# Phase-Fluctuating 3D Bose-Einstein Condensates in Elongated Traps

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We find that in very elongated 3D trapped Bose gases, even at temperatures far below the BEC transition temperature $T_c$, the equilibrium state will be a 3D condensate with fluctuating phase (quasicondensate). At sufficiently low temperatures the phase fluctuations are suppressed and the quasicondensate turns into a true condensate. The presence of the phase fluctuations allows for extending thermometry of Bose-condensed gases well below those established in current experiments.

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Phase coherence properties are among the most interesting aspects of Bose-condensed gases. Since the discovery of Bose-Einstein condensation (BEC) in trapped ultracold clouds of alkali atoms [1], various experiments have proved the presence of phase coherence in trapped condensates. The MIT group [2] has found the interference of two independently prepared condensates, once they expand and overlap after switching off the traps. The MIT [3], NIST [4], and Munich [5] experiments provide evidence for the phase coherence of trapped condensates through the measurement of the phase coherence length and/or single particle correlations.

These results support the usual picture of BEC in 3D gases. In equilibrium, the fluctuations of density and phase are important only in a narrow temperature range near the BEC transition temperature $T_c$. Outside this region, the fluctuations are suppressed and the condensate is phase coherent. This picture precludes the interesting physics of phase-fluctuating condensates, which is present in 2D and 1D systems (see [6,7] and references therein).

In this Letter we show that the phase coherence properties of 3D Thomas-Fermi (TF) condensates depend on their shape. In very elongated 3D condensates, the axial phase fluctuations are found to manifest themselves even at temperatures far below $T_c$. Then, as the density fluctuations are suppressed, the equilibrium state will be a *condensate with fluctuating phase* (quasicondensate) similar to that in 1D trapped gases [7]. Decreasing $T$ below a sufficiently low temperature, the 3D quasicondensate gradually turns into a true condensate.

The presence and the temperature dependence of axial phase fluctuations in sufficiently elongated 3D condensates suggests a principle of thermometry for Bose-condensed gases with indiscernible thermal clouds. The idea is to extract the temperature from a measurement of the axial phase fluctuations, for example, by measuring the single-particle correlation function. This principle works for quasicondensates or for any condensate that can be elongated adiabatically until the phase fluctuations become observable.

So far, axial phase fluctuations have not been measured in experiments with cigar-shaped condensates. We discuss the current experimental situation and suggest how one should select the parameters of the cloud in order to observe the phase-fluctuating 3D condensates.

We first consider a 3D Bose gas in an elongated cylindrical harmonic trap and analyze the behavior of the single-particle correlation function. The natural assumption of the existence of a true condensate at $T = 0$ automatically comes out of these calculations. In the TF regime, where the mean-field (repulsive) interparticle interaction greatly exceeds the radial ($\omega_\rho$) and axial ($\omega_z$) trap frequencies, the density profile of the zero-temperature condensate has the well-known shape $n_0(\rho, z) = n_{0m}(1 - \rho^2/R^2 - z^2/L^2)$, where $n_{0m} = \mu/g$ is the maximum condensate density, with $\mu$ being the chemical potential, $g = 4\pi\hbar^2 a/m$, $m$ is the atom mass, and $a > 0$ is the scattering length. Under the condition $\omega_\rho \gg \omega_z$, the radial size of the condensate, $R = (2\mu/m\omega_\rho^2)^{1/2}$, is much smaller than the axial size $L = (2\mu/m\omega_z^2)^{1/2}$.

Fluctuations of the density and phase of the condensate, in particular, at finite $T$, are related to elementary excitations of the cloud. The density fluctuations are dominated by the excitations with energies of the order of $\mu$. The wavelength of these excitations is much smaller than the radial size of the condensate. Hence, the density fluctuations have the ordinary 3D character and are small. Therefore, one can write the total field operator of atoms as $\hat{\psi}(\mathbf{r}) = \sqrt{n_0(\mathbf{r})}\exp[i\hat{\phi}(\mathbf{r})]$, where $\hat{\phi}(\mathbf{r})$ is the operator of the phase. The single-particle correlation function is then expressed through the mean square fluctuations of the phase (see, e.g., [8]):

