PHILIPS CTT Hogeschool van Utrecht

Systeem en Regeltechniek

FMT / Mechatronica

Deel 6: Extra regeltechniek

Blok 14: Terugblik / Evaluatie

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Cursus Systeem en Regeltechniek

Overzicht

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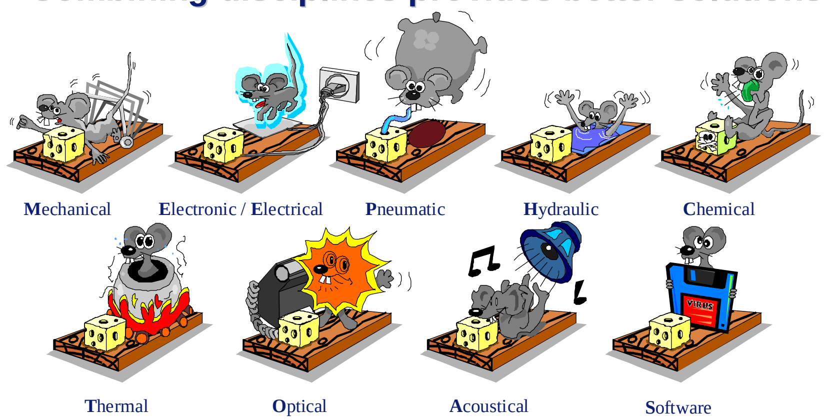
Mechatronics: Combining disciplines provides better solutions



- ***Know*** **all modern technologies and their applications**
- **Be able to work out** ***system solutions***

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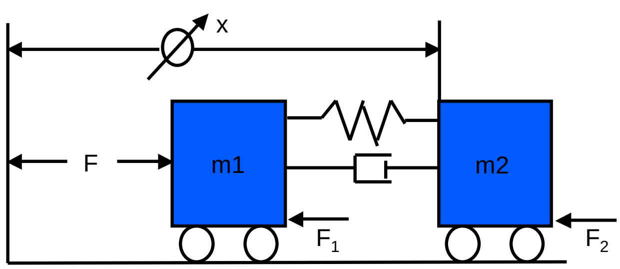
Introduction

Mechanics & Dynamics



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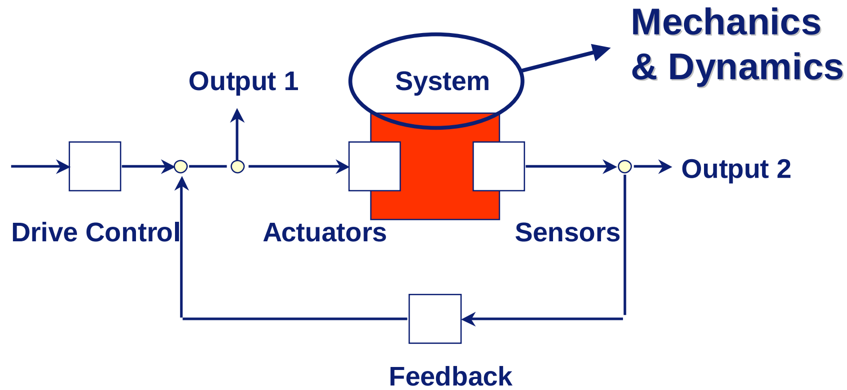
The role of control



Mechatronic System Approach:
Active elements, control of stiffness and damping

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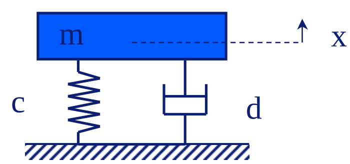
Free motion (translation)

$$m \cdot \ddot{x} + d \cdot \dot{x} + c \cdot x = 0$$



$\Updownarrow$

$$\ddot{x} + \frac{d}{m} \cdot \dot{x} + \frac{c}{m} \cdot x = 0 \quad \Leftrightarrow \quad \frac{m}{c} \cdot \ddot{x} + \frac{d}{c} \cdot \dot{x} + x = 0$$

