Homework Stochastic Simulation (2017) - First set

The deadline of this homework set is Friday October 13 at 15:00. For the programming exercises, you are free to use any programming language or mathematical software package (Maple/Matlab/Mathematica/R/anything) as long as you

- include all code in your deliverables, and
- for random number generation purposes, you use standard built-in methods to generate standard uniform samples. For generating samples from any other distribution, you'll have to construct them yourself using the standard uniform samples.

Please compile answers and code in a single PDF document, and send it by e-mail to Jan-Pieter Dorsman (j.l.dorsman 'at' uva...).

Should you have any questions about the homework exercises, please reach out to one of the lecturers during the lecture.

Exercise 1 Please use your programming language or mathematical software package to perform the following steps for n large enough:

1. Draw n samples from a two-dimensional random variable $U = (U_1, U_2)$, where U_1 and U_2 are mutually independent and both uniformly distributed on $[-1, 1]$. Call your samples $U^{(1)}, U^{(2)}, \ldots$, where $U^{(i)} = (U_1^{(i)}, U_2^{(i)})$.
2. Numerically compute $\frac{4}{n}\sum_{i=1}^{n} \mathbb{1}_{\{(U_1^{(i)})^2+(U_2^{(i)})^2\leq 1\}}$.

What value do you get? Explain your findings.

Exercise 2 Make exercise II.2.1 of [AG].

Exercise 3 Make exercise II.2.8 of [AG].

Exercise 4 Make exercise II.2.10 of [AG].

Exercise 5 Write a discrete event simulation program to perform a single run for the following reliability model. An electronic system has K independent components, which are all functioning. The life times (L) of the components are mutually independent, and each of them has a Weibull distribution: $\mathbb{P}(L \leq t) = 1 - e^{-(\lambda t)^{\alpha}} \mathbb{1}_{\{t\geq 0\}}$. Whenever a component fails, it will be taken into repair instantaneously. This repair takes a units of time with probability p and b units of time with probability $1 - p$. After a repair, a component functions as if it were a completely new component. The repair times are mutually independent and also independent of the life times. The electronical system as a whole goes down when none of the components is functioning anymore. Management wants to know what the expected time is until the first failure. Take $K = 4$, $\alpha = 1.01$, $\lambda = 0.249$, $a = \frac{1}{2}$, $b = \frac{3}{2}$ and $p = \frac{1}{2}$. What is your estimate? How reliable do you deem your estimate?