$$\langle\hat{\psi}^\dagger(\mathbf{r})\hat{\psi}(\mathbf{r}')\rangle = \sqrt{n_0(\mathbf{r})n_0(\mathbf{r}')}\exp\{-\langle[\delta\hat{\phi}(\mathbf{r},\mathbf{r}')]^2\rangle/2\}, \tag{1}$$

with $\delta\hat{\phi}(\mathbf{r},\mathbf{r}') = \hat{\phi}(\mathbf{r}) - \hat{\phi}(\mathbf{r}')$. The operator $\hat{\phi}(\mathbf{r})$ is given by (see, e.g., [9])

$$\hat{\phi}(\mathbf{r}) = [4n_0(\mathbf{r})]^{-1/2}\sum_\nu f_\nu^+(\mathbf{r})\hat{a}_\nu + \text{H.c.}, \tag{2}$$

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where $\hat{a}_\nu$ is the annihilation operator of the excitation with quantum number(s) $\nu$ and energy $\epsilon_\nu$, $f_\nu^+ = u_\nu + v_\nu$, and the $u, v$ functions of the excitations are determined by the Bogolyubov–de Gennes equations.

The excitations of elongated condensates can be divided into two groups: "low energy" axial excitations with energies $\epsilon_\nu < \hbar\omega_\rho$, and "high energy" excitations with $\epsilon_\nu > \hbar\omega_\rho$. The latter have 3D character as their wavelengths are smaller than the radial size $R$. Therefore, as in ordinary 3D condensates, these excitations can provide only small phase fluctuations. The low-energy axial excitations have wavelengths larger than $R$ and exhibit a pronounced 1D behavior. Hence, one expects that these excitations give the most important contribution to the long-wave axial fluctuations of the phase.

The solution of the Bogolyubov–de Gennes equations for the low-energy axial modes gives the spectrum $\epsilon_j = \hbar\omega_z\sqrt{j(j+3)/4}$ [10], where $j$ is a positive integer. The wave functions $f_j^+$ of these modes have the form

$$f_j^+(\mathbf{r}) = \sqrt{\frac{(j+2)(2j+3)gn_0(\mathbf{r})}{4\pi(j+1)R^2L\epsilon_j}}\, P_j^{(1,1)}\left(\frac{z}{L}\right), \quad (3)$$

where $P_j^{(1,1)}$ are Jacobi polynomials. Note that the contribution of the low-energy axial excitations to the phase operator (2) is independent of the radial coordinate $\rho$.

Relying on Eqs. (2) and (3), we now calculate the mean square axial fluctuations of the phase at distances $|z - z'| \ll R$. As in 1D trapped gases [7], the vacuum fluctuations are small for any realistic axial size $L$. The thermal fluctuations are determined by the equation

$$\langle[\delta\hat{\phi}(z,z')]^2\rangle_T = \sum_{j=1}^{\infty} \frac{\mu(j+2)(2j+3)}{15(j+1)\epsilon_j N_0} \times \left[P_j^{(1,1)}\left(\frac{z}{L}\right) - P_j^{(1,1)}\left(\frac{z'}{L}\right)\right]^2 N_j, \quad (4)$$

with $N_0 = (8\pi/15)n_{0m}R^2L$ being the number of Bose-condensed particles, and $N_j$ the equilibrium occupation numbers for the excitations. The main contribution to the sum over $j$ in Eq. (4) comes from several lowest excitation modes, and at temperatures $T \gg \hbar\omega_z$ we may put $N_j = T/\epsilon_j$. Then, in the central part of the cloud ($|z|, |z'| \ll L$) a straightforward calculation yields

$$\langle[\delta\hat{\phi}(z,z')]^2\rangle_T = \delta_L^2|z - z'|/L, \quad (5)$$

where the quantity $\delta_L^2$ represents the phase fluctuations on a distance scale $|z - z'| \sim L$ and is given by

$$\delta_L^2(T) = 32\mu T/15N_0(\hbar\omega_z)^2. \quad (6)$$

Note that at any $z$ and $z'$ the ratio of the phase correlator (4) to $\delta_L^2$ is a universal function of $z/L$ and $z'/L$:

$$\langle[\delta\hat{\phi}(z,z')]^2\rangle_T = \delta_L^2(T)f(z/L, z'/L). \quad (7)$$

In Fig. 1 we present the function $f(z/L) \equiv f(z/L, -z/L)$ calculated numerically from Eq. (4).

