Exercises *Simpson's Paradox*

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2. Simpson's Paradox: resolution

This exercise continues where exercise 1 ended. We will use the formalism of Causal Bayes Nets to resolve Simpson's paradox.

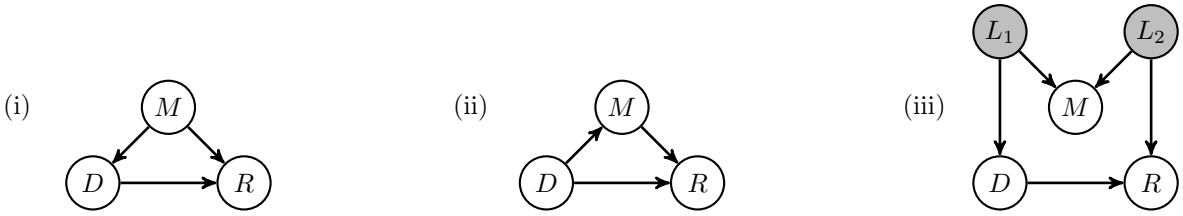

Figure 1: Different hypothetical Causal Bayes Nets, where R stands for *Recovery*, D for taking the *Drug*, and M has different interpretations in cases (i), (ii) and (iii).

Suppose you believe that the causal model in Figure 1(i) applies, where M denotes gender of the patient (male/female).

4a. Apply the back-door criterion to obtain a formula that expresses $p(r \mid \mathrm{do}(D = d))$ *in terms of observable quantities (i.e., in terms of marginal or conditional distributions where the* do*-operator does not appear).*

4b. Is $p(r \mid \mathrm{do}(D = d)) = p(r \mid d)$ *in this case?*

4c. What would be your advice for a patient with unknown gender?

Now suppose that instead, you believe the causal model in Figure 1(ii) to apply. Intuitively, this would be quite unlikely, as we know that most drugs don't change gender, but we could have used a slightly different story where the variable M has a different interpretation (for example, "blood pressure"), and then this causal structure would also be a plausible one.

5a. Again, use the back-door criterion to express $p(r \mid \mathrm{do}(D = d))$ *in terms of observable quantities.*

5b. Is $p(r \mid \mathrm{do}(D = d)) = p(r \mid d)$ *in this case?*

5c. What would be your advice for a patient with unknown M *(say, blood pressure) in this case?*

Finally, suppose that you believe that the causal model in Figure 1(iii) applies.

6a. Invent an interpretation of M and the two latent variables L_1, L_2 yourself that could match the causal model in Figure 1(iii).

6b. Express $p(r \mid \mathrm{do}(D = d))$ in terms of observable *quantities.*

6c. Is $p(r \mid \mathrm{do}(D = d)) = p(r \mid d)$ in this case?

6d. Again, what would be your advice for a patient with unknown M in this case?

Conclusion: whether or not you should prescribe the drug depends on which causal model you believe to apply to this situation. The fact that different causal models will lead to different conclusions is not paradoxical. You can read more about this paradox in [Pearl, 1999].

References

[Pearl, 1999] Pearl, J. (1999). Simpson's paradox: An anatomy. Technical Report R-264, UCLA Cognitive Systems Laboratory.