Focusing of Bose-Einstein condensates in free flight

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We discuss focusing of Bose-Einstein condensates after release from a confining potential. It is shown that in the presence of a linear inward velocity field along the axial direction an elongated condensate will contract axially while expanding in the radial direction under conditions of free flight. We discuss the conditions under which such a focus can be observed and show that this is virtually impossible for collisionless thermal clouds. This difference allows an effective separation of the condensate from the surrounding thermal cloud. The focal size reflects the momentum uncertainty in the condensate.

1.1 Introduction

Since the first observation of Bose Einstein condensation (BEC) coherent atom optics has developed into an important field [1], providing tools for the investigation of macroscopic quantum phenomena in dilute atomic gases below their critical temperature (T_C). Many properties of quantum gases can be extracted by studying the interference between overlapping Bose-Einstein condensates after expansion from magnetic or optical traps [2]. With atom interferometry and quantum information processing as long term goals, atom waveguides as well as atom chips are being developed [3, 4, 5]. Mirrors, beam splitters and beam shaping optics of various types have been demonstrated [6, 7, 8]. Bloch *et al.* [9] demonstrated the focusing of an atom laser beam by a harmonic potential. Focusing in free flight has recently been observed in Amsterdam [10] during the ballistic expansion of an elongated condensate after release from a Ioffe-Pritchard quadrupole trap.

In this contribution we discuss the focusing method introduced in ref. [10]. We show that this method provides focusing for a condensate while the surrounding thermal cloud is not focused. This enables improved observation of small condensate fractions near T_C as well as small thermal fractions well below T_C. From the focal size an estimate for the phase coherence length can be obtained.

1.2 Focusing principle

To describe the principle of focusing of a condensate in free flight (see Fig.1.1a), we consider a cloud of atoms confined in an axially symmetric harmonic trapping potential with angular frequencies ω_z (axial) and ω_ρ (radial) and small aspect ratio $\beta \equiv \omega_z/\omega_\rho \ll 1$. We presume the cloud to dilate periodically in shape with angular frequency ω_Q in such a way that a linear velocity field $v_z(z) = -\alpha_z(t)z$ is present along the z-axis, . At time $t = 0$ the gas is released from the trap by the sudden removal of the trapping potential. For $t \leq 0$ the axial size, normalized to its value at release, is given in linear response by

$$b_z(t) = 1 - a_z \sin \omega_Q t, \tag{1.1}$$

where a_z is the rescaled axial amplitude of the oscillation. For the oscillation shown in Fig.1.1b, the axial size at $t = 0$ is contracting and we look for a focus at some later time $t > 0$.