$\Updownarrow$ $\Updownarrow$

$$\ddot{x} + 2\beta\omega_e \cdot \dot{x} + \omega_e^2 \cdot x = 0 \quad \Leftrightarrow \quad \frac{1}{\omega_e^2} \cdot \ddot{x} + \frac{2\beta}{\omega_e} \cdot \dot{x} + x = 0$$

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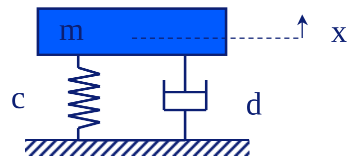
Basic principles modelling mass-spring-systems

Resonance frequency & damping

$$\frac{m}{c}\cdot\ddot{x}+\frac{d}{c}\cdot\dot{x}+x=0$$

$$\frac{1}{\omega_e^2}\cdot\ddot{x}+\frac{2\beta}{\omega_e}\cdot\dot{x}+x=0$$



Resonance frequency: $\omega_e=\sqrt{\frac{c}{m}},\quad f_e=\frac{1}{2\pi}\sqrt{\frac{c}{m}}$

Relative damping: $\beta=\frac{d}{2\sqrt{m\cdot c}}=\frac{d}{2\cdot m\cdot\omega_e}\leq 1$

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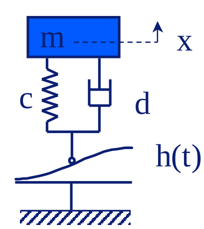
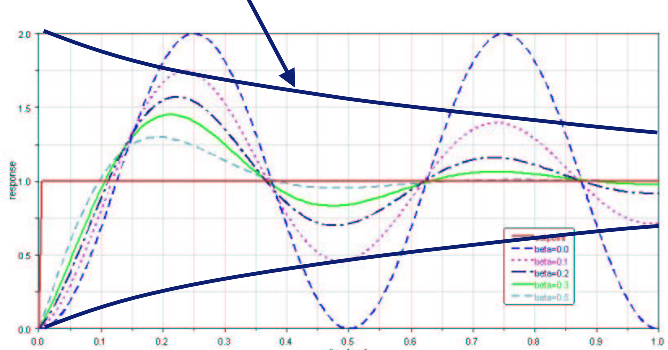
Basic principles modelling mass-spring-systems

Step response of mass-spring-system

$$x=1-e^{-\beta\omega_e t}\cdot\cos(\omega_e\sqrt{1-\beta^2}\cdot t)$$





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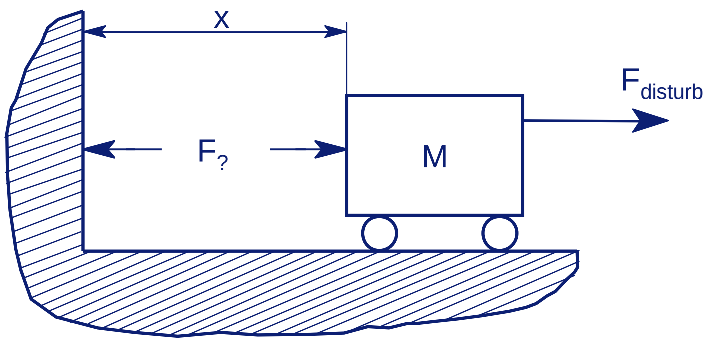
The controller as spring-damper combination

Controlling the position of a mass



Control Objectives:

- Getting there
- Staying there ←

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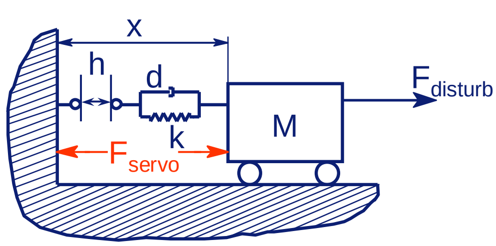
The controller as spring-damper combination

Second control objective 'getting there':



Spring-damper force F: $F = -k(h - x) - d(\dot{h} - \dot{x})$

Controller: $F_{servo} = k_p(x_s - x) + k_v(\dot{x}_s - \dot{x})$

(x_s: setpoint)

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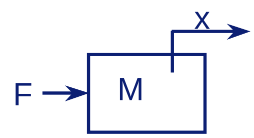
Frequency Domain Description

