



Universiteit Utrecht

# Estimating causal effects of policy interventions

SIKS May 31<sup>st</sup> 2023



*Oisín Ryan*

*With thanks to EJ van Kesteren*



UMC Utrecht

# Context: “Policy Evaluations”

**Evaluating** what **the effect** of implementing a particular **policy** or **intervention** was on some outcome of interest

## Examples:

- What was the effect of raising the maximum speed limit on motorways in the Netherlands on road deaths?
- Did introducing an after-school programme in disadvantaged neighbourhoods lead to improved educational outcomes in children from that neighbourhood?

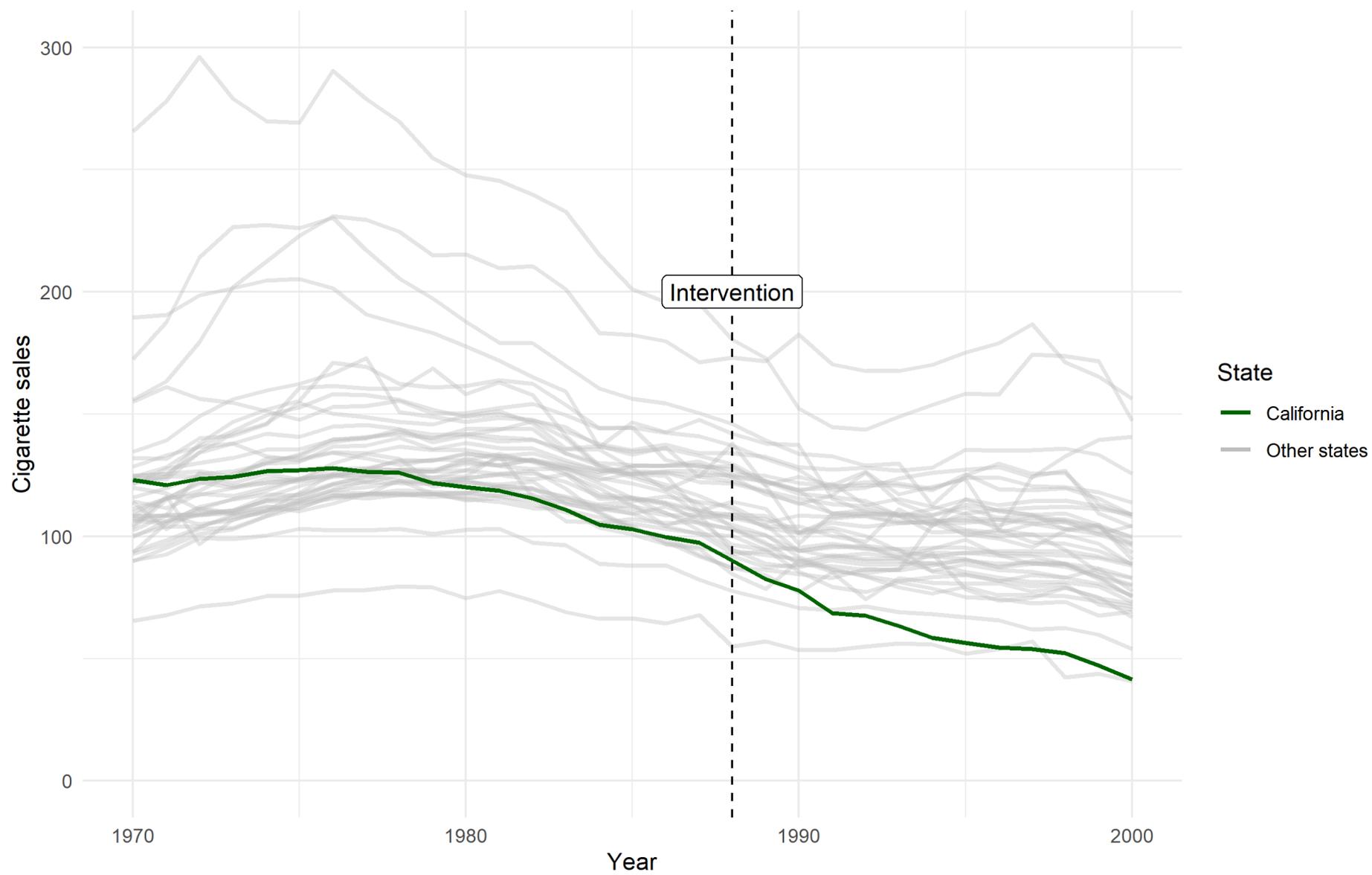
# Running Example: Proposition 99

- A famous example in policy evaluation literature

*Abadie, A., Diamond, A., & Hainmueller, J. (2010). Synthetic control methods for comparative case studies: **Estimating the effect of California's tobacco control program**. Journal of the American statistical Association, 105(490), 493-505.*

- In 1988, the state of California imposed a 25% tax on tobacco cigarettes
- Did this intervention successfully reduce cigarette sales in California?

Panel data for proposition 99



# Causal Policy Evaluation

## Basic Structure:

- We have one (or more) **unit(s)** which we observe **before** and **after** some intervention or action
- Did the intervention produce a change in the outcome for that unit?

# Causal Policy Evaluation

## Basic Structure:

- We have one (or more) **unit(s)** which we observe **before** and **after** some intervention or action
- Did the intervention produce a change in the outcome for that unit?

Many different methods/approaches which differ in:

- The **amount** and **type** of information they use
  - Amount of time-points and amount of potential “control” units
- The specific **statistical approach** they take
- The types of **assumptions** they rely on for identification

# # Control Units



None

Some

# Time-Points



Two

Pre-Post

Diff-in-Diff

Many

Interrupted  
Time-Series  
(ITS)

Synthetic  
Control

Two	Pre-Post	Diff-in-Diff
Many	Interrupted Time-Series (ITS)	Synthetic Control

# This Lecture

**Policy evaluation** through the lens of potential outcomes

We will consider the case where:

- we have **one unit** observed **repeatedly over time**
- at some point in time ( $T_0$ ) an **intervention** takes place

**Pre-intervention** we observe  $Y_t^0$  and **post-intervention**  $Y_t^1$

<i>Time</i>	$Y_t$	$A_t$
1	7	0
2	9	0
3	6	0
4	5	0
5	6	0
6	2	1
7	3	1
8	1	1
...	...	...
$T$	2	1

<i>Time</i>	$Y_t$	$A_t$	$Y_t^0$	$Y_t^1$
1	7	0	7	<i>NA</i>
2	9	0	9	<i>NA</i>
3	6	0	6	<i>NA</i>
4	5	0	5	<i>NA</i>
5	6	0	6	<i>NA</i>
6	2	1	<i>NA</i>	2
7	3	1	<i>NA</i>	3
8	1	1	<i>NA</i>	1
...	...	...	...	...
<i>T</i>	2	1	<i>NA</i>	2

# Causal Effects of Policies

We want to estimate the **causal effect of the policy intervention**

We think about this as the difference between

(a) the **observed outcome** after the policy was introduced

(b) What the outcome **would have been** without the intervention

$$CE_t = Y_t^1 - Y_t^0$$

where  $t > T_0$  (i.e., the post-intervention time period)

<i>Time</i>	$Y_t$	$A_t$	$Y_t^0$	$Y_t^1$
1	7	0	7	<i>NA</i>
2	9	0	9	<i>NA</i>
3	6	0	6	<i>NA</i>
4	5	0	5	<i>NA</i>
5	6	0	6	<i>NA</i>
6	2	1	<i>NA</i>	2
7	3	1	<i>NA</i>	3
8	1	1	<i>NA</i>	1
...	...	...	...	...
<i>T</i>	2	1	<i>NA</i>	2

<i>Time</i>	$Y_t$	$A_t$	$Y_t^0$	$Y_t^1$
1	7	0	7	NA
2	9	0	9	NA
3	6	0	6	NA
4	5	0	5	NA
5	6	0	6	NA
6	2	1	NA	2
7	3	1	NA	3
8	1	1	NA	1
...	...	...	...	...
<i>T</i>	2	1	NA	2