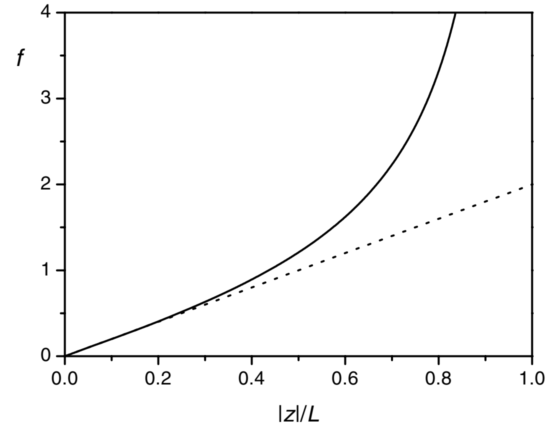


FIG. 1. The function $f(z/L)$. The solid curve shows the result of our numerical calculation, and the dotted line is $f(z) = 2|z|/L$ following from Eq. (5).

The phase fluctuations decrease with temperature. As the TF chemical potential is $\mu = (15N_0 g/\pi)^{2/5} \times (m\bar{\omega}^2/8)^{3/5}$ ($\bar{\omega} = \omega_\rho^{2/3}\omega_z^{1/3}$), Eq. (6) can be rewritten in the form

$$\delta_L^2 = (T/T_c)(N/N_0)^{3/5}\delta_c^2, \quad (8)$$

where $T_c \approx N^{1/3}\hbar\bar{\omega}$ is the BEC transition temperature and $N$ is the total number of particles. The presence of the 3D BEC transition in elongated traps requires the inequality $T_c \gg \hbar\omega_\rho$ and, hence, limits the aspect ratio to $\omega_\rho/\omega_z \ll N$. The parameter $\delta_c^2$ is given by

$$\delta_c^2 = \frac{32\mu(N_0 = N)}{15N^{2/3}\hbar\bar{\omega}}\left(\frac{\omega_\rho}{\omega_z}\right)^{4/3} \propto \frac{a^{2/5}m^{1/5}\omega_\rho^{22/15}}{N^{4/15}\omega_z^{19/15}}. \quad (9)$$

Except for a narrow interval of temperatures just below $T_c$, the fraction of noncondensed atoms is small and Eq. (8) reduces to $\delta_L^2 = (T/T_c)\delta_c^2$. Thus, the phase fluctuations can be important at large values of the parameter $\delta_c^2$, whereas for $\delta_c^2 \ll 1$ they are small on any distance scale and one has a true Bose-Einstein condensate. In Fig. 2 we present the quantity $\delta_c^2$ for the parameters of various experiments with elongated condensates.

In the Konstanz [13] and Hannover [14] experiments the ratio $T/T_c$ was smaller than 0.5. In the recent experiment [17] the value $\delta_c^2 \approx 3$ has been reached, but the temperature was very low. Hence, the axial phase fluctuations were rather small in these experiments and they were dealing with true condensates. The last statement also holds for the ENS experiment [15] where $T$ was close to $T_c$ and the Bose-condensed fraction was $N_0/N \approx 0.75$.