Frequency Domain solution for equation of motion

$$\boxed{F = M\ddot{x}}$$



1) Choose a sinusoidal input:

$$F = \hat{F}\sin(\omega t)$$

2) Then for a linear system:

$$x = \hat{x}\sin(\omega t + \varphi)$$
$$\dot{x} = \omega\hat{x}\cos(\omega t + \varphi)$$
$$\ddot{x} = -\omega^2\hat{x}\sin(\omega t + \varphi)$$

$$\hat{x} = ?; \varphi = ?$$

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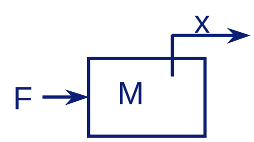
Frequency Domain Description

Frequency Domain solution for equation of motion

$$\boxed{F = M\ddot{x}}$$



3) Solution:

$$\hat{F}\sin(\omega t) = -M\omega^2\hat{x}\sin(\omega t + \varphi)$$

$$\boxed{\begin{aligned} \varphi &= \angle H = -180^\circ \\ \hat{x} &= |H|\hat{F} = \frac{\hat{F}}{M\omega^2} \end{aligned}}$$

$$\boxed{H = \frac{x}{F} = -\frac{1}{M\omega^2}} \Leftarrow$$ **Frequency Response Function !**

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Transfer Function

So we have: $(Ms^2+ds+k)x(s)=F(s)$

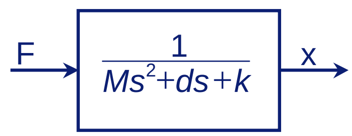
This leads to the Transfer Function H(s):

$$H(s)=\frac{x(s)}{F(s)}=\frac{1}{Ms^2+ds+k}$$

Note that in Laplace domain:

x(s) = H(s)·F(s) So: *output* = *system* x *input* !!!

In block representation:



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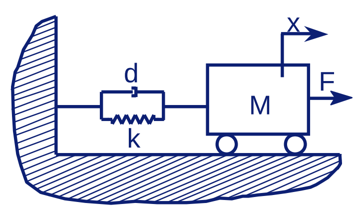
The Bode plot of a mass-spring-system



Transfer Function:

$$H(s)=\frac{x(s)}{F(s)}=\frac{1}{Ms^2+ds+k}$$

$$H(s)=\frac{1/M}{s^2+2\beta\omega_o s+\omega_o^2}$$

Recall:

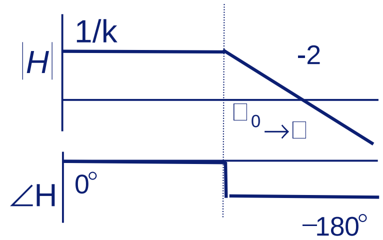
$$\omega_o=\sqrt{\frac{k}{M}}$$ Eigenfrequency

$$\beta=\frac{d}{2\sqrt{Mk}}$$ Relative damping

The Bode plot of a mass-spring-system

Transfer Function:

$$H(s) = \frac{x(s)}{F(s)} = \frac{1}{Ms^2 + ds + k}$$



Asymptotes in Bode plot:

$s \rightarrow 0 \quad \Rightarrow \quad H \rightarrow 1/k \quad \Rightarrow \quad |H| \rightarrow 1/k$
$\angle H \rightarrow 0^o$

$s \rightarrow \infty \quad \Rightarrow \quad H \rightarrow 1/Ms^2 \quad \Rightarrow \quad |H| \rightarrow 1/M\omega^2$
$\angle H \rightarrow 180^o$

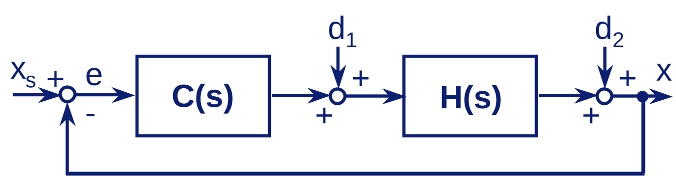
Break point:

$\log(1/k) = \log(1/M\omega^2) = \log(1/M) - 2\log(\omega)$

$$\omega = \omega_o = \sqrt{\frac{k}{M}}$$

Why feedback?