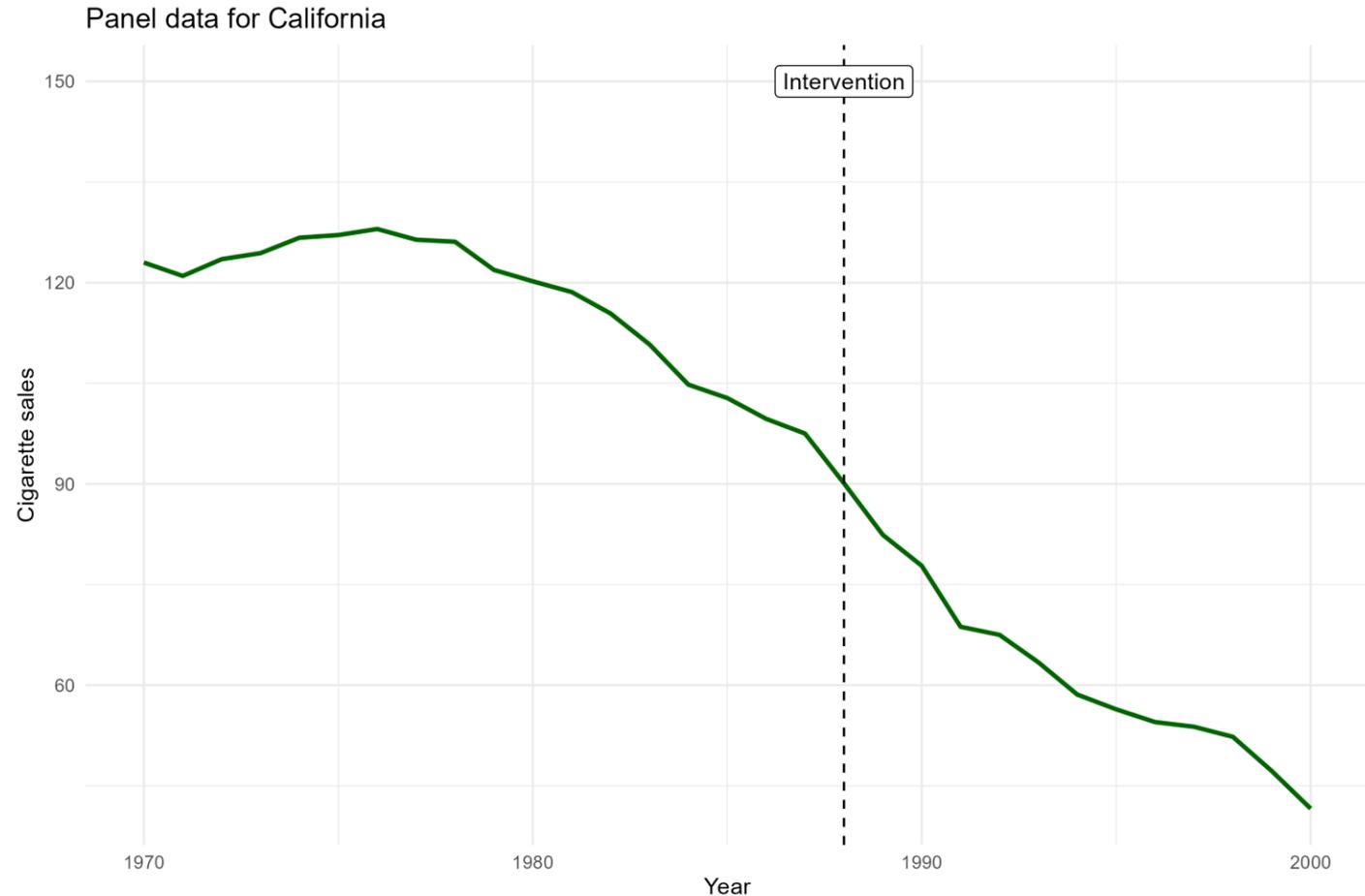
*The problem of estimating the effect of a policy intervention is equivalent to the problem of estimating  $Y_t^0$*

*Abadie, A. (2021). Using synthetic controls: Feasibility, data requirements, and methodological aspects. Journal of Economic Literature, 59(2), 391-425.*

# Pre-Post Estimator

# Pre-post estimator

We use only the cigarette sales time series for California



# Pre-post estimator

- We want to estimate the following quantity:

$$CE_{post} = E[Y_{post}^1] - E[Y_{post}^0]$$

- But we cannot observe  $Y_{post}^0$ !
- Solution(!): replace  $E[Y_{post}^0]$  by  $E[Y_{pre}^0]$ , which is observable

$$CE_{post} = E[Y_{post}^1] - E[Y_{pre}^0]$$

# Pre – Post analysis

<i>Time</i>	$Y_t$	$A_t$	$Y_t^0$	$Y_t^1$
1	7	0	7	NA
2	9	0	9	NA
3	6	0	6	NA
4	5	0	5	NA
5	6	0	6	NA
6	2	1	NA	2
7	3	1	NA	3
8	1	1	NA	1
...	...	...	...	...
$T$	2	1	NA	2

A blue vertical box highlights the  $Y_t^0$  column for rows 1 through 5. A blue arrow points from this box to the label  $\bar{Y}_{pre}^0$ .

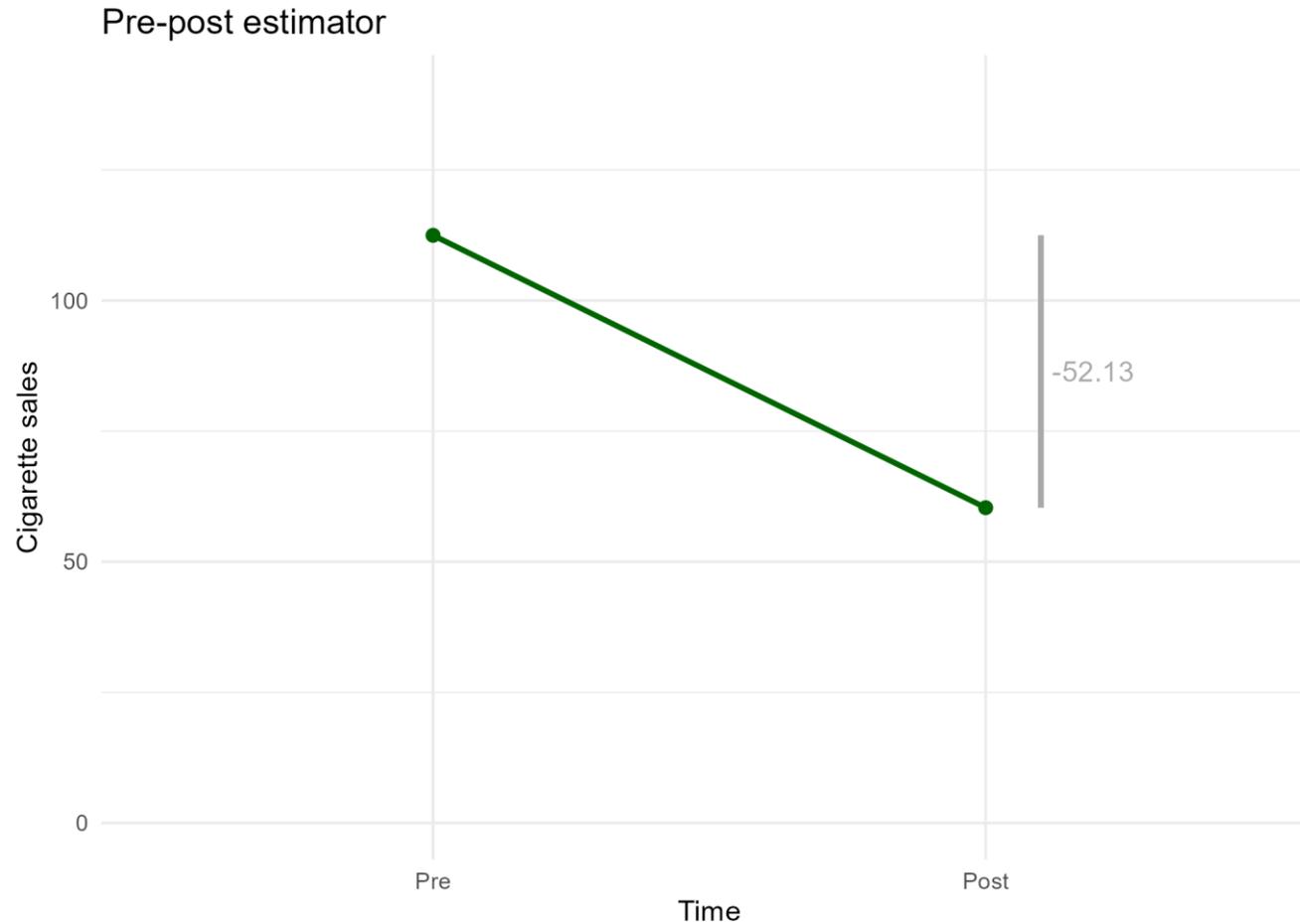
A red vertical box highlights the  $Y_t^1$  column for rows 6 through  $T$ . A red arrow points from this box to the label  $\bar{Y}_{post}^1$ .

# Pre – Post analysis

<i>Time</i>	$Y_t$	$A_t$	$Y_t^0$	$Y_t^1$
1	7	0	7	NA
2	9	0	9	NA
3	6	0	6	NA
4	5	0	5	NA
5	6	0	6	NA
6	2	1	NA	2
7	3	1	NA	3
8	1	1	NA	1
...	...	...	...	...
$T$	2	1	NA	2

$$\widehat{CE}_{post} = \bar{Y}_{post}^1 - \bar{Y}_{post}^0$$

# Pre-post estimator



Can estimate uncertainty by using a regression model on the disaggregated data, correcting SEs for unmodelled autocorrelation

# Pre-post estimator

Most important / strict assumption:

## No trend in time

- Remember: we assumed  $\bar{Y}_{post}^0 = \bar{Y}_{pre}^0$
- We assume the pre-post difference is caused by intervention **only**
- If trend exists, then the effect of trend and of intervention cannot be distinguished