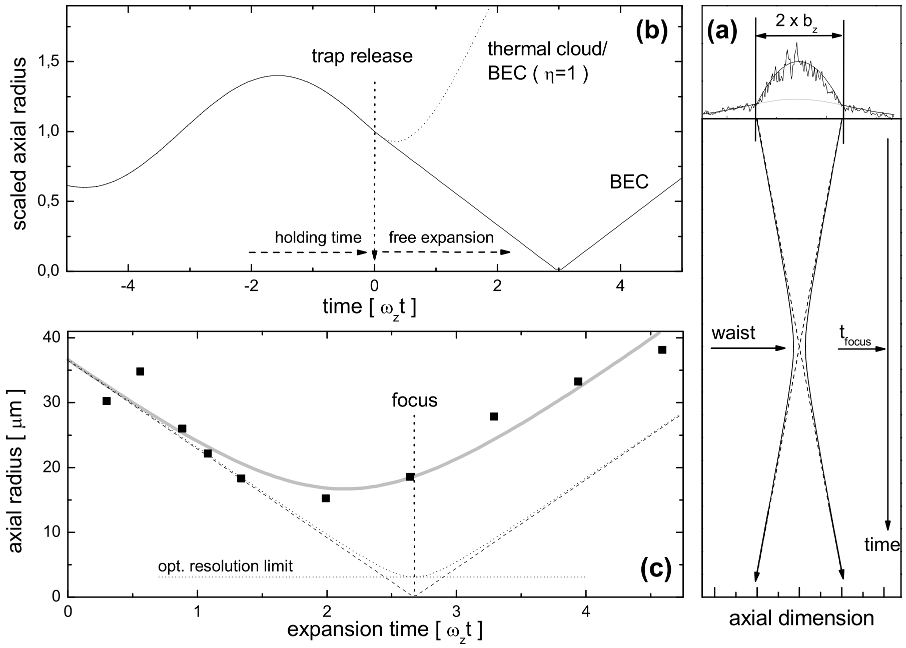

Figure 1.1: a) BEC-focusing observed as a contraction of the Thomas-Fermi size as a function of time; b) Evolution of the axial size before and after trap release at $t = 0$ for an oscillation amplitude $a_z = 0.4$. Solid line: condensate evolution for $\zeta = 0.41$; Dotted line: evolution of collisionless thermal cloud or oscillator ground state ($\eta = 1$) for $\zeta = 0.41$; c) Black squares: experimental data of ref. [10]; Grey curve: fit of Eq.(1.5) to the black squares, corresponding to $\zeta = 0.37$ and $\eta = 0.20$; Dashed line: plot of Eq.(1.3) for $\zeta = 0.37$. Dotted line: as dashed line but showing the optical resolution limit (see [18]) of ref. [10].

Let us first consider a pure Bose-Einstein condensate driven on the low-frequency mode of a quadrupole shape oscillation for which $\omega_Q \approx 1.58\,\omega_z$ [11, 12]. At $t = 0$ the axial size is given by the equilibrium Thomas-Fermi radius, $L_z = \left[2\mu/m\omega_\rho^2\right]^{1/2}$, where μ is the chemical potential of the gas and m the atomic mass. Within the Thomas-Fermi approximation the evolution of the axial and radial sizes of the cloud is given by the scaling equations [11, 13]

$$\ddot{b}_i = \frac{\omega_i^2}{b_i b_z b_\rho^2}, \qquad \text{with } i \in \{z, \rho\}, \tag{1.2}$$

subject to the initial conditions $b_z(0) = b_\rho(0) = 1$, $\dot{b}_z(0) = -a_z\omega_Q$ and $\dot{b}_\rho(0) \simeq 0$. As $\beta \ll 1$ we find to a good approximation for the radial expansion $b_\rho(t) = [1+\omega_\rho^2 t^2]^{1/2}$. The axial expansion at $t \gg 1/\omega_\rho$ is given by

$$b_z(t) = |1 - \zeta\omega_z t|\,, \tag{1.3}$$

where the contraction parameter ζ is defined as $\zeta = (a_z\omega_Q/\omega_z - \beta\pi/2)$. The result is shown as the solid line in Fig.1.1b. Hence, for $\zeta > 0$ the axial size decreases to produce a (one-dimensional) focus at time $t_{\text{focus}} = 1/(\zeta\omega_z)$. This is the case if the axial contraction velocity at release, $-a_z\omega_Q$, dominates over the axial expansion velocity $\beta\pi\omega_z/2$ induced by the 'kick' during the initial stages ($t \lesssim 1/\omega_\rho$) of the expansion, i.e. $a_z > \beta$. As the radial size remains finite and L_z decreases, around t_{focus} the chemical potential is restored and the focus reaches a minimum size $b_z(t_{\text{focus}}) = 2\beta^2$, independent of the value of a_z. This result is obtained by using the approximation $b_\rho \approx b_\rho(t_{\text{focus}}) \approx \omega_\rho t_{\text{focus}}$ and integrating Eq.(1.2) for $i = z$. Matching the resulting slope $\dot{b}_z \approx [2\omega_z^2 b_\rho^{-2}(t_{\text{focus}}) b_z^{-1}(t_{\text{focus}})]^{1/2}$ with the contraction velocity $\dot{b}_z(1/\omega_\rho \lesssim t \ll t_{\text{focus}})$ we get the mentioned result. The compression can be very tight, e.g. $2\beta^2 \approx 4\times 10^{-3}$ for the conditions of ref. [10]. In such cases the optical resolution of the imaging system used for detection is likely to limit the minimum observable focal size as was reported in ref. [9].

1.3 Focal broadening

Beyond a certain expansion time the kinetic energy of the original condensate can no longer be neglected as it gives rise to spreading of the condensate wavefunction. This effect may be accounted for by writing

$$b_z(t) \approx \left[(1-\zeta\omega_z t)^2 + \eta^2\omega_z^2 t^2\right]^{1/2}, \tag{1.4}$$

where η is a parameter determining the size of the focal waist. Note that for $\eta = 1$, Eq.(1.4) represents the spreading of a minimum uncertainty wavepacket released under conditions of axial contraction. Notice further that in this case *no* appreciable focusing is observed (see dotted line in Fig.1.1b) except for shape oscillations driven far outside $(\zeta > 1)$ the linear regime. In general, Eq.(1.4) gives rise to substantial focusing only if $\zeta > \eta$ (at $\zeta = \eta$ the condensate is compressed by 30%). The condition $\zeta > \eta$ is satisfied for elongated Thomas-Fermi condensates at $T = 0$, because the momentum spread is strongly reduced compared to that of the oscillator ground state. This situation is described by approximating the waist parameter with the value $\eta = \hbar\omega_z/2\mu$. Then, for $t < t_0 = 2\beta^2 m L_z^2/\hbar$, the spreading can be neglected even with respect to the compression minimum $2\beta^2 L_z$. For the conditions of ref. [10] we calculate $t_0 \approx 7$ ms.