Feedback with disturbances:



Recall: $H_c(s) = \frac{x(s)}{x_s(s)} = \frac{C(s) \cdot H(s)}{1 + C(s) \cdot H(s)}$

So: $x \approx x_s$ if: $C(s) \cdot H(s) >> 1$

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Further introduction in control theory

Why feedback?

The aim of feedback is...

Disturbance Suppression

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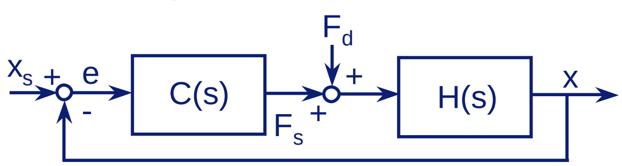
Further introduction in control theory

Four important transfer functions



Open Loop: $H_o(s)=\frac{x}{e}=C(s)H(s)$

Closed Loop: $H_c(s)=\frac{x}{x_s}(s)=\frac{C(s)H(s)}{1+C(s)H(s)}$

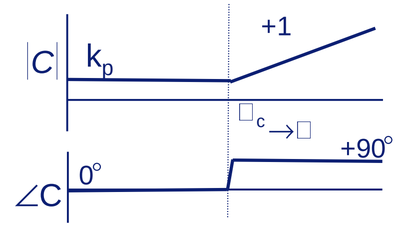
Sensitivity: $S(s)=\frac{e}{x_s}(s)=\frac{1}{1+C(s)H(s)}$

Process Sensitivity: $H_{ps}(s)=\frac{x}{F_d}(s)=\frac{H(s)}{1+C(s)H(s)}$

The Bode plot of a PD controller

Transfer Function:

$$C(s) = \frac{F}{e}(s) = (k_p + k_v s)$$



Asymptotes in Bode plot:

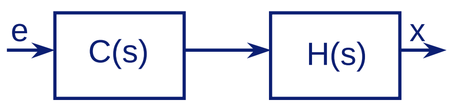
Amplitude: $|C| = \sqrt{k_p^2 + k_v^2 \omega^2}$

$s \to 0 \quad \Rightarrow \quad C \to k_p \quad \Rightarrow \quad |C| \to k_p$, $\angle C \to 0^o$

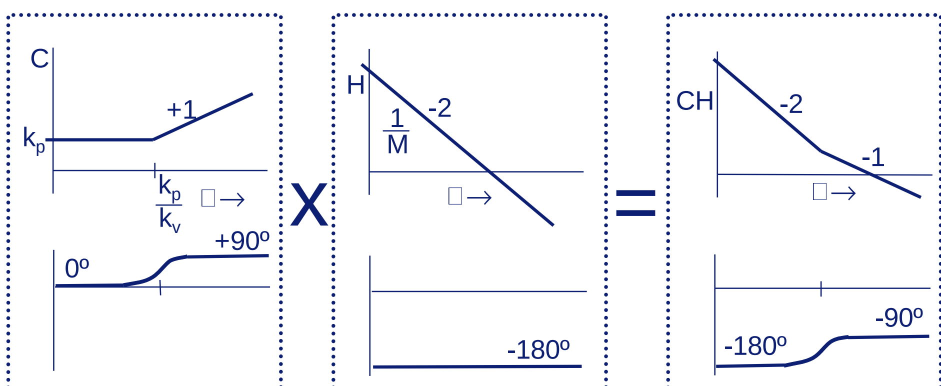
$s \to \infty \quad \Rightarrow \quad C \to k_v s \quad \Rightarrow \quad |C| \to k_v \omega$, $\angle C \to 90^o$

Break point: $\log k_p = \log k_v + \log \omega \quad \Rightarrow \quad \omega_c = \frac{k_p}{k_v}$