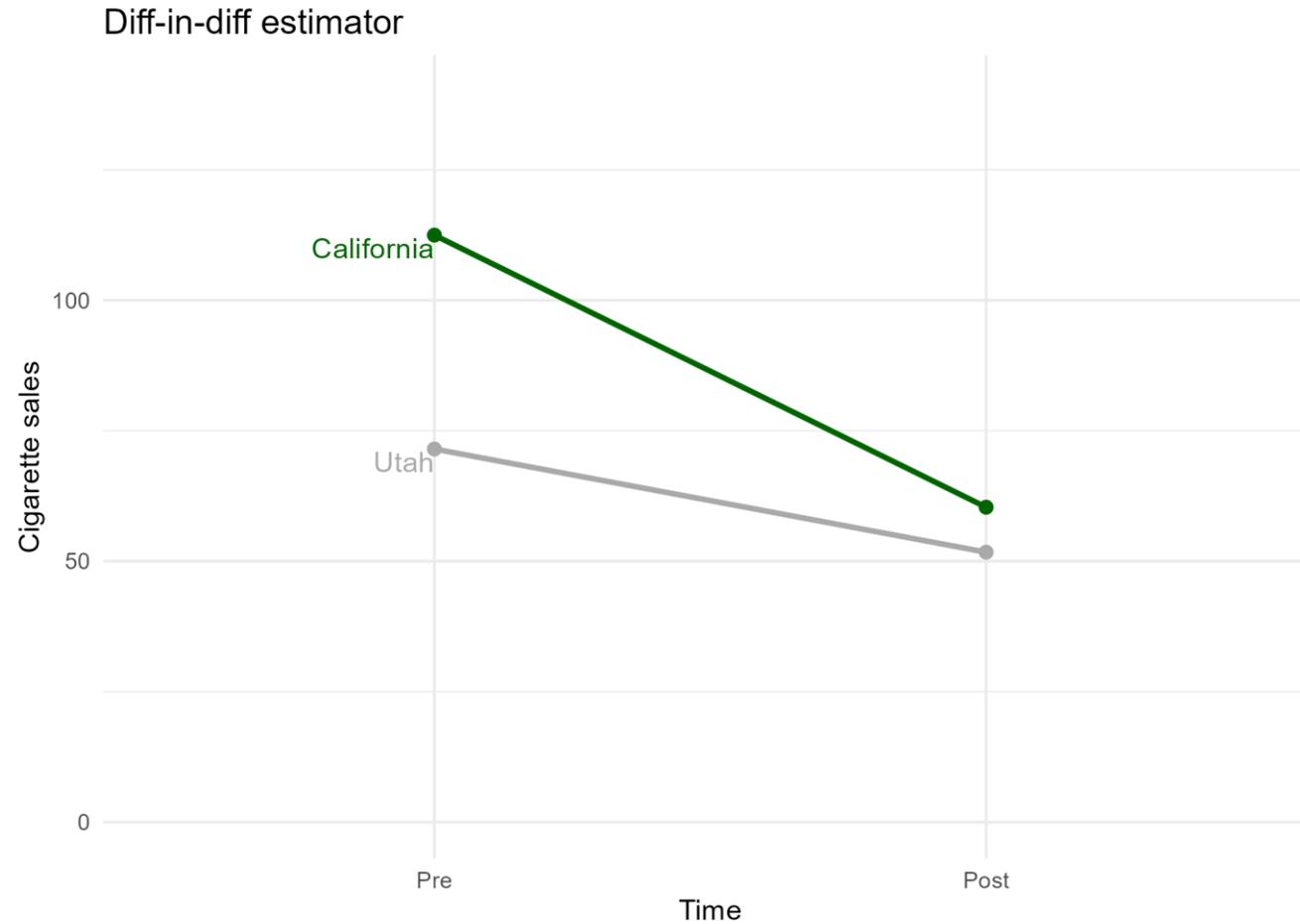
# **Difference-in-Differences**

*„transparent and often at least superficially plausible”*

*Angrist, J. D. and Krueger, A. B. (1999). Empirical strategies in labor economics. In Handbook of labor economics, volume 3, pages 1277–1366. Elsevier.*

<i>Time</i>	$Y_t$	$A_t$	$Y_t^0$	$Y_t^1$	$C_{1t}$
1	7	0	7	<i>NA</i>	2
2	9	0	9	<i>NA</i>	6
3	6	0	6	<i>NA</i>	4
4	5	0	5	<i>NA</i>	2
5	6	0	6	<i>NA</i>	1
6	2	1	<i>NA</i>	2	3
7	3	1	<i>NA</i>	3	2
8	1	1	<i>NA</i>	1	4
...	...	...	...	...	...
<i>T</i>	2	1	<i>NA</i>	2	3

# Difference-in-differences



# Difference-in-differences

- Like before, we want to estimate the following quantity:

$$\overline{CE}_{post} = \bar{Y}_{post}^1 - \bar{Y}_{post}^0$$

- Now, we assume there is an effect of time:  $\beta \cdot Time$
- We can represent unobservable  $\bar{Y}_{post}^0$  as

$$\bar{Y}_{post}^0 = \bar{Y}_{pre}^0 + \beta \cdot Time$$

# Difference-in-differences

- But the trend  $\beta \cdot Time$  is also unobservable!
- Solution: assume equal trends for Utah and California

$$\beta \cdot Time = (\bar{C}_{post}^0 - \bar{C}_{pre}^0)$$

- Thus, our model for the counterfactual is:

$$\bar{Y}_{post}^0 = \bar{Y}_{pre}^0 + (\bar{C}_{post}^0 - \bar{C}_{pre}^0)$$

# Difference-in-differences

- Plugging this into the causal effect equation:

$$\overline{CE}_{post} = (\bar{Y}_{post}^1 - \bar{Y}_{pre}^0) - (\bar{C}_{post}^0 - \bar{C}_{pre}^0)$$

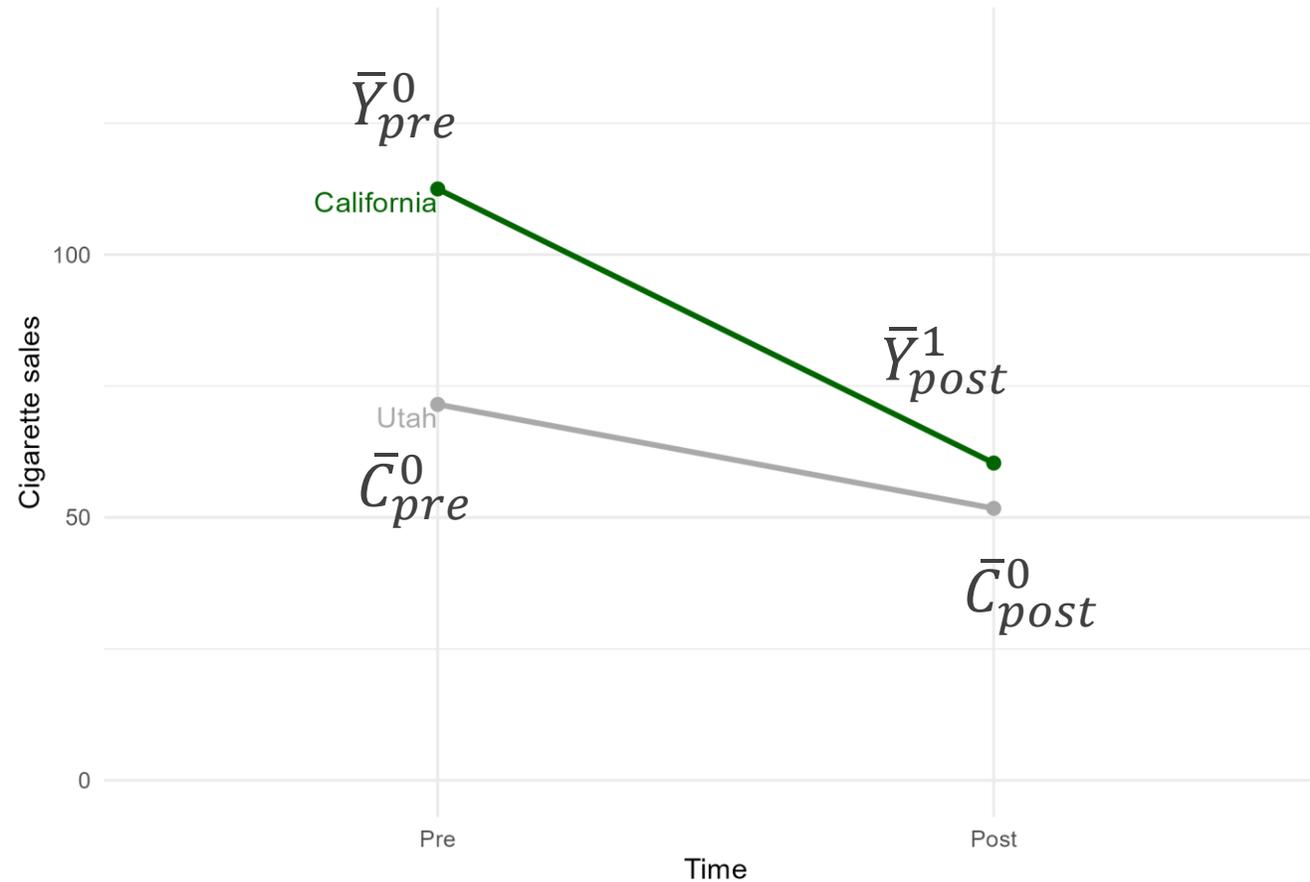
- Difference in differences!

$$\widehat{CE}_{post} = (\bar{Y}_{post} - \bar{Y}_{pre}) - (\bar{C}_{post} - \bar{C}_{pre})$$

