- Stability in time domain
- Poles / Zeros
- Stability in frequency domain
  - Poles of closed loop should be in RHP
- The Nyquist plot
- The Nyquist stability criterion
  - Leave (-1,0) at left hand of open loop H(s)
  - Leave sufficient stability margins (gain/phase)

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