Intermezzo: Multiplication of Bode plots



$$\frac{x(s)}{e(s)} = C(s)H(s)$$



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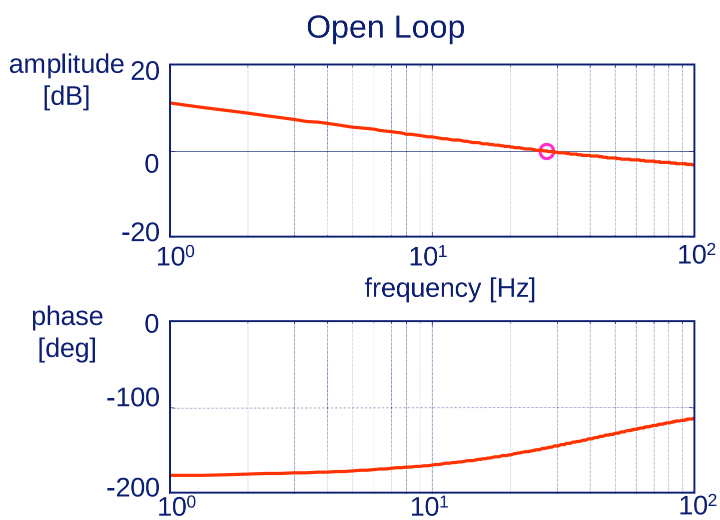
The PD controller as spring-damper combination

The Bandwidth Concept - Second order system



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The PD controller as spring-damper combination

The Bandwidth Concept - 'Definition'

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Bandwidth: 0 dB crossing open loop

Intermezzo: poles and zeros

$$H(s) = K \frac{(s - z_1)\,(s - z_2)\,\ldots\,(s - z_m)}{(s - p_1)\,(s - p_2)\,\ldots\,(s - p_n)}$$

⇐ zeros
⇐ poles

Zeros are roots of the numerator polynomial

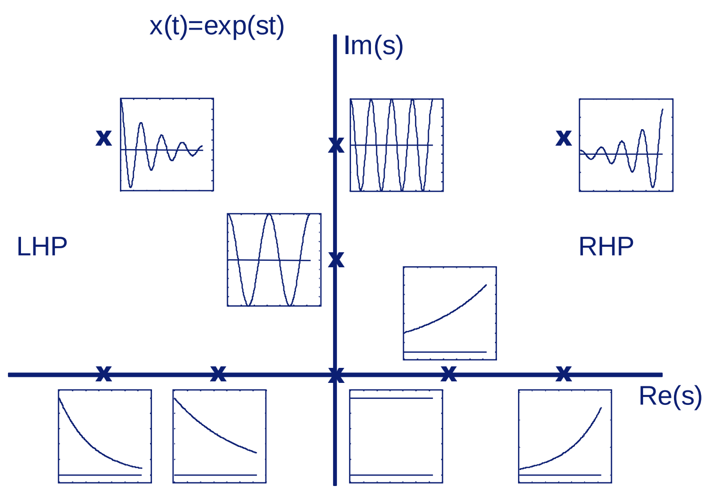
(values of *s* for which numerator becomes zero)

Poles are roots of the denominator polynomial

(values of *s* for which denominator becomes zero)