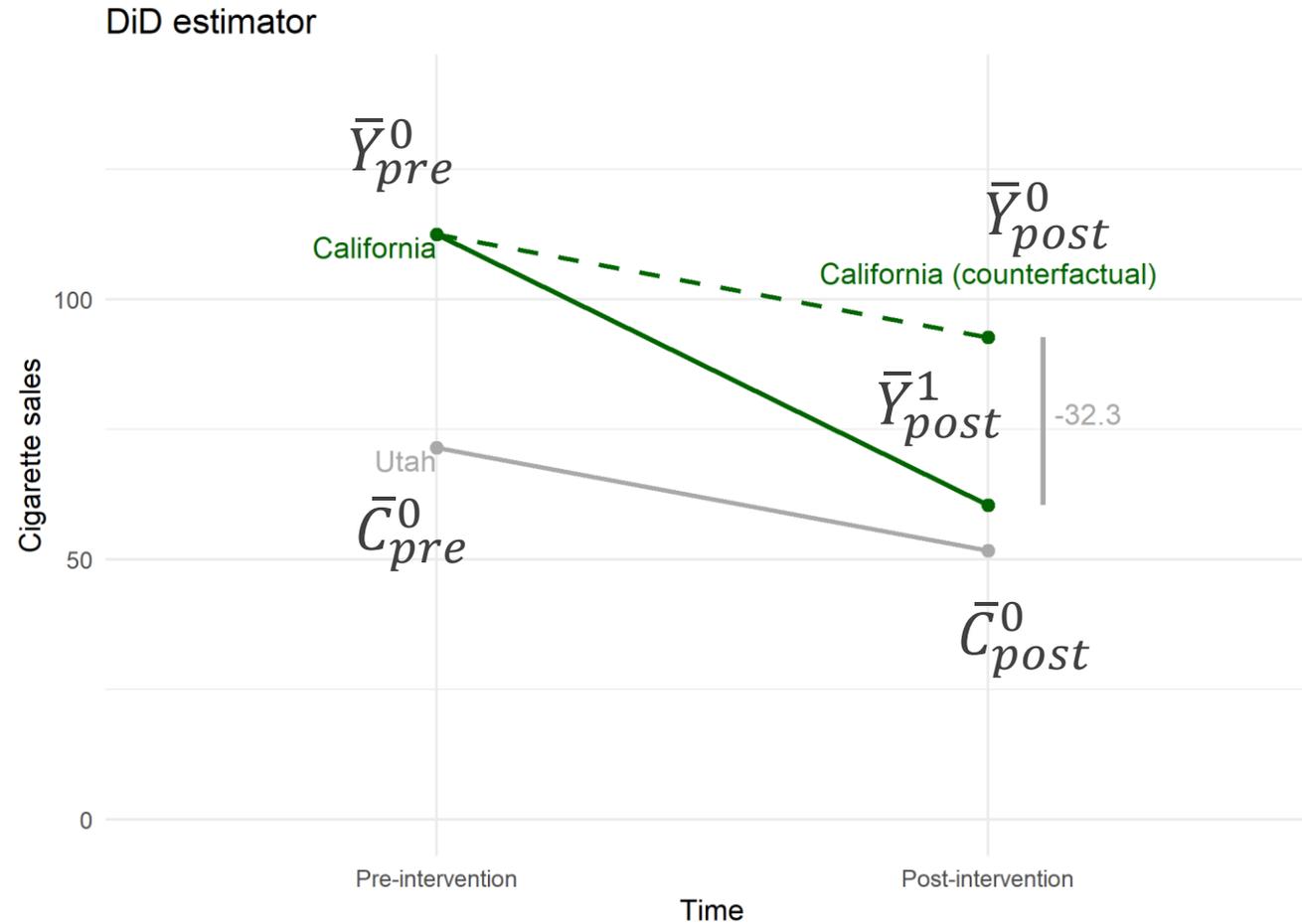
# Difference-in-differences



# Difference-in-differences



# Difference-in-differences



# Most important assumptions

## Parallel trends

$$\beta \cdot Time = (\bar{C}_{post}^0 - \bar{C}_{pre}^0)$$

Time effect is the same for the treated and the control unit

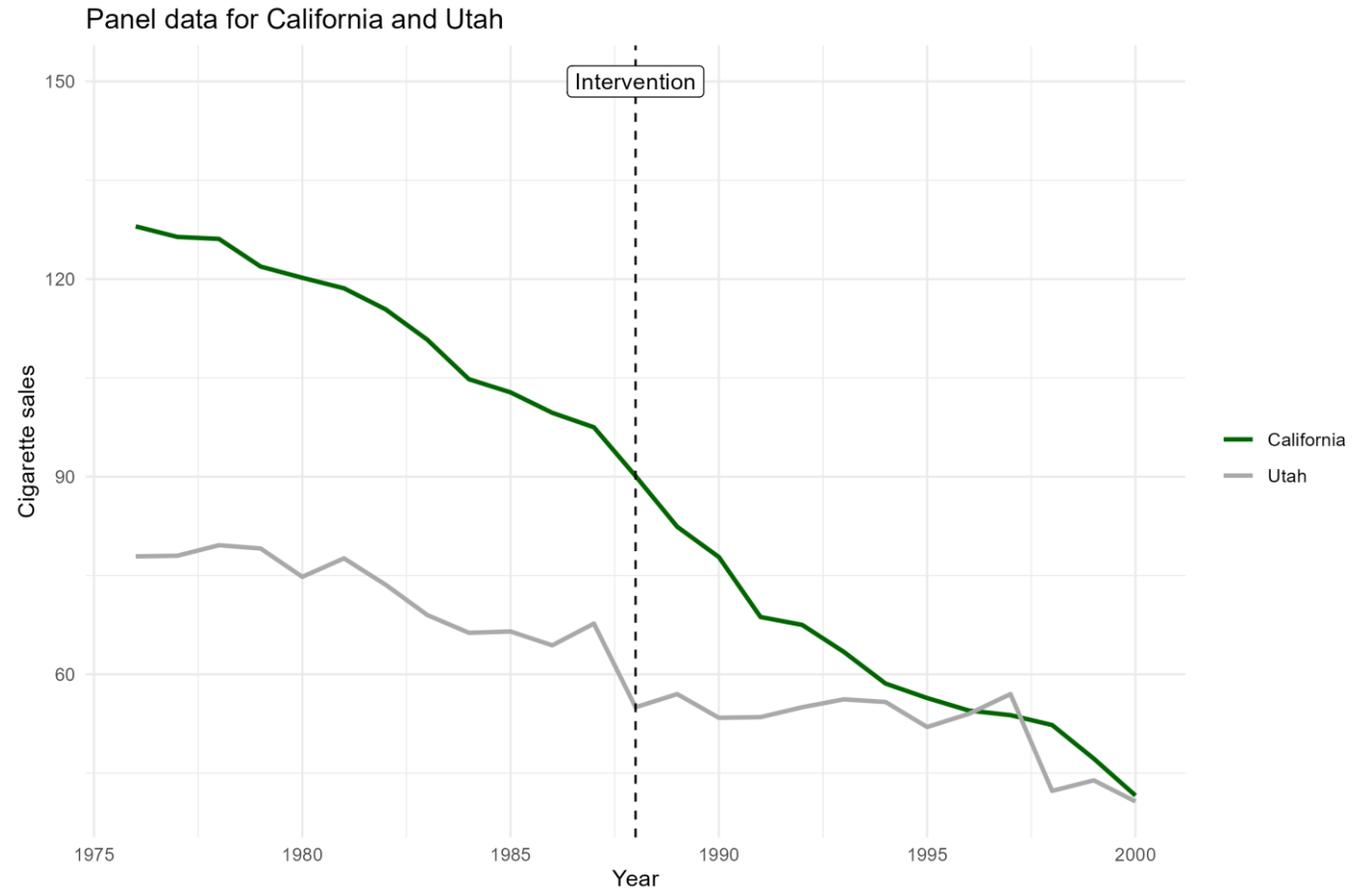
## No interference / spillover

$$\bar{C}_{post} = \bar{C}_{post}^0$$

The control unit is not affected by the intervention

# Most important assumptions

- Can we assume parallel trends?
- Superficially plausible?



# Practical: pre-post & DiD

Work in pairs/groups!  
[tinyurl.com/2s3hn8pj](https://tinyurl.com/2s3hn8pj)



# # Control Units



None

Some

# Time-Points



Two

Pre-Post

Diff-in-Diff

Many

Interrupted  
Time-Series  
(ITS)

Synthetic  
Control

Two	Pre-Post	Diff-in-Diff
Many	Interrupted Time-Series (ITS)	Synthetic Control

# **Synthetic Control**

*„arguably the most important innovation in the policy evaluation literature in the last 15 years”*

*Athey, S., & Imbens, G. W. (2017). The state of applied econometrics: Causality and policy evaluation. Journal of Economic perspectives, 31(2), 3-32.*

# Basic idea

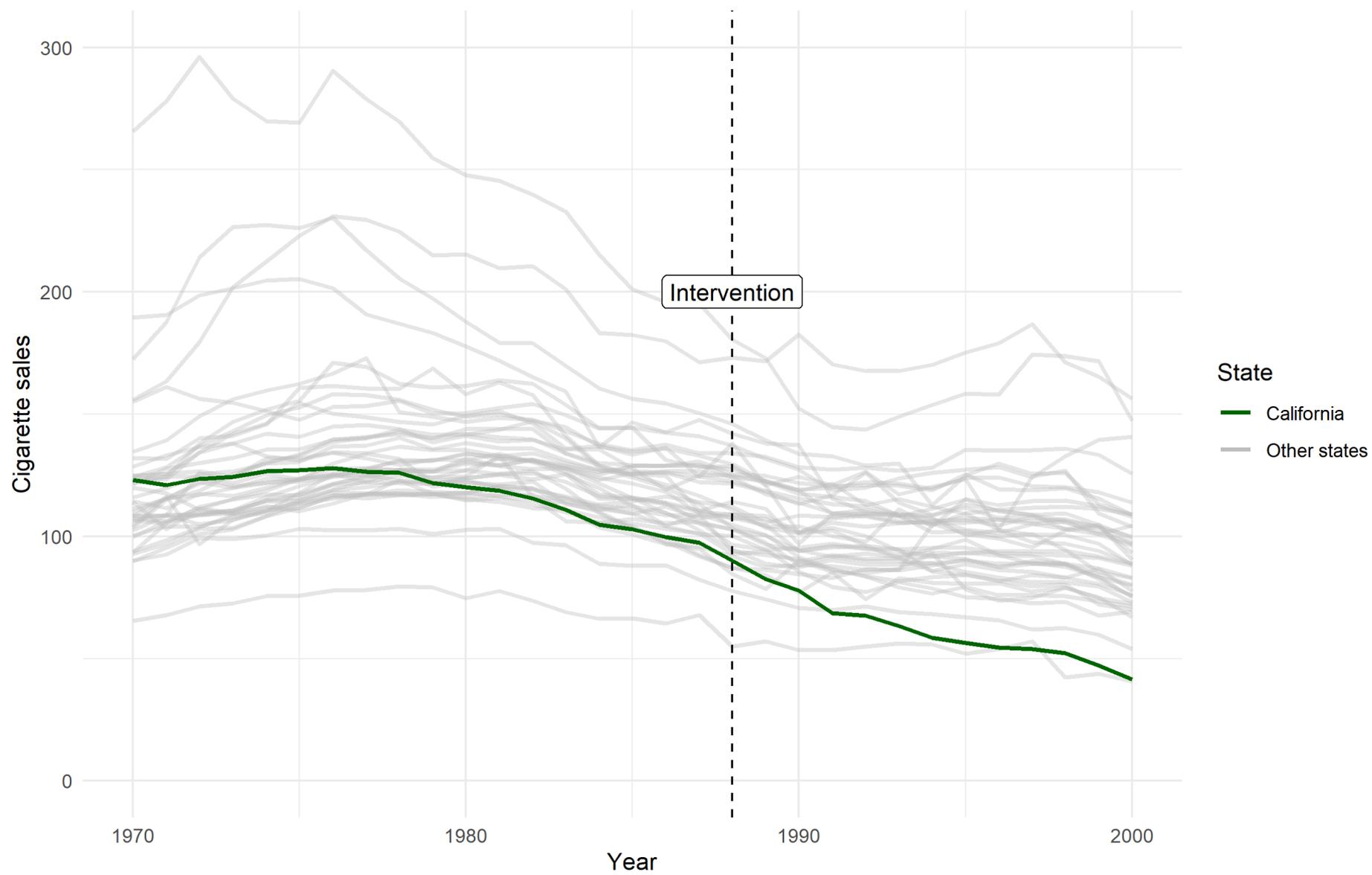
With diff-in-diff we used a control unit to attempt a correction for unmeasured time-varying confounders (e.g., macroeconomic situation in U.S.A.)

