Zij $f, g : \mathbb{N} \rightarrow \mathbb{R}$. We zeggen “$f(n)$ is $O(g(n))$” als er $c \in \mathbb{R}$ en $n_0 \in \mathbb{N}$ bestaan met $c > 0$ en $n_0 \geq 1$ zo dat

$$f(n) \leq c \times g(n) \text{ voor } n \geq n_0$$
1
Algorithm A uses $10n \log n$ operations, while algorithm B uses $n^2$ operations. Determine the value $n_0$ such that A is better than B for $n \geq n_0$.

2
Algorithm A uses $40n^2$ operations, while algorithm B uses $2n^3$ operations. Determine the value $n_0$ such that A is better than B for $n \geq n_0$.

3
What is the sum of all the even numbers from 0 to $2n$, for any positive integer $n$?
4

Show that the following two statements are equivalent:

1. The running time of algorithm A is $\mathcal{O}(f(n))$
2. In the worst case, the running time of algorithm A is $\mathcal{O}(f(n))$
Give a big-Oh characterization of the running time of $\text{Loop1}(n)$.

**Algorithm Loop1(n):**

1. $s ← 0$
2. for $i ← 1$ to $n$ do
   3. $s ← s + i$
3. end for
6

Give a big-Oh characterization of the running time of \textbf{Loop2(n)}

\textbf{Loop2(n)}

\textbf{Algorithm} \textsc{Loop2}(n):
\begin{verbatim}
p ← 1
for i ← 1 to 2n do
    p ← p \cdot i
end for
\end{verbatim}
Give a big-Oh characterization of the running time of Loop3(n)

Algorithm Loop3(n):
   p ← 1
   for i ← 1 to n^2 do
      p ← p . i
   end for
Give a big-Oh characterization of the running time of Loop4(n)

Algorithm Loop4(n):
    s ← 0
    for i ← 1 to 2n do
        for j ← 1 to i do
            s ← s + i
        end for
    end for
Give a big-Oh characterization of the running time of Loop5(n):

Algorithm Loop5(n):

1. \( s \leftarrow 0 \)
2. for \( i \leftarrow 1 \) to \( n^2 \) do
3.   for \( j \leftarrow 1 \) to \( i \) do
4.     \( s \leftarrow s + i \)
5.   end for
6. end for
10. Show that if $f(n)$ is $\mathcal{O}(g(n))$ and $d(n)$ is $\mathcal{O}(h(n))$, then $f(n) + d(n)$ is $\mathcal{O}(g(n) + h(n))$.

11. Show that $(n + 1)^5$ is $\mathcal{O}(n^5)$

12. Show that $2^{n+1}$ is $\mathcal{O}(2^n)$

13. Laat zien dat $3n^2 + 10$ is $\mathcal{O}(n^2)$