Schedule

Monday, 6 March

9:30 Welcome coffee/tea

10:00 Krystal Guo: Strongly regular graphs with a regular point

11:00 Ferdinand Ihringer: Approximately Strongly Regular Graphs

11:30 Antonina P. Khramova: Constructing cospectral hypergraphs

12.00 Lunch on own

13:45 Coffee/tea and business meeting

14:15 Richard Lang

14:45 Mehmet Akif Yildiz: Cycle Partition of (Regular and Directed) Graphs

15:15 15 min break

15.30 7x 5 min lightning talks: Robin, Jan, Relinde, Jesse, Pjotr, Wout, Anna.

16.05 10 min break

16:15 Timothy Budd: Enumeration of lattice walks by winding angle.

Tuesday, 7 March

9:30 Coffee/tea

10:00 Daniel Dadush: Interior point methods are not worse than Simplex

11:00 Pieter Kleer: Sampling from the Gibbs Distribution in Congestion Games

11:30 Céline Swennenhuis: Bin Packing with the help of additive combinatorics

12.00 Lunch on own

13:45 Coffee/tea

14:15 8x 5 min lightning talks: Jelle, Sukanya, Erik-Jan, Krisztina, Michelle, Lorenzo, Gabriëlle, Artem.

14:55 20-min break

15:15 Anurag Bishnoi: Codes, blocking sets and graphs

16:00 Closing borrel

Contents

1 Invited talks

1.1 Strongly regular graphs with a regular point (Krystal Guo)

Arising from Hoffman and Singleton's study of Moore graphs, strongly regular graphs play an important role in algebraic graph theory. Strongly regular graphs can be construct from various geometric objects, such as finite geometries and generalized quadrangles. Certain geometric properties, such as having a regular point, can be studied in the context of graphs, using combinatorial and algebraic tools. We study pseudo-geometric strongly regular graphs whose second subconstituent with respect to a vertex is a cover of a strongly regular graph or a complete graph. By studying the structure of such graphs, we characterize all graphs containing such a vertex, thereby, answering a question posed by Gardiner, Godsil, Hensel, and Royle. As a by-product of our characterisation, we are able to give new constructions of infinite families of strongly regular graphs and compute many small sporadic examples; for example, we find 135478 new strongly regular graphs with parameters (85,20,3,5). This is joint work with Edwin van Dam.

1.2 Enumeration of lattice walks by winding angle (Timothy Budd)

Counting walks on lattices subject to a variety of conditions (on the step sets and boundary conditions) has been a very active topic in enumerative combinatorics, due to many links with probability theory, statistical physics, random geometry and algorithms but also due to the rich algebraic and analytic properties of the corresponding generating functions. In this talk I will discuss the enumeration of walks on the square lattice with prescribed winding angle around the origin. Applications include the enumeration of excursions in cones of various opening angles, statistics of winding angles of random walks and loops, and even some electronic properties of 2d materials in physics.

1.3 Interior point methods are not worse than Simplex (Daniel Dadush)

Whereas interior point methods provide polynomial-time linear programming algorithms, the running time bounds depend on bit-complexity or condition measures that can be unbounded in the problem dimension. This is in contrast with the classical simplex method that always admits an exponential bound. We introduce a new polynomial-time path-following interior point method where the number of iterations also admits a combinatorial upper bound $O(2^n n^{1.5} \log n)$ for an n-variable linear program in standard form. This complements previous work by Allamigeon, Benchimol, Gaubert, and Joswig (SIAGA 2018) that exhibited a family of instances where any path-following method must take exponentially many iterations.

The number of iterations of our algorithm is at most $O(n^{1.5} logn)$ times the number of segments of any piecewise linear curve in the wide neighborhood of the central path. In particular, it matches the number of iterations of any path following interior point method up to this polynomial factor. The overall exponential upper bound derives from studying the 'max central path', a piecewise-linear curve with the number of pieces bounded by the total length of n shadow vertex simplex paths.

This is joint work with Xavier Allamigeon (INRIA / Ecole Polytechnique), Georg Loho (U. Twente), Bento Natura (LSE), Laszlo Vegh (LSE).