n: number of poles = order of the system

Stability of poles in s-plane

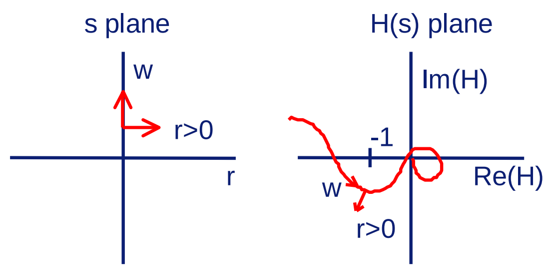


Nyquist stability criterion

Graphical evaluation of stability

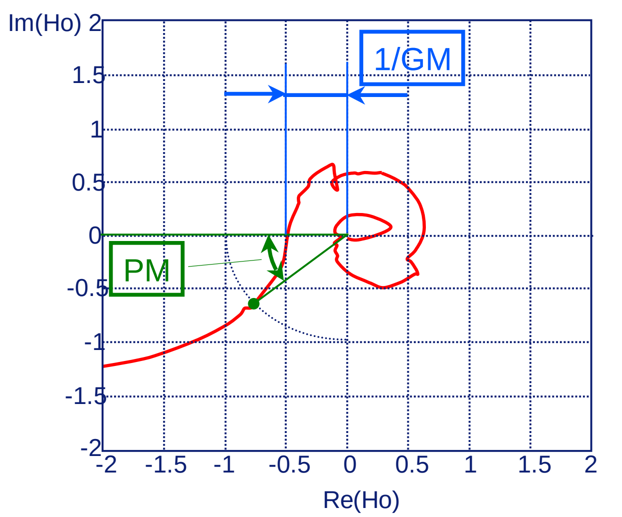
For increasing frequency along the curve of H(jω) in the complex plane, the point (-1,0) should stay at the left hand side of the curve

H(s) with s=r+jω

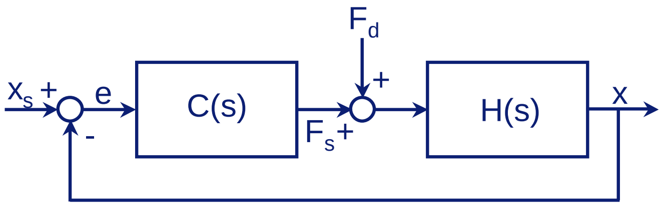


Stability margins

Nyquist plot of open loop reticle stage



Limitations of PD controller

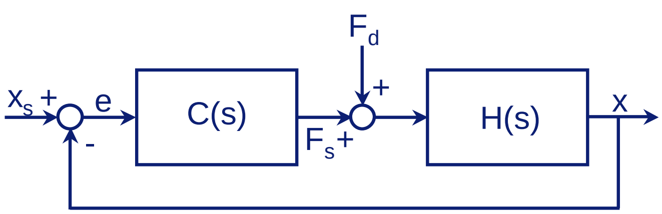


Recall k_p/k_v or PD controller: $C_{PD}(s) = (k_p+k_v s)$

Limitations PD controller:

- Suppression of constant disturbance forces F_d
- Pure differentiating action can not be realised
- Suppression of high-frequent noise in the control loop
- Suppression of resonances in the open loop response

Limitations of PD controller



Solutions - extend PD controller with filters:

- Integral action (PID)
- Lead-lag filter
- Second order Low Pass filter
- Notch filter(s)

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The PID controller in the frequency domain

Overview Filters

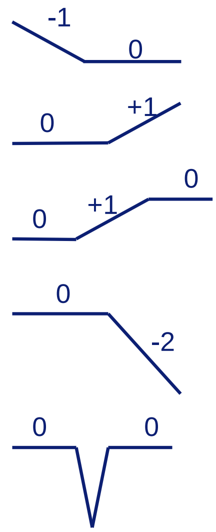
I-action

PD

Lead-lag

2^{nd} order low pass

Notch



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Bandwidth and disturbance suppression

Disturbance suppression

No control: $y_o = Hd$

Closed-loop: $y_c = \frac{H}{1+CH} d$

$$\frac{y_c}{y_o} = \frac{1}{1+CH} = S$$

Benefit of feedback = Sensitivity function

Example: H = 0.01; CH = 10^4; d = 1

$y_o = 0.01$

$S \approx 10^{-4}$

$y_c = \frac{0.01}{1+10^4} 1 \approx 10^{-6}$

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Bandwidth and disturbance suppression

Robustness

Open-loop: $y_o = kHCr$

Relative change: $\dfrac{y_o - \bar{y}_o}{\bar{y}_o} = \dfrac{kHCr - HCr}{HCr} = k - 1$

Closed-loop: $y_c = \dfrac{kCH}{1+kCH} r$

Relative change: $\dfrac{y_c - \bar{y}_c}{\bar{y}_c} = \dfrac{\dfrac{kCH}{1+kCH} r - \dfrac{CH}{1+CH} r}{\dfrac{CH}{1+CH} r} = \dfrac{k-1}{1+kCH}$

$$\frac{\dfrac{y_c - \bar{y}_c}{\bar{y}_c}}{\dfrac{y_o - \bar{y}_o}{\bar{y}_o}} = \frac{\dfrac{k-1}{1+kCH}}{k-1} = \frac{1}{1+kCH} = S$$