The single-particle correlation function is determined by Eq. (1) only if the condensate density $n_0$ is much larger than the density of noncondensed atoms, $n'$. Otherwise, this equation should be completed by terms describing correlations in the thermal cloud. However, irrespective of the relation between $n_0$ and $n'$, Eq. (1) and Eqs. (4)–(6) correctly describe phase correlations in the condensate as long as the fluctuations of the condensate density are

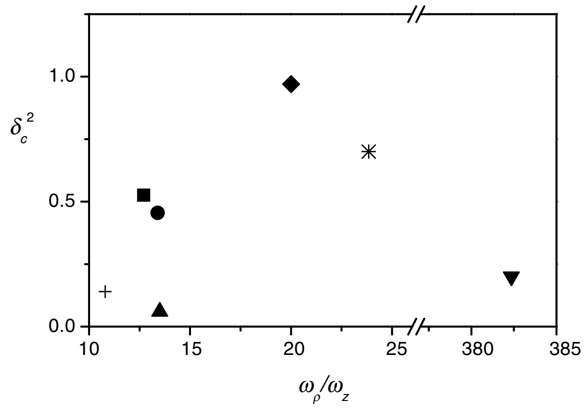


FIG. 2. The parameter $\delta_c^2$ for experiments with elongated condensates. The up and down triangles stand for $\delta_c^2$ in the sodium [11] and hydrogen [12] MIT experiments, respectively. Square, cross, diamond, circle, and star show $\delta_c^2$ for the rubidium experiments at, respectively, Konstanz [13], Munich [5], Hannover [14], ENS [15], and AMOLF [16].

suppressed. This is still the case for $T$ close to $T_c$ and $N_0 \ll N$, if we do not enter the region of critical fluctuations. Then, Eq. (8) gives $\delta_L^2 = (N/N_0)^{3/5}$. At the highest temperatures of the Bose-condensed cloud in the MIT sodium experiment [11], the condensed fraction was $N_0/N \sim 0.1$ and the phase fluctuations were still small.

On the contrary, for $N_0/N \approx 0.06$ in the hydrogen experiment [12], with $\delta_c^2$ from Fig. 2 we estimate $\delta_L^2 \approx 1$. The same or an even larger value of $\delta_L^2$ was reached in the Munich Rb experiment [5] where the gas temperature was varying in a wide interval around $T_c$. In the Rb experiment at AMOLF [16], the smallest observed Bose-condensed fraction was $N_0/N \approx 0.03$, which corresponds to $\delta_L^2 \approx 5$. However, axial fluctuations of the phase have not been measured in these experiments.

We focus our attention on the case where $N_0 \approx N$ and the presence of the axial phase fluctuations is governed by the parameter $\delta_c^2$. For $\delta_c^2 \gg 1$, the nature of the Bose-condensed state depends on temperature. In this case we can introduce a characteristic temperature

$$T_\phi = 15(\hbar\omega_z)^2 N/32\mu \tag{10}$$

at which the quantity $\delta_L^2 \approx 1$ (for $N_0 \approx N$). In the temperature interval $T_\phi < T < T_c$, the phase fluctuates on a distance scale smaller than $L$. Thus, as the density fluctuations are suppressed, the Bose-condensed state is a condensate with fluctuating phase or quasicondensate. The expression for the radius of phase fluctuations (phase coherence length) follows from Eq. (5) and is given by

$$l_\phi \approx L(T_\phi/T). \tag{11}$$

The phase coherence length $l_\phi$ greatly exceeds the correlation length $l_c = \hbar/\sqrt{m\mu}$. Equations (11) and (10) give the ratio $l_\phi/l_c \approx (T_c/T)(T_c/\hbar\omega_\rho)^2 \gg 1$. Therefore, the quasicondensate has the same density profile and local correlation properties as the true condensate. However, the phase coherence properties of quasicondensates are drastically different (see below).

The decrease of temperature to well below $T_\phi$ makes the phase fluctuations small ($\delta_L^2 \ll 1$) and continuously transforms the quasicondensate into a true condensate.

It is interesting to compare the described behavior of the interacting gas for $\delta_c^2 \gg 1$, with the two-step BEC predicted for the ideal Bose gas in elongated traps [18]. In both cases, at $T_c$ the particles Bose condense in the ground state of the radial motion. However, the ideal gas remains noncondensed (thermal) in the axial direction for $T > T_{1D} = N\hbar\omega_z/\ln 2N$ (assuming $T_{1D} < T_c$), and there is a sharp crossover to the axial BEC regime at $T \approx T_{1D}$. The interacting Bose gas below $T_c$ forms the 3D TF (nonfluctuating) density profile, and the spatial correlations become nonclassical in all directions. For $\delta_c^2 \gg 1$, the axial phase fluctuations at $T \sim T_c$ are still large, and one has a quasicondensate which continuously transforms into a true condensate at $T$ below $T_\phi$. Note that $T_\phi$ is quite different from $T_{1D}$ of the ideal gas.

