Guide Lattice Gauge Theory Course 2005

See the errata to the book.

Skipped in the lectures: sections 6.5 and 7.6; only briefly mentioned: sections 8.4-8.7. These sections will not be part of the oral exam. Reading through is still recommended.

List of problems:
(2.i), (3.i)-(3.iii), (3.v), (3.viii), (4.i), (4.ii), (6.i)-(6.vi), (7.i), (8.i).

1. Going through section 4.3, I mentioned that the vertex functions (represented by 1PI diagrams) are the derivatives of the so-called effective action Γ. More on this can be found in many books on quantum field theory, e.g. the book by Peskin and Schroeder[1] (P&S), sections 11.3 and 11.5.

2. A continuum account of the discussion in section 3.10 can be found in P&S section 12.1.

3. The method of renormalized perturbation theory was reviewed in class on 04-04-2005. See section 10.2 in the book by Peskin and Schroeder; the beginning of this section is essentially what I mentioned in class; it then continues with the φ^4 model for illustration, using dimensional regularization. You can compare the result (10.23) in P&S with (3.77) in my book.

4. Problem (3.iv) was treated in class.

   Minimal-Subtraction (MS) is a useful way to introduce a renormalized coupling constant $\lambda(s)$, or as it is usually written, $\lambda(\mu)$, or simpy λ. Other definitions like $\lambda_{\rm R}$ in (3.89) can be expressed in terms of λ by comparing the perturbative series for vertex functions, respectively in the scheme using $\lambda_{\rm R}$ and λ.

   The beta function in minimal subtraction can be read off from the coefficients of the terms linear in $\ln a\mu$ in (3.190), which correspond to the coefficients linear in $1/(d-4)$ in dimensional regularization (see (7.84) in P&S and the equation below it).

   The solution of the partial differential equation (3.192) with the boundary condition $Z_\lambda(\lambda, 0) \equiv 1$ is implicity given by $\lambda_0 = \lambda Z_\lambda(\lambda, s-t)$ and

$$s - t = \int_{\lambda_0}^{\lambda} \frac{dx}{\beta(x)} = -\frac{1}{\beta_1 \lambda} + \frac{1}{\beta_1 \lambda_0} + \frac{\beta_2}{\beta_1^2} \ln \frac{\lambda}{\lambda_0} + O(\lambda, \lambda_0), \qquad (1)$$

   where $s - t = \ln a\mu$.

[1] M.E. Peskin and D.V. Schroeder, An Introduction to Quantum Field Theory, Perseus Books, Reading, Massachusetts, 1995.

5. To show conservation of the current in (4.177), consider

$$\begin{aligned}
\partial_\mu J^\mu(x) &= \int_{\tau_2}^{\tau_1} d\tau \, \frac{dz^\mu(\tau)}{d\tau} \, \partial_\mu \delta^4(x - z(\tau)) \\
&= -\int_{\tau_2}^{\tau_1} d\tau \, \frac{dz^\mu(\tau)}{d\tau} \, \frac{\partial}{\partial z^\mu} \, \delta^4(x - z(\tau)) \\
&= -\int_{\tau_2}^{\tau_1} d\tau \, \frac{d}{d\tau} \, \delta^4(x - z(\tau)) \\
&= -\delta^4(x - z_1) + \delta^4(x - z_2).
\end{aligned}$$

For a closed path $z_1 = z_2$. For a never-ending world line of a point particle (time-like $dz^\mu/d\tau$), z_1 goes to the infinite future and z_2 to the infinite past. In this case we use an arbitrary test function $f(x)$ (which rapidly vanishes for $|\mathbf{x}| + |x^0| \to \infty$) and find

$$\int d^4x \, f(x) \partial_\mu J^\mu(x) = f(z_2) - f(z_1) \to 0.$$