Homework Stochastic Simulation (2018) - First set

The deadline of this homework set is Monday October 8 at 09:00. For the programming exercises, you are free to use any programming language or mathematical software package (Maple/Matlab/Mathematica/R/anything) as long as you

- include all code in your deliverables, and
- for random number generation purposes, you use standard built-in methods to generate standard uniform samples. For generating samples from any other distribution, you'll have to construct them yourself using the standard uniform samples.

You are expected to submit the homework individually. Please compile answers and code in a single PDF document, and send it by e-mail to Nikki Levering (nikki.levering 'at' student.uva...).

Should you have any questions about the homework exercises, please reach out to one of the lecturers during the lecture, or to Nikki via e-mail.

Exercise 1 Please use your programming language or mathematical software package to perform the following steps for n large enough:

1. Draw n samples from a two-dimensional random variable $U = (U_1, U_2)$, where U_1 and U_2 are mutually independent and both uniformly distributed on $[-1, 1]$. Call your samples $U^{(1)}, U^{(2)}, \ldots$, where $U^{(i)} = (U_1^{(i)}, U_2^{(i)})$.
2. Numerically compute $\frac{4}{n}\sum_{i=1}^{n} \mathbb{1}_{\{(U_1^{(i)})^2+(U_2^{(i)})^2\leq 1\}}$.

What value do you get? Explain your findings.

Exercise 2 Make exercise II.2.1 of [AG].

Exercise 3 Make exercise II.2.8 of [AG].

Exercise 4 Make exercise II.2.10 of [AG].

Exercise 5 Write a discrete event simulation program to perform a single run for the following reliability model. An electronic system has K independent components, which are all functioning. The life times (L) of the components are mutually independent, and each of them has a Weibull distribution: $\mathbb{P}(L \leq t) = 1 - e^{-(\lambda t)^\alpha}\mathbb{1}_{\{t\geq 0\}}$. Whenever a component fails, it will be taken into repair instantaneously. This repair takes a units of time with probability p and b units of time with probability $1-p$. After a repair, a component functions as if it were a completely new component. The repair times are mutually independent and also independent of the life times. The electronical system as a whole goes down when none of the components is functioning anymore. Management wants to know what the expected time is until the first failure. Take $K = 4$, $\alpha = 1.01$, $\lambda = 0.249$, $a = \frac{1}{2}$, $b = \frac{3}{2}$ and $p = \frac{1}{2}$. What is your estimate? How reliable do you deem your estimate?