1.4 Codes, blocking sets and graphs (Anurag Bishnoi)

I will survey the recent bounds on linear trifferent codes and minimal codes that we have obtained by using equivalences with finite geometric objects called blocking sets. I will also discuss a new explicit construction of all of these objects, based on expander graphs and asymptotically good codes.

Based on joint work(s) with Noga Alon, Shagnik Das, Jozefien D'haeseleer, Dion Gijswijt, Alessandro Neri, and Aditya Potukuchi.

2 Contributed talks

2.1 Approximately Strongly Regular Graphs (Ferdinand Ihringer)

A graph is called *strongly regular* with parameters (v, k, λ, μ) if it has v vertices, each vertex has degree k, two adjacent vertices have precisely λ common neighbors, and two nonadjacent vertices have precisely μ common neighbors. Obvious divisibility conditions aside, two important existence conditions for strongly regular graphs are known: the *Krein bound* and the *absolute bound*. We generalize the Krein conditions to what we call approximately strongly regular graphs. Here a graph is *approximately strongly regular* with parameters $(v, k, \lambda, \mu; \sigma)$ if it has v vertices, each vertex has degree k, two adjacent vertices have on average λ common neighbors, two nonadjacent vertices have on average μ common neighbors, and σ^2 measures the variance of the sizes of occurring common neighborhoods of two vertices. More precisely, $\frac{1}{vk}\sum_{x\sim y}(|N(x)\cap N(y)|-\lambda)^2 \leq \sigma^2$ and $\frac{1}{v(v-k-1)}\sum_{x\not\sim y}(|N(x)\cap N(y)|-\mu)^2 \leq \sigma^2$ (where $N(x)$ denotes the neighborhood of a vertex).

2.2 Constructing cospectral hypergraphs (Antonina P. Khramova)

Spectral hypergraph theory mainly concerns using hypergraph spectra to obtain structural information about the given hypergraphs. The study of cospectral hypergraphs is important since it reveals which hypergraph properties cannot be deduced from their spectra. In this talk, we show a new method for constructing cospectral uniform hypergraphs using two well-known hypergraph representations: adjacency tensors and neighbor matrices. The method is inspired by WQH-switching (Wang, Qiu, Hu, 2019), also known in the literature as modified GM-switching.

The talk is based on joint work with Aida Abiad (`https://arxiv.org/abs/2211.06087`).

2.3 Tiling Combinatorial Structure (Richard Lang)

We give an asymptotic characterisation of combinatorial structures that contain perfect tilings. There are three natural obstructions to containing a perfect tiling that correspond to cover, space and divisibility barriers. We show that any large enough combinatorial structure, which robustly overcomes each of these obstacles must already contain a perfect tiling. This generalises the geometric theory of hypergraph matching of Keevash and Mycroft. As an application, we recover recent work for codegree conditions, ordered graphs, quasirandom hypergraphs and rainbow transversals.

2.4 Cycle Partition of (Regular and Directed) Graphs (Mehmet Akif Yildiz)

A well-known result of Dirac states that every graph on n vertices with minimum degree at least $n/2$ has a Hamilton cycle. As a generalization of Dirac's result, one can be interested in finding a small collection of cycles that covers the vertex set, rather than finding a Hamilton cycle. Magnant and Martin conjectured that if G is a d-regular graph on n vertices, then $V(G)$ can be partitioned into $n/(d+1)$ cycles, and Gruslys and Letzter verified it for dense graphs. We prove the analogous result for directed graphs and oriented graphs, and I will explain some parts of the proof in this talk. Our result also solves Jackson's long-standing conjecture: every d-regular oriented graph on n vertices with $n < 4d+2$ is Hamiltonian. It is joint work with Allan Lo and Viresh Patel.

2.5 Sampling from the Gibbs Distribution in Congestion Games (Pieter Kleer)

Logit dynamics is a form of randomized game dynamics in which players repeatedly update their strategy, putting relatively more probability mass on strategies that yield a higher improvement in cost. In congestion (or potential) games, logit dynamics gives rise to a (reversible) Markov chain whose stationary distribution is the so-called Gibbs distribution. Unfortunately, convergence to the Gibbs distribution can be very slow. We are therefore interested in the following two questions:

1. Are there other Markov chains that converge quickly to the Gibbs distribution?
2. More general, are there other efficient algorithms for sampling from the Gibbs distribution?

