# STAR Workshop on Random Graphs 2018

Radboud University Nijmegen

12 & 13 April 2018 http://www.math.ru.nl/~rkang/SWRG2018/

(Last updated April 9, 2018.)

We welcome you to Nijmegen for a workshop on random graphs and related topics.

This contains important practical and scientific information about the meeting. To conserve paper, we provide at the meeting printed copies of only the programme. This document has links, via hyperref, to other parts of the document and the web.

§ Practical matters § Programme § Invited lectures § Contributed talks § Participants § Sponsorship §

# **Practical matters**

Almost the entire workshop takes place in the Huygensgebouw (Huygens building) on the campus of Radboud University Nijmegen. All talks are in the auditorium HG00.304, located on the ground floor not far from the pendulum. Here is a map of key locations.

The campus is about 3 kilometres south of the city centre. To get there by bus (stop "Station Heyendaal" or "Huygensgebouw"), use 13, 15, 300 from the east side of the centre, or take the shuttle bus service 10 if nearer the train station. It can be a 10 minute journey if you have a bicycle.

There is eduroam. Remember to configure it at your home institution beforehand.

During the meeting, we provide caffeine, lunch and other refreshments at the right moments. There is a buffet reception on Thursday, and drinks with nibbles on Friday.

Nijmegen can be reached by a 90-minute train journey from Schiphol airport (possibly with a quick change at Utrecht Centraal). Other airports nearby are Eindhoven and Weeze. Consult 9292 to check the overall route. Note that free internet access is usually provided at Schiphol airport, major train stations, and on intercity trains.

# Programme

Embedded below are links to Google maps, as well as to abstracts.

Thursday, 12 April

10:00 - 10:20	Welcome coffee/tea
10:20 - 10:30	Opening by Lutgarde Buydens (Dean of FNWI)
10:30 - 11:30	Leslie Goldberg — The Moran process: An algorithm modelling the
	spread of genetic mutations in populations
11:30 - 11:40	Break
11:40 - 12:30	Two short talks (Betken, Hoorn)
12:30 - 13:30	Lunch
13:30 - 14:30	Eric Cator — The contact process on random graphs: a new approach
14:30 - 14:55	One short talk (Ritter)
14:55 - 15:30	Coffee/tea break (conference photo first)
15:30 - 16:30	Colin McDiarmid — Sampling from a high modularity graph
16:45 - 19:00	Borel (Cultuurcafé) or bicycle ride (starting at Comeniuslaan 4)
19:00 -	Dinner reception (Huize Heyendael)

Friday, 13 April

10:00 - 10:30	Welcome coffee/tea
10:30 - 11:30	Patrice Ossona de Mendez — Structural convergence and <i>H</i> -colorings
	of random graphs
11:30 - 11:40	Break
11:40 - 12:30	Two short talks (Šileikis, Bonnet)
12:30 - 13:30	Lunch
13:30 - 14:30	Frank den Hollander — Exploration on dynamic networks
14:30 - 14:45	One short talk (Kumar)
14:55 - 15:15	Coffee/tea break
15:15 - 15:40	One short talk (Cai)
15:40 - 16:40	Bruce Reed — The Structure of A Random Graph from a Hereditary
	Family
17:15 -	Closing borrel (third floor, between wings 7 and 8)

# Invited lectures

Eric Cator (Radboud University Nijmegen)

"The contact process on random graphs: a new approach".

The contact process has been a major object of study in probability for the last 30 years. Many results were obtained for specific graphs, such as the integer lattice or a tree. In the physics literature, the contact process on random graphs, as a model for infection or information spread, has been very popular, and many mean field approximations were developed. Recently, rigorous results for the configuration model with heavy tailed node distributions were obtained by Chatterjee and Durret, and later by Daniel Valesin, Thomas Mountford and others, showing that the process survives for long times on a subnetwork created by nodes of very high degree. In this talk, we will present a new method of controlling the behaviour of the contact process, based on uniform lower bounds of the number of outgoing links from the set of infected nodes. We are able to prove the metastable behaviour of the contact process on Erdős-Rényi graphs and on configuration model graphs with degree distributions that do not have a heavy tail.

This is joint work with Henk Don.

### Leslie Ann Goldberg (University of Oxford)

"The Moran process: An algorithm modelling the spread of genetic mutations in populations".