- You need a good control unit to make the parallel trends assumption at least superficially plausible
- But how much is Utah really like California?

We can instead use a weighted average of a **donor pool** of control units to create a **synthetic control** unit

- Choose the weights such that control is like California

Panel data for proposition 99



<i>Time</i>	$Y_t$	$A_t$	$Y_t^0$	$Y_t^1$	$C_{1t}$	$C_{2t}$	...	$C_{jt}$
1	7	0	7	NA	2	9	...	6
2	9	0	9	NA	6	9	...	8
3	6	0	6	NA	4	3	...	5
4	5	0	5	NA	2	1	...	4
5	6	0	6	NA	1	2	...	7
6	2	1	NA	2	3	6	...	7
7	3	1	NA	3	2	5	...	6
8	1	1	NA	1	4	6	...	5
...	...	...	...	...	...	...	...	4
$T$	2	1	NA	2	3	4	...	6

# Synthetic Control

<i>Time</i>	$Y_t$	$A_t$	$Y_t^0$	$Y_t^1$	$C_{1t}^0$	$C_{2t}^0$	...	$C_{jt}^0$
1	7	0	7	NA	2	9	...	6
2	9	0	9	NA	6	9	...	8
3	6	0	6	NA	4	3	...	5
4	5	0	5	NA	2	1	...	4
5	6	0	6	NA	1	2	...	7
6	2	1	NA	2	3	6	...	7
7	3	1	NA	3	2	5	...	6
8	1	1	NA	1	4	6	...	5
...	...	...	...	...	...	...	...	4
$T$	2	1	NA	2	3	4	...	6

Estimate Weights

$$Y_t = \sum_{j=1}^J \widehat{w}_j C_{jt} \quad t < T_0$$

# Synthetic Control

Time	$Y_t$	$A_t$	$Y_t^0$	$Y_t^1$	$C_{1t}^0$	$C_{2t}^0$	...	$C_{jt}^0$
1	7	0	7	NA	2	9	...	6
2	9	0	9	NA	6	9	...	8
3	6	0	6	NA	4	3	...	5
4	5	0	5	NA	2	1	...	4
5	6	0	6	NA	1	2	...	7
6	2	1	$\widehat{Y}_6^0$	2	3	6	...	7
7	3	1	$\widehat{Y}_7^0$	3	2	5	...	6
8	1	1	$\widehat{Y}_8^0$	1	4	6	...	5
...	...	...	...	...	...	...	...	4
$T$	2	1	$\widehat{Y}_T^0$	2	3	4	...	6

Estimate Weights

$$Y_t = \sum_{j=1}^J \widehat{w}_j C_{jt} \quad t < T_0$$

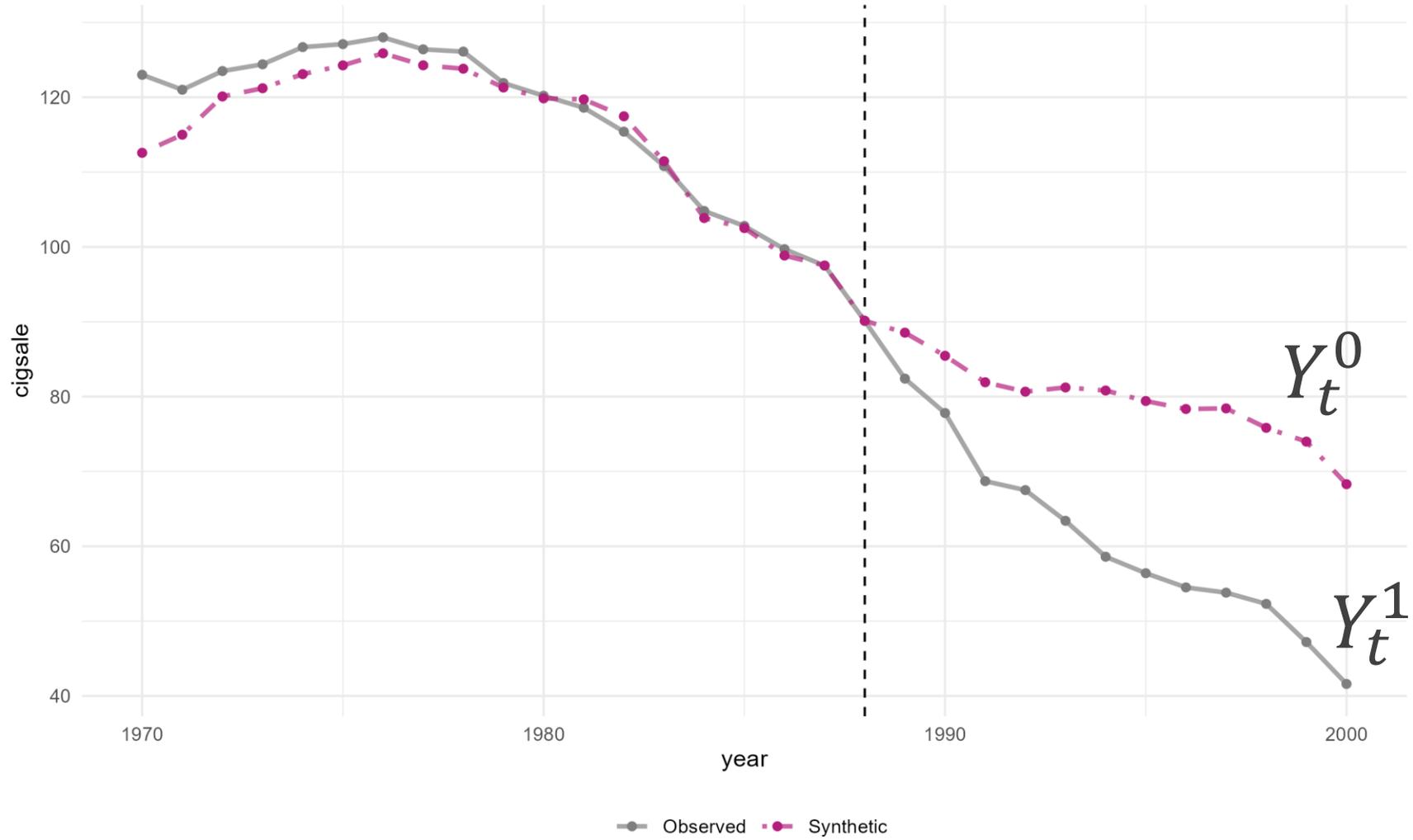


$$\widehat{Y}_t^0 = \sum_{j=1}^J \widehat{w}_j C_{jt} \quad t > T_0$$

Impute counterfactual

$$\widehat{CE}_t = Y_t^1 - \widehat{Y}_t^0$$

Time Series of the synthetic and observed cigsale



Dashed line denotes the time of the intervention.

# Estimating weights

- Choose weights such that the synthetic control **looks like** the treated unit
- Use only pre-intervention data for this
- Weights should be positive and sum to one  
Interpolation constraint / convex hull

# Estimating weights

What does it mean to “look like” California? This is a choice by the researcher!

- Pre-intervention target variable
  - Cigarette sales in certain years
- Pre-intervention **covariates**
  - Population composition
  - Average income of population
  - Price of cigarettes
  - Beer consumption

# Estimating weights

- Simultaneous estimation of two weights
  - Unit weights  $w_j$   
How important is each donor pool unit  $j$ ?
  - Variable weights  $v_h$   
How important is each variable  $p$ ?
- Choose  $w$  to minimize  $v$ -weighed multivariate Euclidean distance between treated and synthetic control pre-intervention

$$\hat{w}_j = \min_{w_j} \|v \cdot (X_T - w^T X_D)\|$$

- Like nearest neighbours matching!

# Estimating weights

How to choose  $v_h$ ?