In this talk, we will provide answers to these questions for the simple class of load balancing games using tools from discrete mathematics. We will also discuss some extensions to more general classes of games.

2.6 Bin Packing with the help of additive combinatorics (Céline Swennenhuis)

In the Bin Packing problem one is given n items with different weights and m bins with a given capacity; the goal is to distribute the items over the bins without exceeding the capacity. Björklund, Husfeldt and Koivisto (SICOMP 2009) presented an $O(2^n)$ time algorithm for Bin Packing. We show that for every m there exists a constant $s_m > 0$ such that an instance of Bin Packing with m bins can be solved in $O((2 - s_m)^n)$ time. Before our work, such improved algorithms were not known even for m equals 4.

A key step in our approach is a new result in Littlewood-Offord theory on the additive combinatorics of subset sums. (Joint work with Jesper Nederlof, Jakub Pawlewicz, Karol Węgrzycki, appeared at SODA'21)

3 Lightning talks (Monday)

3.1 Cospectrality results on generalized Johnson and Grassmann graphs (Robin Simoens)

Can we characterize a graph by its spectrum? In 2003, van Dam and Haemers conjectured that the answer is positive for almost all graphs. This conjecture, which plays a special role in the graph isomorphism problem, has only been solved for some specific families of graphs. In this talk, we use a switching method to prove that the answer is negative for some graph classes in the Johnson and Grassman schemes.

3.2 A geometric construction of a strongly regular graph non-isomorphic to NO+(8,2) (Jan De Beule)

The graph $\mathrm{NO}^+(8,2)$ is a strongly regular graph with parameters $(120, 63, 30, 36)$. In their recent book – Strongly Regular Graphs – Brouwer and Van Maldeghem mention the existence of a non-isomorphic, strongly regular graph with the same parameters, admitting Sym(7) as automorphism group. In this talk we discuss a geometric construction of this graph using a quadratic form of Witt index 4 on $\mathrm{GF}(2)^8$. Indeed it turns out to be possible to modify the adjacencies of $\mathrm{NO}^+(8,2)$ in such a way to obtain the non-isomorphic graph with the same parameters? The up to isomorphism unique ovoid of the triality quadric plays a central role. This is joint work with Sam Adriaensen, Robert Bailey and Morgan Rodgers.

3.3 Mathematics at the military (Relinde Jurrius)

Did you know Den Helder hosts a kind of mini technical university, where military officers in training study for a bachelors degree in engineering? No? Let me introduce my work place to you!

3.4 Approximation Ineffectiveness of a Tour-uncrossing Heuristic (Jesse van Rhijn)

We analyze a tour-uncrossing heuristic for the Travelling Salesperson Problem, showing that its worst-case approximation ratio is $\Omega(n)$ and its average-case approximation ratio is $\Omega(\sqrt{n})$ in expectation. We furthermore evaluate the approximation performance of this heuristic numerically on average-case instances, and find that it performs far better than the average-case lower bound suggests. This indicates a shortcoming in the approach we use for our analysis, which is a rather common approach in the analysis of local search heuristics.

3.5 Continuity of the Shannon capacity on graphs (Pjotr Buys)

For integers $p \geq 2q \geq 2$ let $E_{p/q}$ denote the graph with vertex set $\mathbb{Z}_p$ and distinct u, v are connected iff either $u - v$ or $v - u$ is strictly less than q modulo p. The asymptotic spectrum of graphs consists of graph parameters with interesting properties (examples are the Lovasz theta function and the fractional clique cover number). Together with David the Boer and Jeroen Zuiddam we proved that the functions $\mathbb{Q}_{\geq 2} \to \mathbb{R}$ given by $p/q \mapsto F(E_{p/q})$, where F either denotes the Shannon capacity or any element of the asymptotic spectrum on graphs, are right-continuous. The question remains whether they are also left-continuous.