The Moran process, as studied by Lieberman, Hauert and Nowak, is a randomised algorithm modelling the spread of genetic mutations in populations. The algorithm runs on an underlying graph where individuals correspond to vertices. Initially, one vertex (chosen uniformly at random) possesses a mutation, with fitness r > 1. All other individuals have fitness 1. During each step of the algorithm, an individual s chosen with probability proportional to its fitness, and its state (mutant or non-mutant) is passed on to an out-neighbour which is chosen uniformly at random. If the underlying graph is strongly connected then the algorithm will eventually reach fixation, in which all individuals are mutants, or extinction, in which no individuals are mutants. This talk will be a survey about what we know about this algorithm. It will include joint work with Galanis, Goebel, Lapinskas and Richerby and also joint work with Lapinskas, Lengler, Meier, Panagiotou and Pfister.

### Frank den Hollander (Leiden University)

"Exploration on dynamic networks".

Search algorithms on networks are important tools for the organisation of large data sets. A key example is Google PageRank, which assigns a numerical weight to each element of a hyperlinked set of documents, such as the World Wide Web, with the purpose of measuring its relative importance within the set. The weighting is achieved by exploration. A hyperlink counts as a vote of support. The PageRank of a page is defined recursively, and depends on the number and the weight of all the pages that link to it. A page that is linked to by many pages with a high rank receives a high rank itself.

Real-world networks are modelled as graphs, consisting of a set of vertices and a set of edges connecting pairs of vertices. Complex networks are modelled as random graphs, where the vertices and the edges are chosen randomly, according to an appropriate probability distribution. Search algorithms, in turn, are modelled as random walks, which move along the network by randomly picking an edge incident to the vertex that is currently visited and jumping to the vertex at the other end. The mixing time of a random walk on a random graph is the time it needs to approach its stationary distribution (also called equilibrium distribution). The characterisation of the mixing time has been the subject of intensive study. One of the key motivations is that it gives information about the geometry of the graph.

Many real-world networks are dynamic in nature. It is therefore natural to study random walks on dynamic random graphs. In this talk we focus on one particular example, namely, a random graph with prescribed degrees (also called the configuration model). We investigate what happens to the mixing time of the random walk when at each unit of time a certain fraction of the edges is randomly rewired. We identify three regimes in the limit as the graph becomes large: fast, moderate, slow dynamics. These regimes exhibit surprising behaviour.

Joint work with Luca Avena (Leiden), Hakan Guldas (Leiden) and Remco van der Hofstad (Eindhoven).

#### Colin McDiarmid (University of Oxford)

"Sampling from a high modularity graph".

For a given graph G, modularity gives a score to each vertex partition, with higher values indicating that the partition better captures community structure in G. The modularity  $q^*(G)$  (where  $0 \le q^*(G) < 1$ ) of the graph G is defined to be the maximum over all vertex partitions of the modularity value. Given the prominence of modularity in community detection, we wish to understand it mathematically.

Suppose that there is an unknown underlying graph G on a large vertex set, and we can test only a proportion of the possible edges to check whether they are present in G. If G has high modularity, is the observed graph G' likely to have high modularity? In other words, if G' has low modularity can we assert with some confidence that G has low modularity?

We shall see that this is indeed the case, in a natural model where we test edges at random. Before doing so, we shall sketch some background results on modularity, and in particular on the modularity of random graphs. This talk is based on joint work with Fiona Skerman.

#### Patrice Ossona de Mendez (CNRS)

"Structural convergence and H-colorings of random graphs".

In a first part we study the asymptotic properties of random graphs, and particularly the convergence properties of sequences of random graphs. We prove several convergence theorems and conjecture that a sequence of random graph with edge probability p(n) is FO-convergent with high probability if and only if G(n, p(n))satisfies a zero-one law.

In a second part, for random graphs with edge probability d/n, we prove that for countably many non-trivial *H*-coloring problems there is a linear time algorithm, which — for a random graph in G(n, d/n) — either exhibits a small obstruction to *H*-coloring, or computes a *H*-coloring, or answers that it cannot solve the problem, the two first cases arising with high probability

Joint work with J. Nešetřil.

#### Bruce Reed (CNRS/IMPA)

"The Structure of A Random Graph from a Hereditary Family".