For similar reasons it is virtually impossible to focus a collisionless thermal cloud. To illustrate this we consider a simple Boltzmann gas at temperature T with an oscillation described for $t < 0$ by Eq.(1.1) and released from the trap at $t = 0$. If the collisional mean free path is much larger than the radial size of the cloud, the expansion proceeds ballistically and the momentum of the individual atoms is conserved (free expansion). The scaled axial size evolves according to

$$b_z(t) = \left[(1-a_z\omega_Q t)^2 + \omega_z^2 t^2\right]^{1/2}, \tag{1.5}$$

which represents the convolution of two gaussians: the density profile of equilibrium width $l_z = [2k_BT/m\omega_z^2]^{1/2}$ and the velocity distribution of equilibrium width $\alpha_z = [2k_BT/m]^{1/2}$ which is locally shifted by the imposed velocity field $v_z(z) = -a_z\omega_Q z$. In the absence of a shape oscillation $(a_z = 0)$ this expression reduces to the well-know result used in time-of-flight analysis of collisionless thermal clouds [2, 14]. Presuming the same value of $\zeta = a_z\omega_Q/\omega_z$ as for the condensate (i.e. the solid line in Fig.1.1b), the thermal cloud is represented by the dotted line in Fig.1.1b.

Returning to elongated condensates we point out that at temperatures above the phase fluctuation temperature, $T > T_\phi = 15\,(\hbar\omega_z)^2 N/32\mu$, equilibrium phase fluctuations will dominate the focal broadening [15, 16, 17]. In this case the waist parameter may be approximated by $\eta \approx (L_z/L_\phi)^2\hbar\omega_z/\mu \approx (l_h/L_\phi)^2$, where L_ϕ is the phase coherence length and $l_h = [\hbar/m\omega_z]^{1/2}$ the axial harmonic oscillator length.

1.4 Applications of BEC focusing

The first application concerns the focus observed by Shvarchuck *et al.* [10] and reproduced in Fig.1.1c. The grey line represents a fit of Eq.(1.4) to the data and yields $\zeta = 0.37$ and $\eta = 0.20$.

In this experiment the focus is strongly broadened, $b_z(t_{\text{focus}}) \gg 2\beta^2$ exceeding the optical resolution limit of $3.3\ \mu$m (see Fig.1.1c) [18]. Hence, for the conditions of this experiment we may write $\eta \approx (l_h/L_\phi)^2$ and find a phase coherence length of $L_\phi \approx 0.45\ l_h \approx 1\mu$m.

Condensate focusing offers improved detection of small condensate fractions. Near the focus the axial condensate size is compressed by a factor $1/b_z(t_{\text{focus}})$. In time-of-flight absorption imaging, the signal-to-noise can be improved accordingly by choosing the time of detection equal to t_{focus}. This is advantageous, particularly close to T_C where the condensate fraction is small and has to be detected against the background of a large thermal cloud.

Condensate focusing also provides some advantage in detecting small thermal clouds as the separation time of the two components is reduced. Therefore, in time-of-flight absorption imaging detection can be shifted to shorter expansion times when the drop in optical density of the thermal cloud ($D_{\text{th}} \propto 1/\omega_\rho \omega_z t_{\text{sep}}^2$) is less and an improvement in the signal-to-noise ratio of a factor of two can be obtained.

1.5 Conclusion

We have demonstrated, how BEC focusing leads to compression of the Bose-Einstein condensed fraction of an ultracold sample, while leaving the non-condensed part unaffected. This separation allows easier observation of both components. We also showed how a detailed analysis of the focal size can provide information on the phase coherence properties of a condensate in the trap. The concepts described in this contribution can be extended to two dimensions. It will be interesting to see to what extent this can be made into a practical tool in coherent atom optics.

1.6 Acknowledgements

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[18] The optical resolution limit of the imaging system was measured with a positive 1951 USAF resolution target. A stripe pattern was defined as resolved if the transmitted intensity was modulated at least by 20%. The smallest resolved pattern had a repetition period of 6.9 μm, which corresponds to a resolution of $3.3\,\mu$m (1/e halfwidth; note that black sqares in Fig.1.1c also represent halfwidths).