Benefit of feedback
= Sensitivity function

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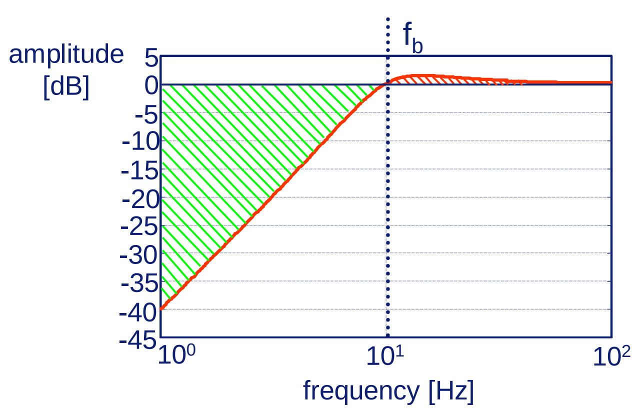
Bandwidth and disturbance suppression

Design for performance

Bode Sensitivity Integral



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Bandwidth and disturbance suppression

Loop Shaping Procedure

1. Stabilize the plant:
   Add lead/lag with zero = BW/3 and pole = BW*3, adjust gain to set stability or add a pure PD with breakpoint at the BW.
2. Add low-pass filter:
   Choose poles = BW*6.
3. Add notch, if necessary, or apply any other kind of first or second order filter and shape the loop.
4. Add integral action:
   Choose zero = BW/5.
5. Increase BW:
   Increase gain and adapt zeros/poles of integral action, lead/lag and other filters.

During steps 2-5: check all relevant transfer functions and relate to disturbance spectrum.

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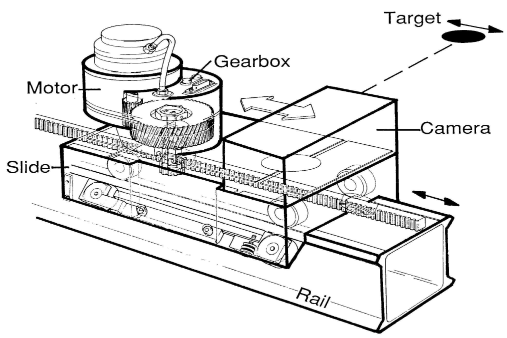
Exercise



Moving target: **3 Hz, 3mm**

Allowed tracking error: **40µm**

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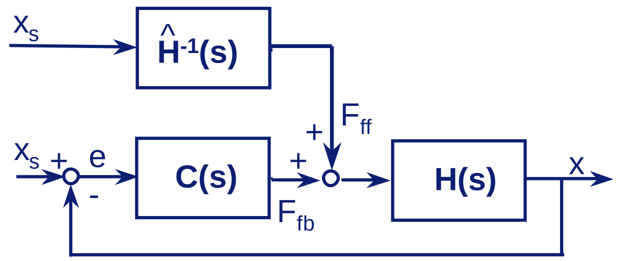
Setpoints and feedforward tuning

Feedback plus feedforward



$e(s) = 0$ if: $\hat{H}(s) = H(s)$

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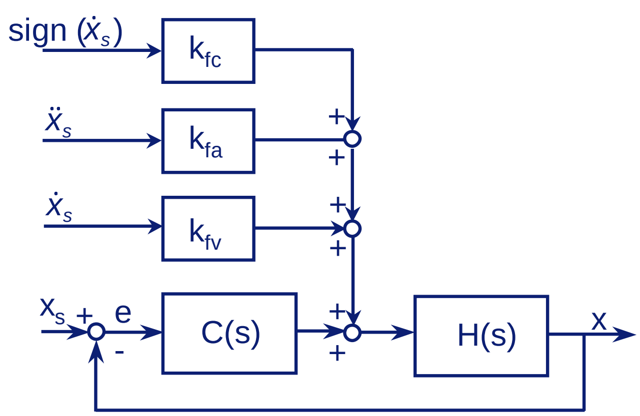
Setpoints and feedforward tuning