### Contributed talks

Carina Betken (University of Osnabrück),

"Fluctuations in the preferential attachment graph via Stein's method".

We look at the indegree of a uniformly chosen vertex in a preferential attachment random graph, where the probability that a newly arriving vertex n connects to an older vertex i is proportional to a sublinear function f of the indegree of vertex i at that time. Using Stein's method we provide rates of convergence for the total variation distance between this degree distribution and an asymptotic power-law distribution, which is derived in [?], as the number of vertices tends to  $\infty$ .

Gilles Bonnet (Ruhr University Bochum),

"Maximal degree in a Poisson-Delaunay graph".

Consider a random Delaunay graph constructed on a Poisson point process in  $\mathbb{R}^d$ ,  $d \geq 2$ . We investigate its maximal degree over all nodes in a cubic window of volume n, as n goes to infinity. The exact order of this maximum is provided in any dimension. In the particular setting d = 2, we show that this quantity is concentrated on two consecutive integers with high probability. An extension of this result is discussed when  $d \geq 3$ . One of the main difficulties in such setting is to deal with the strong local geometric constraints of the model. We overcome it by showing the absence of cluster of points with high degree.

Xing Shi Cai (Uppsala University),

"Inversions in split trees and conditional Galton–Watson trees".

We study I(T), the number of inversions in a tree T with its vertices labeled uniformly at random, which is a generalization of inversions in permutations. We first show that the cumulants of I(T) have explicit formulas involving the k-total common ancestors of T (an extension of the total path length). Then we consider  $X_n$ , the normalized version of  $I(T_n)$ , for a sequence of trees  $T_n$ . For fixed  $T_n$ 's, we prove a sufficient condition for  $X_n$ to converge in distribution. As an application, we identify the limit of  $X_n$  for complete b-ary trees. For  $T_n$  being split trees, we show that  $X_n$  converges to the unique solution of a distributional equation. Finally, when  $T_n$ 's are conditional Galton–Watson trees, we show that  $X_n$  converges to a random variable defined in terms of Brownian excursions. Pim van den Hoorn (Northeastern University),

"Limit theorems for assortativity and clustering in the configuration model with scale-free degrees".

The correlation between the degrees of connected nodes and the fraction of triangles are important characteristics for the structure of complex networks. In this work we study the behavior of Pearson's correlation coefficient, as a measure for degree-degree correlations, in the erased configuration model and the clustering coefficient in both the standard and erased configuration model. We consider the case where the degrees follow a general scalefree degree distribution with exponent  $1 < \gamma < 2$ , i.e. the distribution has finite mean and infinite variance.

It is known that Pearson's correlation coefficient in the erased configuration model tends to zero as the graph size tend to infinite. Similarly, the clustering coefficient in the configuration model tends to zero whenever  $\gamma > 4/3$ , whereas it tends to infinity for  $\gamma < 4/3$ . However, up to some preliminary bounds, the exact scaling of these quantities with the network size where unknown. Moreover, a corresponding result for the clustering coefficient in the erased configuration model was never proved. In this work, we classify the rate of convergence, in terms of the graph size and exponent  $\gamma$ , for the measures mention above. To achieve this, we find the right scaling, such that the rescaled measures converge in distribution to some random variable. In addition we also classify the distribution of this random variable, showing that it involves a collection of coupled random variables with stable distributions. An essential tool for our proofs is a new result for the scaling of the number of removed edges in the erased configuration model with scale-free degree distributions, which is of independent interest.

Besides classifying the asymptotic behavior of both measures, our results also provide insights into the effects imposed by the constraint that the graph is simple, on both degree-degree correlations and clustering. We show that the limit distribution of Pearson's correlation coefficient is only supported on the negative real numbers, establishing a first rigorous proof of structural negative correlations. We also see that the clustering coefficient always converges to zero in the erased configuration model and its rate of convergence is always faster than for the standard model. Moreover, the scaling in the erased model is non-monotonic in terms of  $\gamma$  and achieves its maximum at  $\gamma = \sqrt{4/3}$ , reflecting the trade off between the scaling of the number of triangles in the graph and the total number of possible triangles.

Joint work with: Remco van der Hofstad, Nelly Litvak and Clara Stegehuis.

#### Pankaj Kumar (Copenhagen Business School),

"Deep Graph Kernels for Blockchain: Inferring Bitcoin Transaction Dynamics".