3.6 New quantum invariants of planar knotoids (Wout Moltmaker)

I will discuss a generalization of knots called knotoids, their applications, and efforts towards their classification. In particular I will discuss universal quantum invariants of knotoids on the plane and show how they are used to help in the classification of planar knotoids. I will also give examples of computing a particular universal quantum invariant using a computer implementation.

3.7 Clique Dynamics of Finite or Infinite Locally Cyclic Graphs with $\delta \geq 6$ (Anna M. Limbach)

We prove that the clique graph operator k is divergent on a (not necessarily finite) locally cyclic graph G (i. e. $N_G(v)$ is a circle for every vertex v) with minimum degree $\delta(G) \geq 6$ if and only if the universal cover of G contains arbitrarily large triangular-shaped subgraphs. For finite G, this is equivalent to G being 6-regular.

The clique graph G of a graph G has the maximal complete subgraphs of G as vertices and its edges are given by non-empty intersections. The $(n + 1)$-st iterated clique graph is inductively defined as the clique graph of the n-th iterated clique graph. If all iterated clique graphs of G are pairwise non-isomorphic, the graph G is called k-divergent; otherwise, it is k-convergent.

Locally cyclic graphs with $\delta \geq 6$ which induce simply connected simplicial surfaces are isomorphic to their universal covers. On this graph class, we prove our claim by explicit construction of the iterated clique graphs. After that, we show that locally cyclic graphs with $\delta \geq 6$ are k-convergent if and only if their universal covers are k-convergent. This way, we can drop the condition of simple connectivity.

This talk is based on joint work with Markus Baumeister and Martin Winter.

4 Lightning talks (Tuesday)

4.1 Complexity Framework For Forbidden Subgraphs (Jelle Oostveen)

For any finite set $\mathcal{H} = \{H_1, \ldots, H_p\}$ of graphs, a graph is $\mathcal{H}$-subgraph-free if it does not contain any of $H_1, \ldots, H_p$ as a subgraph. We give a meta-classification for $\mathcal{H}$-subgraph-free graphs: assuming a problem meets three conditions, it is "efficiently solvable" if $\mathcal{H}$ contains a disjoint union of one or more paths and subdivided claws, and is "computationally hard" otherwise. The conditions are that the problem should be efficiently solvable on graphs of bounded treewidth, computationally hard on subcubic graphs, and computational hardness is preserved under edge subdivision. We illustrate the broad applicability of our meta-classification by obtaining a dichotomy between polynomial-time solvability and NP-completeness for many well-known partitioning, covering and packing problems, network design problems and width parameter problems. For other problems, we obtain a dichotomy between almost-linear-time solvability and having no subquadratic-time algorithm (conditioned on some hardness hypotheses). Along the way, we uncover and resolve several open questions from the literature, while adding many new ones.

4.2 Complexity of problems easy on subcubic graphs (Sukanya Pandey)

We proposed a framework for the complexity classification of problems that satisfy the following three conditions:

1. they are efficiently computable on graph classes of bounded treewidth
2. they are computationally hard on the class of subcubic graphs
3. they remain computationally hard on subdivisions of subcubic graphs

We call the problems that satisfy all three conditions C123-problems. This talk discusses the complexity of problems that satisfy the first condition but violate the second, on graph classes excluding certain subdivided stars as subgraphs. A few notable examples of these problems are feedback vertex set, independent feedback vertex set, and coloring. In particular, we show that certain classes of connected, H-subgraph-free graphs that are not subcubic must have bounded treewidth. We show this for the cases when $H = S_{1,1,1,r}$ and $H = S_{1,1,q,r}$. The immediate consequence of these theorems is that the problems under consideration are polynomial-time solvable on the aforementioned graph classes. We also discuss the NP-completeness of Feedback Vertex Set and Independent Feedback Vertex Set on $S_{2,2,2,2}$-subgraph-free graphs, leaving $S_{1,p,q,r}$-subgraph-free graphs as the only open case remaining for these problems.