## Simple

Use inverse of variance of each variable  $h$

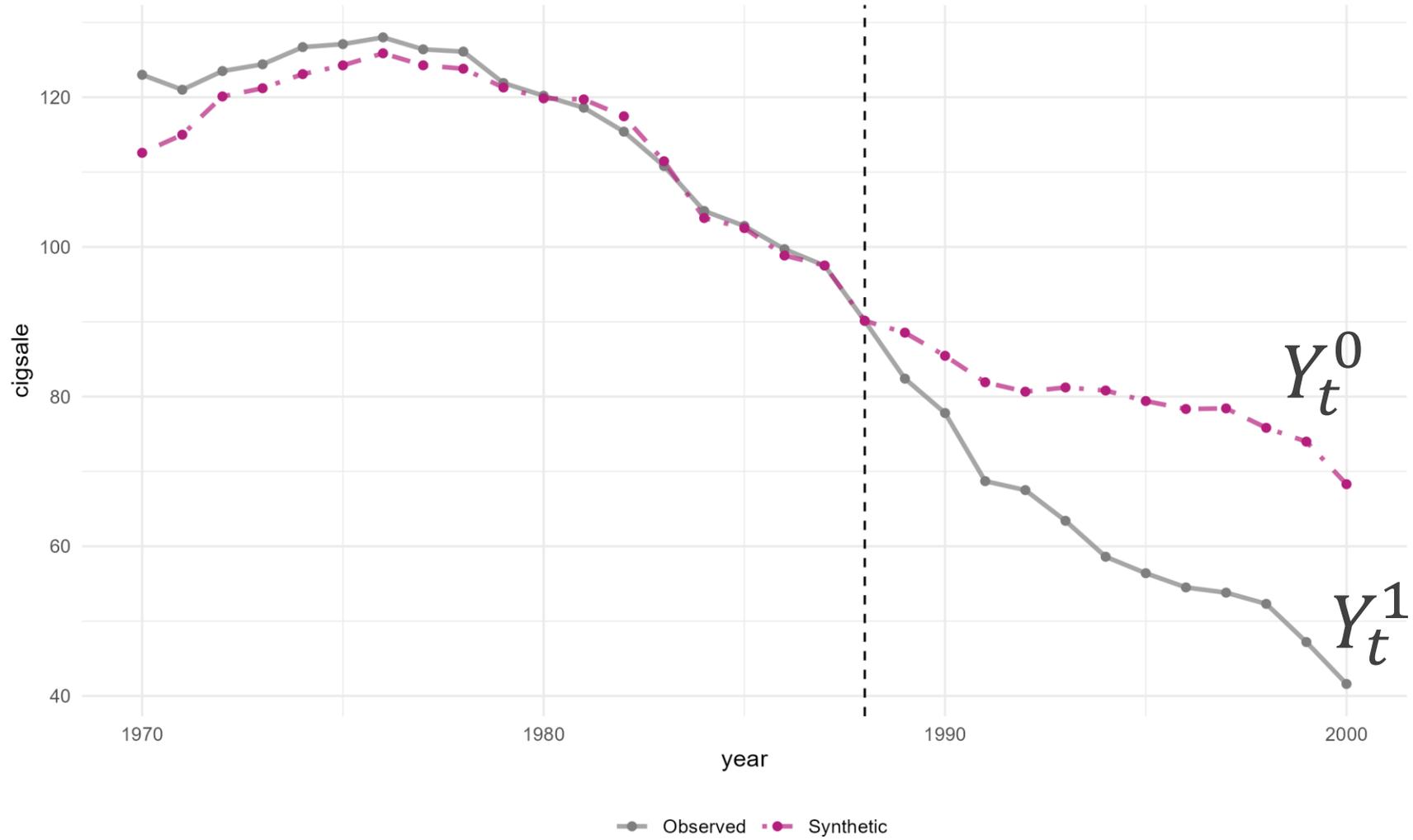
Like scaling the variables and then using unweighted Euclidean distance matching

## Complex

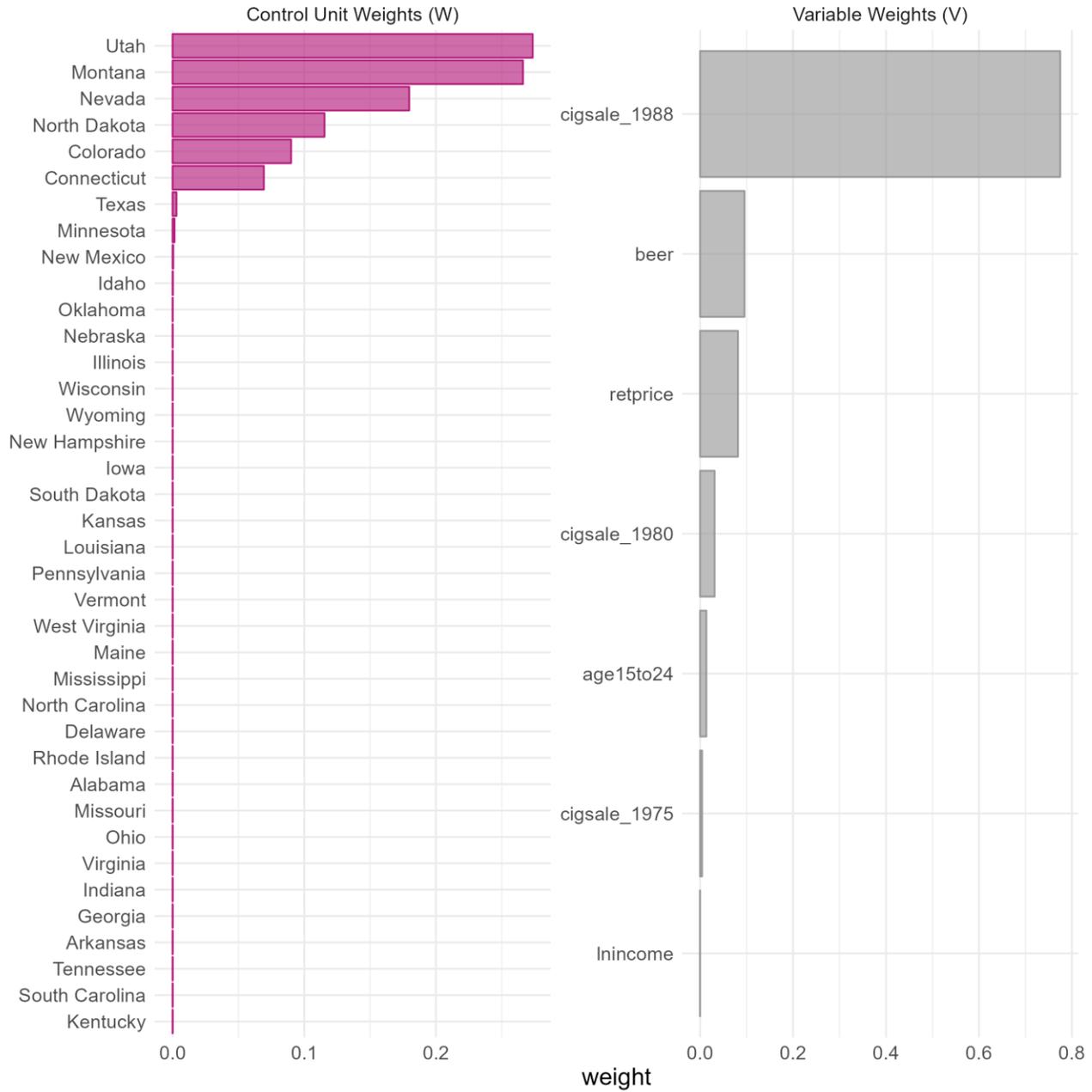
Choose  $v$  such that root mean squared prediction error (RMSPE) on pre-intervention target variable is minimized

Increased importance of good pre-intervention prediction

Time Series of the synthetic and observed cigsale



Dashed line denotes the time of the intervention.



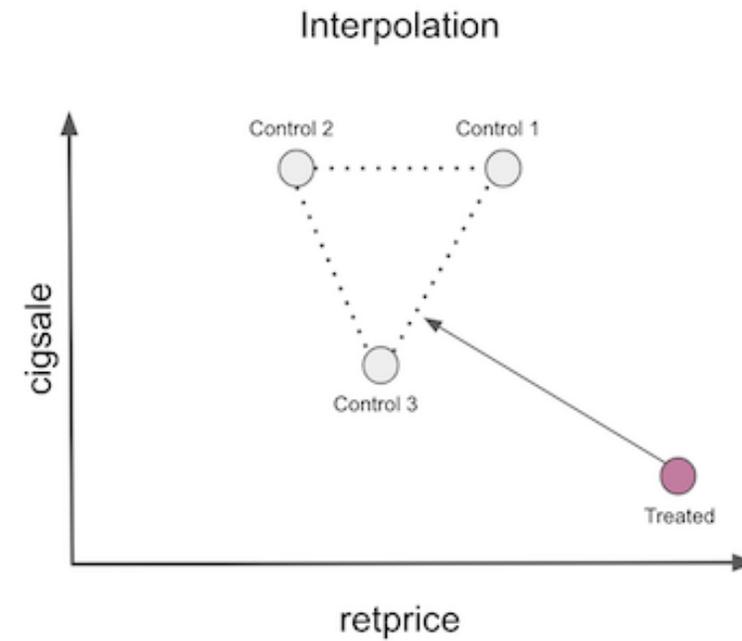
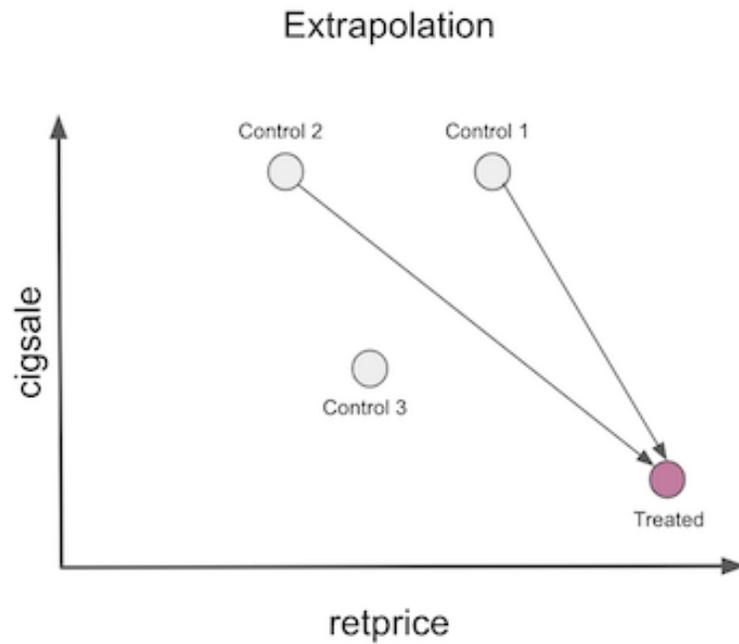
# Convex hull condition