With readily available blockchain, log of all transactions that were ever verified on the Bitcoin network, I attempt to investigate how structural changes in the network accompany significant changes in the exchange price of bitcoins. By extending the quite novel framework for graph kernels inspired by latest advancements in natural language processing and deep learning, I propose, deep graph propagation kernels. The unseen deep framework in the literature takes account of attributed graphs with continuous values. The deep graph kernels outperforms its best base graph kernels variants in terms of capturing correlation between network structure and market price. This defines a baseline to predict the price of BTC/USD exchange, which leverage on deep learning learning framework to extract feature space of blockchain. Daniel Ritter (LMU Munich), "Financial Contagion on a Generalized Stochastic Block Model".

As spectacularly demonstrated during the latest financial crisis starting in 2007, network effects play a central role in the stability of the financial system. Since then researchers have developed several approaches for dealing with this so-called "systemic risk". One of them focuses on direct exposures (such as loans) between financial institutions and models the resulting network structure as a random graph. Firstly, this allows to capture uncertainty about the real network both in the future and in the present. Secondly, for large networks this approach enables asymptotic analytic results regarding stability of the system. Previous research has studied among others directed versions of the configuration model [1] and the Chung-Lu random graph [2] as models for the financial network. One persistent problematic assumption is that the distributions of the exposures may only depend on characteristics of the exposed institution and not of the exposing one. We overcome this issue by introducing a block structure so that exposures depend on the types (blocks) of both involved institutions. Further, this generalized Stochastic Block Model allows for the description of various interesting network patterns such as Core-Periphery, which can prove useful in a variety of contexts.

[1] H. Amini, R. Cont and A. Minca. Resilience to Contagion in Financial Networks. Mathematical Finance 26(2):329-365, 2016. [2] N. Detering, T. Meyer-Brandis, K. Panagiotou and D. Ritter. Managing Default Contagion in Inhomogeneous Financial Networks. arXiv:1610.09542, 2016.

Matas Šileikis (Czech Academy of Sciences),

"Counterexamples to the Demarco-Kahn conjecture on the upper tail of subgraph counts".

Given a fixed graph H, what is the (exponentially small) probability that the number  $X_H$  of copies of H in the binomial random graph G(n, p) is at least twice its mean? In 2011 DeMarco and Kahn determined the order of  $-\log P(X_H > 2EX_H)$  for general p = p(n) when H is a clique and conjectured what it should be for general H. In this note we show that their conjecture is false for an infinite family of graphs H. This is joint work with Lutz Warnke.

# **Participants**

Invited speakers:

Eric Cator (Nijmegen), Leslie Ann Goldberg (Oxford), Frank den Hollander (Leiden), Colin McDiarmid (Oxford), Patrice Ossona de Mendez (CNRS), Bruce Reed (CNRS/IMPA).

Registered participants:

Bas Terwijn (Nijmegen), Xing Shi Cai (Uppsala), Stijn Cambie (Nijmegen), Lena Yuditsky (Karlsruhe), Pim van der Hoorn (Northeastern), Wouter Cames van Batenburg (Nijmegen/ULB), Carina Betken (Osnabrück), Stephan Bussmann (Osnabrück), Matas Šileikis (Czech Academy of Sciences), Max Hahn-Klimroth (Frankfurt), Rémi de Joannis de Verclos (Nijmegen), Pankaj Kumar (Copenhagen Business School), Matthias Reitzner (Osnabrück), László Kozma (TU Eindhoven), Hassane Bouzahir (Ibn Zohr), David Sabonis (TU Munich/Copenhagen), Carla Groenland (Oxford), Daniel Ritter (Munich), Gilles Bonnet (Ruhr University Bochum), William Weimin Yoo (Leiden), Markus Schepers (Groningen), Henk Don (Nijmegen), Ben Hansen (Groningen), Nicolas Grelier (IMT Atlantique), Davis Issac (MPI Saarbrücken), Aida Abiad (Maastricht).

Organisers:

Ross Kang (Nijmegen) and Tobias Müller (Groningen).

# Sponsorship

The meeting enjoys support from the Netherlands mathematics cluster "Stochastics – Theoretical and Applied Research" (STAR), Nijmegen's Institute for Mathematics, Astrophysics and Particle Physics (IMAPP) and Radboud University Nijmegen.