Friction feedforward - Implementation



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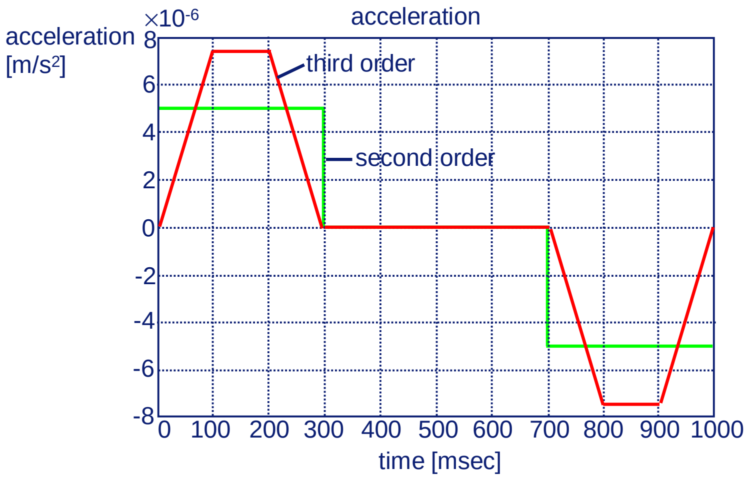
Setpoints and feedforward tuning

Third order setpoint

Jerk rectangular instead of acceleration



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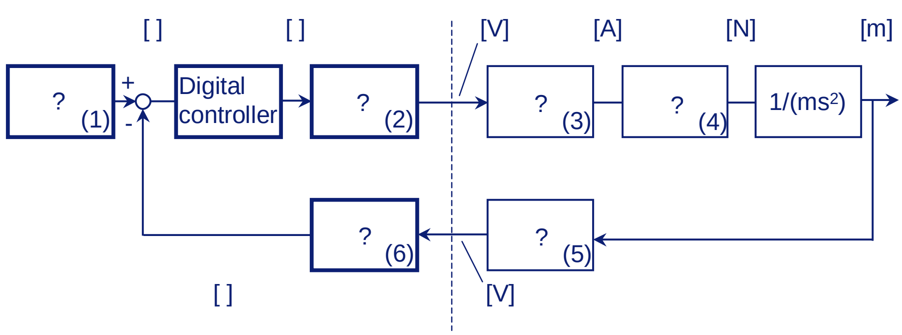
Digital implementation effects

Exercise: building blocks in digitally controlled system



What elements do the question marks represent?

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Digital implementation effects

Cause and effects of delay

Delay in digital system caused by:

- Time needed to calculate new actions (force) as response to new inputs (position errors), called T_c
- Lag effect in the DA conversion (ZOH)

So, in total delay complies to:

- $T_{delay} = T_c + 0.5 \cdot T_s$

Remark:

- Transfer function of delay with time T: $H_{delay}(s) = e^{-Ts}$
- FRF of delay with time T: $H_{delay}(\omega) = e^{-j\omega T}$

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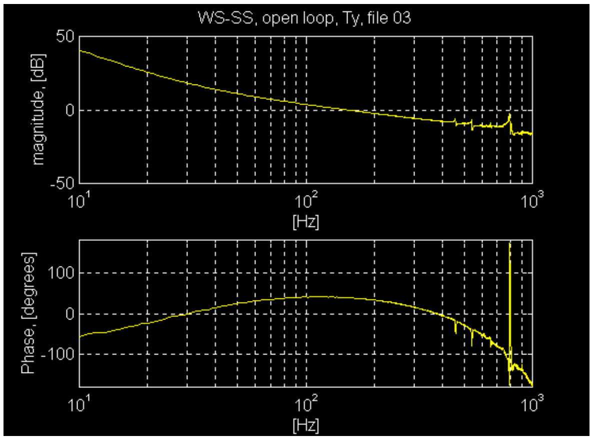
Digital implementation effects

Cause and effects of delay



At what sample rate would the phase margin be 5 degrees larger?

Evaluatie

- Relevantie voor dagelijkse praktijk?
- Aansluiting op andere vakken?
- Niveau / moeilijkheidsgraad?
- Snelheid van behandelen?
- Balans theorie / praktijk / oefeningen?
- Onderwerpen gemist?
- Onderwerpen overbodig?