4.3 Steiner Forest on H-subgraph-free graphs (Erik Jan van Leeuwen)

In this talk, we consider the complexity of Steiner Forest on graphs that exclude a fixed graph H as a subgraph. Recently, a framework was proposed to classify the complexity of problems on H-subgraph-free graphs. Unfortunately, Steiner Forest does not fit this framework. We discuss why this is the case as well as our progress on obtaining a separate classification for Steiner Forest.

4.4 Detecting and Counting Small Patterns in Unit Disc Graphs (Krisztina Szilagyi)

We investigate the parameterized complexity of the task of counting and detecting occurrences of small patterns in unit disk graphs: Given an n-vertex unit disk graph with an embedding of ply p (that is, the graph is represented as intersection graph with disks of unit size, and each point is contained in at most p disks) and a k-vertex unit disk graph P, count the number of (induced) copies of P in G. For general patterns P, we give an $2^{O(pk/\log k)}n^{O(1)}$ time algorithm for counting pattern occurrences. We show this is tight, even for ply $p = 1$ and $k = n$: any $2^{o(n/\log n)}n^{O(1)}$ time algorithm violates the Exponential Time Hypothesis (ETH).

Joint work with Jesper Nederlof.

4.5 Convergence of the number of period sets in strings (Michelle Sweering)

Consider words of length n. The set of all periods of a word of length n is a subset of $\{0, 1, 2, \ldots, n-1\}$. However, any subset of $\{0, 1, 2, \ldots, n-1\}$ is not necessarily a valid set of periods. In a seminal paper in 1981, Guibas and Odlyzko have proposed to encode the set of periods of a word into an n long binary string, called an autocorrelation, where a one at position i denotes a period of i. They considered the question of recognizing a valid period set, and also studied the number of valid period sets for length n, denoted κ_n. They conjectured that $\ln(\kappa_n)$ asymptotically converges to a constant times $\ln^2(n)$. If improved lower bounds for $\ln(\kappa_n)/\ln^2(n)$ were proposed in 2001, the question of a tight upper bound has remained opened since Guibas and Odlyzko's paper. Here, we exhibit an upper bound for this fraction, which implies its convergence and closes this long standing conjecture. Moreover, we extend our result to find similar bounds for the number of correlations: a generalization of autocorrelations which encodes the overlaps between two strings.

4.6 On the enumeration of Krom formulas (Lorenzo Sauras-Altuzarra)

In the fourth volume his celebrated work "The Art of Computer Programming", Donald Knuth treats Boolean functions (i.e. operations on the binary field) and the enumeration of some of its subclasses. We will discuss some observations on the enumeration of one of these kinds, the so-called Krom functions (also known as 2-CNFs).

4.7 Recognising trees with missing cards (Gabriëlle Zwaneveld)

The unlabeled subgraphs $G - v$ are called cards of G and the deck of G is the multiset $\{G - v : v \in v(G)\}$. Wendy Myrvold showed that a non-connected graph and a connected graph both on n vertices have at most $\lfloor \frac{n}{2} \rfloor + 1$ in common and she found (infinite) families of trees and non-connected forests for which this bound is strict. I proved that for sufficiently large n a tree T and a unicyclic graph G on n vertices have at most $\lfloor \frac{n}{2} \rfloor + 1$ common cards, implying that based on any $\lfloor \frac{n}{2} \rfloor + 2$ of cards of a graph one can determine whether that graph is a tree.

4.8 Endless evolution and the structure of biological fitness landscapes (Artem Kaznatcheev)

Experiments show that biological fitness landscapes can have a rich combinatorial structure due to epistasis. The structure of this epistatic interactions between genes can be succinctly represented as instances of valued constraint satisfaction problems (VCSPs). For each VCSP instance, we can look at the corresponding fitness graph as a DAG with edges between mutationally-adjacent genotypes directed from the lower to the higher fitness genotype. From this perspective, seeing open-ended evolution (i.e., evolutionary dynamics that require more than a polynomial number of fitness-increasing steps to reach a local fitness peak) becomes a question of long directed paths to the sinks (i.e., peaks) in the fitness graph. I will give a brief overview of this connection between theoretical evolutionary biology and combinatorics, and discuss some results on how treewidth of the VCSP instances relate to the diameter and longest-path in the corresponding fitness graph.