Let us now demonstrate that 3D elongated quasicondensates can be achieved for realistic parameters of trapped gases. As found above, the existence of a quasicondensate requires large values of the parameter $\delta_c^2$ given by Eq. (9). Most important is the dependence of $\delta_c^2$ on the aspect ratio of the cloud $\omega_\rho/\omega_z$, whereas the dependence on the number of atoms and on the scattering length is comparatively weak. Figure 3 shows $T_c/T_\phi = \delta_c^2$, $\mu/T_\phi$, and the temperature $T_\phi$ as functions of $\omega_\rho/\omega_z$ for rubidium condensates at $N = 10^5$ and $\omega_\rho = 500$ Hz. Comparing the results for $\delta_c^2$ in Fig. 3 with the data in Fig. 2, we see that 3D quasicondensates can be obtained by transforming the presently achieved BEC's to more elongated geometries corresponding to $\omega_\rho/\omega_z \gtrsim 50$.

One can distinguish between quasicondensates and true BEC's in various types of experiments. By using the Bragg spectroscopy method developed at MIT one can measure the momentum distribution of particles in the trapped gas and extract the coherence length $l_\phi$ [3]. The use of two (axially) counterpropagating laser beams to absorb a photon from one beam and emit it into the other one, results in axial momentum transfer to the atoms which have momenta at Doppler shifted resonance with the beams. These atoms form a small cloud which will axially separate from the rest of the sample provided the mean free path greatly exceeds the axial size $L$. The latter condition can be assured by applying the Bragg excitation after abruptly switching off the radial confinement of the trap. The axial momentum distribution is then conserved if the dynamic evolution of the cloud does not induce axial velocities. According to the scaling approach [19], this is the case for the axial frequency decreasing as $\omega_z(t) = \omega_z(0)[1 + \omega_\rho^2 t^2]^{-1/2}$.

In "juggling" experiments described in [7] and similar to those at NIST and Munich [4,5], one can directly measure the single-particle correlation function. The latter is

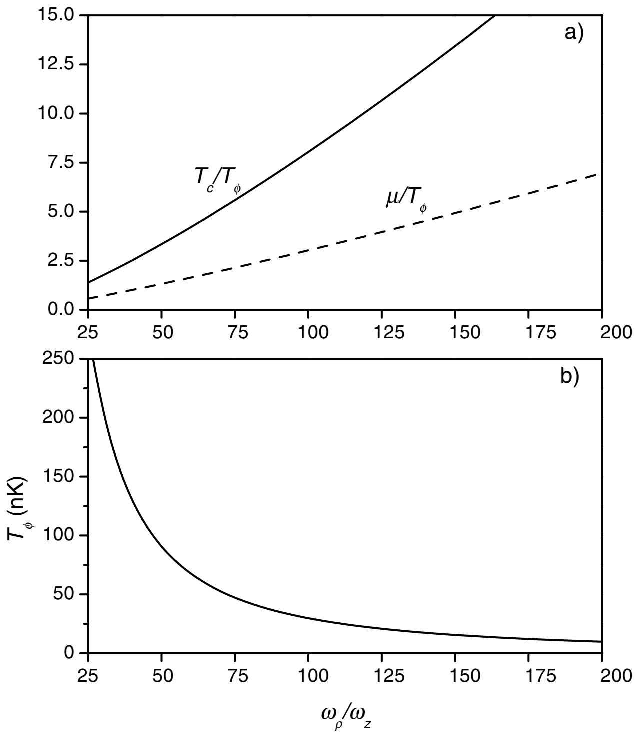


FIG. 3. The ratios $T_c/T_\phi = \delta_c^2$ and $\mu/T_\phi$ in (a) and the temperature $T_\phi$ in (b), versus the aspect ratio $\omega_\rho/\omega_z$ for trapped Rb condensates with $N = 10^5$ and $\omega_\rho = 500$ Hz.

obtained by repeatedly ejecting small clouds of atoms from the parts $z$ and $z'$ of the sample and averaging the pattern of interference between them in the detection region over a large set of measurements. As follows from Eqs. (6) and (7), for $z' = -z$ the correlation function depends on temperature as $\exp[-\delta_L^2(T)f(z/L)/2]$, where $f(z/L)$ is given in Fig. 1.