**Distribution of covariate and target variables in donor pool should cover treated unit**

- It should be possible to interpolate the target unit values pre-intervention using the donor pool units

This is a way of ensuring that the **target unit** is not **too different** from the units that go into the synthetic control

# Interpolation



# Key Assumption

## **No interference / spillover:**

The donor pool units do not receive any intervention effect

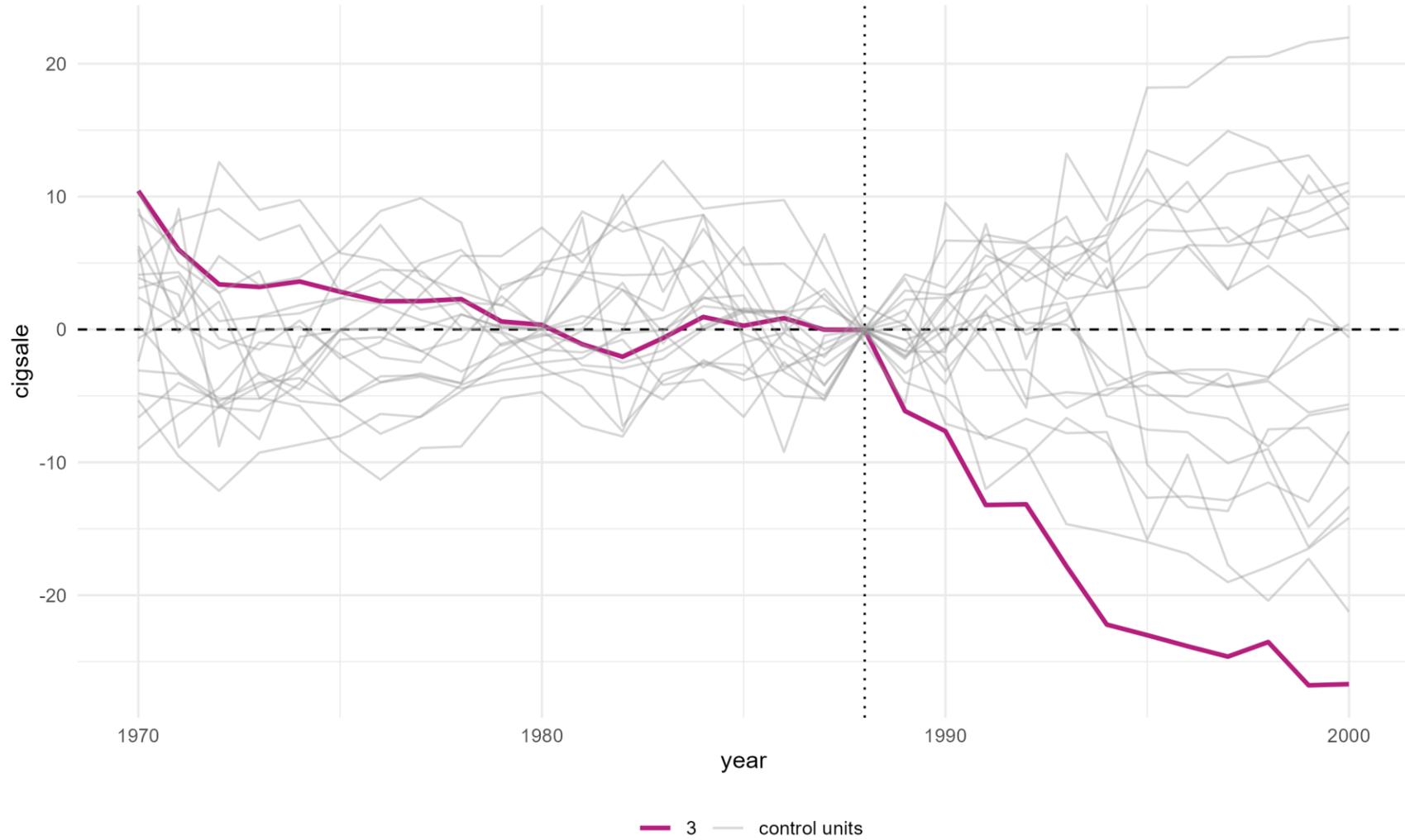
## Example spillover effects

- Californians living near the border may buy their cigarettes in states across the border
- Other states may pass laws similar to on California

# How to quantify uncertainty?

- Most common method: **permutation test**
- Apply synthetic control method many times, once for each unit in the donor pool
- These units have no intervention effect
- Create reference/null distribution of  $Y_t^0$
- Compare target unit's counterfactual to reference distribution
- Obtain a permutation p-value

### Difference of each 'state' in the donor pool



Pruned all placebo cases with a pre-period RMSPE exceeding two times the treated unit's pre-period RMSPE.

# There are many choices

- Which units in the donor pool?
- Which control variables?
- What should my weights optimize?
- How many nonzero unit weights should I get?
- What settings do I give to the nonlinear optimizer?

“researcher degrees of freedom”

# There are many choices

- These choices influence your causal estimate  $\widehat{CE}_t$
- Think of your causal estimate as “conditional” on the “model” (choices)
- Investigate the impact of different choices through robustness checks / sensitivity analysis

# **Interrupted Time Series (in brief)**

# # Control Units



None

Some

# Time-Points



Two

Pre-Post

Diff-in-Diff

Many

Interrupted  
Time-Series  
(ITS)

Synthetic  
Control

Two	Pre-Post	Diff-in-Diff
Many	Interrupted Time-Series (ITS)	Synthetic Control

# Interrupted Time Series

Another popular method which is built on **predicting** the counterfactual

Here the prediction is based on a **time-forward** or **forecasting** model

- I.e. we use **past** pre-intervention data to **impute** the missing counterfactual at each point in time

# Interrupted Time Series

<i>Time</i>	$Y_t$	$A_t$	$Y_t^0$	$Y_t^1$
1	7	0	7	NA
2	9	0	9	NA
3	6	0	6	NA
4	5	0	5	NA
5	6	0	6	NA
6	2	1	$\widehat{Y}_6^0$	2
7	3	1	$\widehat{Y}_7^0$	3
8	1	1	$\widehat{Y}_8^0$	1
...	...	...	...	...
$T$	2	1	$\widehat{Y}_T^0$	2

Train a forecasting model, e.g.  
 $\widehat{Y}_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \dots + \beta * Time$

Make forecasts

$$\widehat{CE}_t = Y_t^1 - \widehat{Y}_t^0$$

# Building a forecasting model

Much of the challenge of this approach is in choosing an appropriate **forecasting** model

These can be very simple or very complex, e.g.:

- We can forecast by fitting a **growth curve** which would model the overall time trend

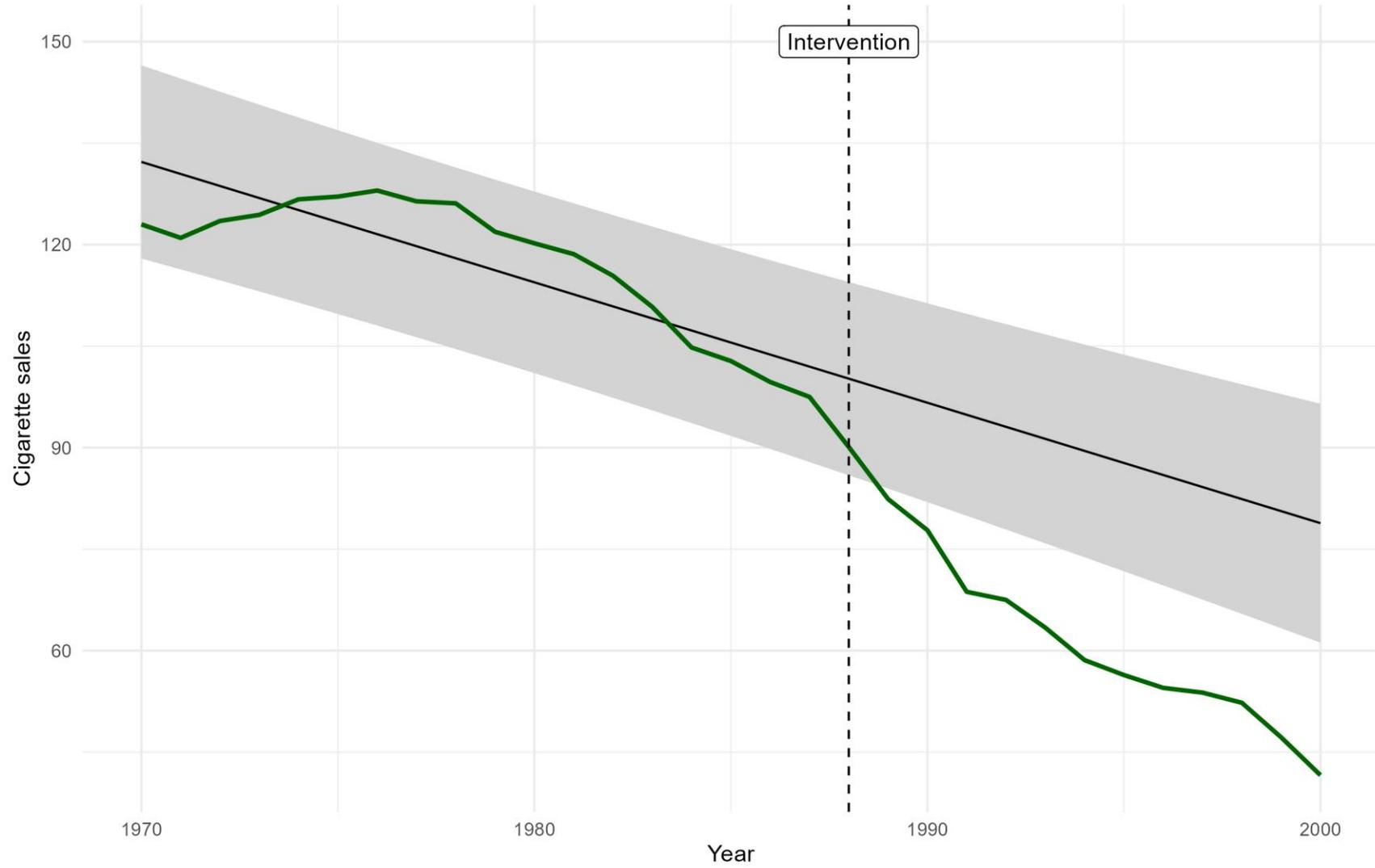
$$Y_t = \beta_0 + \beta_1 \text{Time} + e_t$$

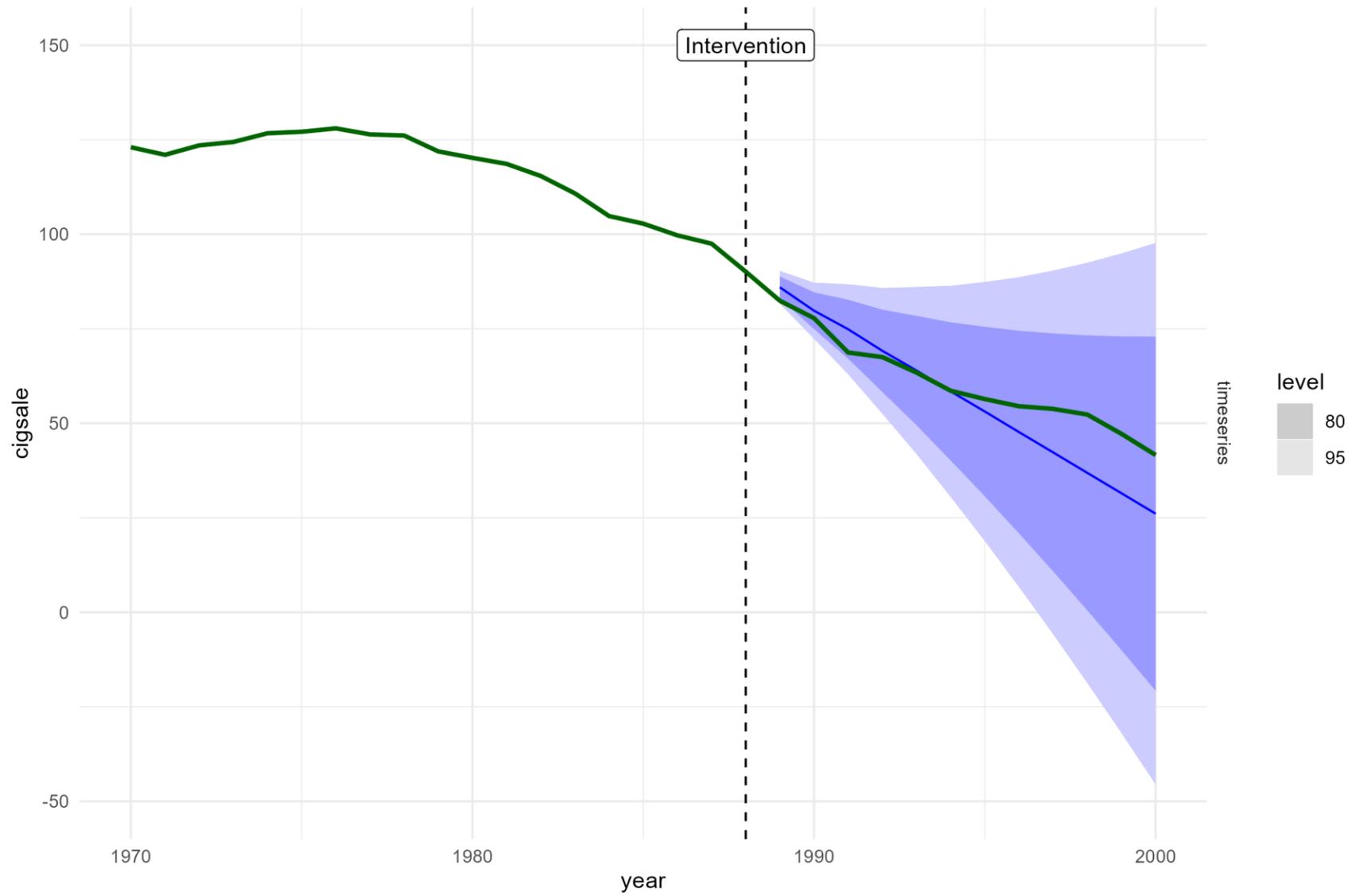
- We can forecast by using **time-series models** that model **autocorrelation**

$$Y_t = \phi_1 Y_{t-1} + e_t \quad Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + e_t \quad Y_t - Y_{t-1} = \gamma e_{t-1} + e_t$$

*e.g. ARIMA models can account for autocorrelation and time trends*

Panel data for California





# Key Assumptions

Compared to pre-post, we do not assume away the trend, but instead model it directly

But our inferences about the causal effect are entirely dependent on being able to fit **an appropriate forecasting model**

- i.e. one that correctly captures the trend(s) and autocorrelation structures in the data

In practice, this may be very difficult

# Discussion

# Summary

Different policy evaluation methods have been developed across a variety of different disciplines

- Economics, Epidemiology/Public health, Social Sciences

These methods differ in terms of:

- The **amount** and **type** of information they use
  - Amount of time-points and amount of potential “control” units
- The specific **statistical approach** they take
- The types of **assumptions** they make

Using potential outcomes, we can more readily understand and compare these different methods

# # Control Units



None

Some

# Time-Points



Two

Pre-Post

Diff-in-Diff

Many

Interrupted  
Time-Series  
(ITS)

Synthetic  
Control

Two	Pre-Post	Diff-in-Diff
Many	Interrupted Time-Series (ITS)	Synthetic Control

# So which method is best?

The answer **in part** depends on what type and amount of data you have

- But this is the **easy part**

The answer in practice depends on **domain knowledge**

- The **hard part** is to figure out which **assumptions** you need for causal inference and whether they are reasonable in your particular use case
- It may simply not be possible in some cases!
- E.g. DiD won't work if trends are not parallel; synthetic control won't work if there is interference between units (no matter how much data you have!)
- Often, methods which are “data hungry” can relax some assumptions, but:

**There is no free lunch!**

**Thanks!**

oisinryan.org

UU: Special Interest Group in Causal Data Science

# Useful References

## Difference in Differences

Angrist, J. D., & Krueger, A. B. (1999). Empirical strategies in labor economics. In Handbook of labor economics (Vol. 3, pp. 1277-1366). Elsevier.

Angrist, J. D., & Pischke, J. S. (2009). Mostly harmless econometrics: An empiricist's companion. Princeton university press.

Caniglia, E. C., & Murray, E. J. (2020). Difference-in-difference in the time of cholera: a gentle introduction for epidemiologists. *Current epidemiology reports*, 7, 203-211.

## Interrupted Time Series

Bernal, J. L., Cummins, S., & Gasparrini, A. (2017). Interrupted time series regression for the evaluation of public health interventions: a tutorial. *International journal of epidemiology*, 46(1), 348-355.

Bernal, J.L, Cummins, S., & Gasparrini, A. (2019). Difference in difference, controlled interrupted time series and synthetic controls. *International journal of epidemiology*, 48(6), 2062-2063.

# Useful References

## Synthetic Control

Abadie, A., Diamond, A., & Hainmueller, J. (2010). Synthetic control methods for comparative case studies: Estimating the effect of California's tobacco control program. *Journal of the American Statistical Association*, 105(490), 493-505.

Abadie, A. (2021). Using synthetic controls: Feasibility, data requirements, and methodological aspects. *Journal of Economic Literature*, 59(2), 391-425.

## CausalImpact

Brodersen, K. H., Gallusser, F., Koehler, J., Remy, N., & Scott, S. L. (2015). Inferring causal impact using Bayesian structural time-series models. *The Annals of Applied Statistics*, 247-274.

Linden, A. (2018). Combining synthetic controls and interrupted time series analysis to improve causal inference in program evaluation. *Journal of evaluation in clinical practice*, 24(2), 447-453.

<http://google.github.io/CausalImpact/CausalImpact.html>

# Useful References

## Synthetic DiD

Arkhangelsky, D., Athey, S., Hirshberg, D. A., Imbens, G. W., & Wager, S. (2021). Synthetic difference-in-differences. *American Economic Review*, 111(12), 4088-4118.

## More on Causal Policy Evaluation

Free online course materials made by Andrew Heiss

*Program Evaluation for Public Service*

<https://evalf22.classes.andrewheiss.com/content/>