The phase fluctuations are very sensitive to temperature. From Fig. 3 we see that one can have $T_\phi/T_c < 0.1$, and the phase fluctuations are still significant at $T < \mu$, where only a tiny indiscernible thermal cloud is present.

This suggests a principle for thermometry of 3D Bose-condensed gases with indiscernible thermal clouds. If the sample is not an elongated quasicondensate by itself, it is first transformed to this state by adiabatically increasing the aspect ratio $\omega_\rho/\omega_z$. This does not change the ratio $T/T_c$ as long as the condensate remains in the 3D TF regime. Second, the phase coherence length $l_\phi$ or the single-particle correlation function are measured. These quantities depend on temperature if the latter is of the order of $T_\phi$ or larger. One thus can measure the ratio $T/T_c$ for the initial cloud, which is as small as the ratio $T_\phi/T_c$ for the elongated cloud.

We believe that the studies of phase coherence in elongated condensates will reveal many new interesting phenomena. The measurement of phase correlators will allow one to study the evolution of phase coherence in the course of the formation of a condensate out of a nonequilibrium thermal cloud.

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*Note added.*—When completing the paper we were informed that 3D quasicondensates were observed in Hannover [20].

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[1] M. H. Anderson *et al.,* Science **269**, 198 (1995); K. Davis *et al.,* Phys. Rev. Lett. **75**, 3969 (1995); C. C. Bradley *et al.,* Phys. Rev. Lett. **75**, 1687 (1995).
[2] M. R. Andrews *et al.,* Science **275**, 637 (1997).
[3] J. Stenger *et al.,* Phys. Rev. Lett. **82**, 4569 (1999).
[4] E. W. Hagley *et al.,* Science **283**, 1706 (1999).
[5] I. Bloch *et al.,* Nature (London) **403**, 166 (2000).
[6] D. S. Petrov, M. Holzmann, and G. V. Shlyapnikov, Phys. Rev. Lett. **84**, 2551 (2000).
[7] D. S. Petrov, G. V. Shlyapnikov, and J. T. M. Walraven, Phys. Rev. Lett. **85**, 3745 (2000).
[8] V. N. Popov, *Functional Integrals in Quantum Field Theory and Statistical Physics* (Reidel, Dordrecht, 1983).
[9] S. I. Shevchenko, Sov. J. Low Temp. Phys. **18**, 223 (1992).
[10] S. Stringari, Phys. Rev. A **58**, 2385 (1998).
[11] D. M. Stamper-Kurn *et al.,* Phys. Rev. Lett. **81**, 500 (1998).
[12] D. G. Fried *et al.,* Phys. Rev. Lett. **81**, 3811 (1998).
[13] U. Ernst *et al.,* Europhys. Lett. **41**, 1 (1998).
[14] K. Bongs *et al.,* Phys. Rev. Lett. **83**, 3577 (1999).
[15] J. Söding *et al.,* Appl. Phys. B **69**, 257 (1999).
[16] K. Dieckmann, Ph.D. thesis, FOM Institute for Atomic and Molecular Physics, Amsterdam, 2001.
[17] K. Bongs *et al.,* Phys. Rev. A **63**, 031602(R) (2001).
[18] N. J. van Druten and W. Ketterle, Phys. Rev. Lett. **79**, 549 (1997).
[19] Y. Castin and R. Dum, Phys. Rev. Lett. **77**, 5315 (1996); Yu. Kagan, E. L. Surkov, and G. V. Shlyapnikov, Phys. Rev. A **55**, R18 (1997).
[20] K. Sengstock (private communication).