

EVERY HAUSDORFF COMPACTIFICATION OF A LOCALLY COMPACT SEPARABLE SPACE IS A GA COMPACTIFICATION

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1. Introduction. In [4], De Groot and Aarts constructed Hausdorff compactifications of topological spaces to obtain a new intrinsic characterization of complete regularity. These compactifications were called *GA compactifications* in [5] and [7]. A characterization of complete regularity was earlier given by Frink [3], by means of Wallman compactifications, a method which led to the intriguing problem of whether every Hausdorff compactification is a Wallman compactification. An analogous question was posed by A. B. Paalman de Miranda; can every Hausdorff compactification of a Tychonoff space be obtained as a *GA* compactification? We will give a partial answer to this question, suggesting that the answer will be yes. This paper is organized as follows: in the second section we will recall the definition of *GA* compactifications and we will characterize the class of *GA* compactifications of a given topological space. Using an analogous characterization of Wallman compactifications, given by Steiner [11], it then follows that every Wallman compactification is a *GA* compactification. In the third section we will show that every Hausdorff compactification of a locally compact separable space is a *GA* compactification. In fact we have a more general result from which this is a corollary.

2. GA compactifications. Let X be a topological space and let ζ be a subbase for the closed subsets of X . Then ζ is defined to be a

- (i) *T_1 -subbase* if for each $x \in X$ and $S \in \zeta$ such that $x \notin S$ there exists a $T \in \zeta$ with $x \in T$ and $S \cap T = \emptyset$;
- (ii) *weakly normal subbase* if for each $S, T \in \zeta$ with $S \cap T = \emptyset$ there exists a finite cover \mathcal{U} of X by elements of ζ such that each element of \mathcal{U} meets at most one of S and T ;
- (iii) *normal subbase* if for each $S_0, T_0 \in \zeta$ with $S_0 \cap T_0 = \emptyset$ there exist $S_1, T_1 \in \zeta$ with $S_1 \cup T_1 = X$ and $S_1 \cap T_0 = \emptyset = S_0 \cap T_1$.

Note that a normal subbase is a weakly normal subbase.

A subsystem $\mathcal{U} \subset \zeta$ is called a *linked system* if every two of its members meet. A *maximal linked system* is a linked system not properly contained in

another linked system. By Zorn's lemma, every linked system is contained in at least one maximal linked system. Define

$$\lambda_\zeta(X) = \{\mathcal{U} \subset \zeta \mid \mathcal{U} \text{ is a maximal linked system}\}.$$

If $A \subset X$ then define $A^+ = \{\mathcal{U} \in \lambda_\zeta(X) \mid \exists M \in \mathcal{U} : M \subset A\}$. The set $\lambda_\zeta(X)$ can be topologized by taking $\zeta^+ = \{S^+ \mid S \in \zeta\}$ as a closed subbase. With this subbase $\lambda_\zeta(X)$ is called *the superextension of X relative to ζ* . Superextensions have been studied in [6; 9 and 12]. It is easy to see that every superextension is compact (in fact, $\lambda_\zeta(X)$ is *supercompact*, i.e. it admits an open subbase \mathcal{U} such that each covering of $\lambda_\zeta(X)$ by elements of \mathcal{U} contains a subcover of two elements of \mathcal{U}). The subbase ζ^+ has the property that each linked subsystem of $\mathcal{U} \subset \zeta^+$ has a nonempty intersection, as can easily be seen. A closed subbase with this property is called *binary*. If ζ is a T_1 -subbase, then X can be embedded in $\lambda_\zeta(X)$ by the natural embedding i defined by $i(x) = \{S \in \zeta \mid x \in S\}$. We will always identify X and $i(X)$. The *GA* compactification $\beta_\zeta(X)$ relative to ζ now is the closure of X in $\lambda_\zeta(X)$. As a matter of fact De Groot and Aarts obtained $\beta_\zeta(X)$ in another way; however in [6] it was shown that the compactification $\beta_\zeta(X)$ as defined above is equivalent to the compactification they defined in [4].

LEMMA 2.1: *Let ζ be a closed T_1 -subbase for the topological space X . Then the following properties are equivalent:*

- (i) $\beta_\zeta(X)$ is Hausdorff.
- (ii) ζ is weakly normal.
- (iii) $\{S^+ \cap \beta_\zeta(X) \mid S \in \zeta\}$ is weakly normal.

Proof. (i) \Rightarrow (ii). Assume that $\beta_\zeta(X)$ is Hausdorff and take $S_0, S_1 \in \zeta$ such that $S_0 \cap S_1 = \emptyset$. As $S_0 \cap S_1 = \emptyset$ it follows that $(S_0^+ \cap \beta_\zeta(X)) \cap (S_1^+ \cap \beta_\zeta(X)) = \emptyset$ and hence there exist open $U_0, U_1 \subset \beta_\zeta(X)$ such that $S_i \subset U_i (i = 0, 1)$ and $U_0 \cap U_1 = \emptyset$. Then $\beta_\zeta(X) \setminus U_i$ is closed in $\beta_\zeta(X)$ and as $\beta_\zeta(X)$ is closed in $\lambda_\zeta(X)$ it is closed in $\lambda_\zeta(X)$ too ($i = 0, 1$). As ζ^+ is a closed subbase for the compact space $\lambda_\zeta(X)$, there exist $T_{ij} \in \zeta$ and $T_{ij}' \in \zeta$ ($i, j \in \{1, 2, \dots, n\}$) such that

$$(i) \beta_\zeta(X) \setminus U_0 \subset \bigcup_{i=1}^n \bigcap_{j=1}^n T_{ij}^+; \quad \beta_\zeta(X) \setminus U_1 \subset \bigcup_{i=1}^n \bigcap_{j=1}^n T_{ij}'^+.$$

$$(ii) \bigcup_{i=1}^n \bigcap_{j=1}^n T_{ij}^+ \cap S_0^+ = \emptyset = \bigcup_{i=1}^n \bigcap_{j=1}^n T_{ij}'^+ \cap S_1^+.$$

Notice that a finite intersection of finite unions of subbase elements also can be represented as a finite union of finite intersections of subbase elements. As ζ is binary, for each $i \in \{1, 2, \dots, n\}$ there exists a $j_0 \in \{1, 2, \dots, n\}$ such that $T_{ij_0}^+ \cap S_0^+ = \emptyset$ and a $j_1 \in \{1, 2, \dots, n\}$ such that $T_{ij_1}'^+ \cap S_1^+ = \emptyset$, and therefore we may assume that there exist $T_i \in \zeta$ ($i \in \{1, 2, \dots, n\}$) and

$T_i' \in \zeta$ ($i \in \{1, 2, \dots, n\}$) such that

$$(i) \beta_\zeta(X) \setminus U_0 \subset \bigcup_{i=1}^n T_i^+; \quad \beta_\zeta(X) \setminus U_1 \subset \bigcup_{i=1}^n T_i'^+.$$

$$(ii) \bigcup_{i=1}^n T_i^+ \cap S_0^+ = \emptyset = \bigcup_{i=1}^n T_i'^+ \cap S_1^+.$$

Then $X \subset \beta_\zeta(X) \subset \bigcup_{i=1}^n T_i^+ \cup \bigcup_{i=1}^n T_i'^+$ and consequently

$$X = \bigcup_{i=1}^n (T_i^+ \cap X) \cup \bigcup_{i=1}^n (T_i'^+ \cap X) = \bigcup_{i=1}^n T_i \cup \bigcup_{i=1}^n T_i'.$$

Moreover it is obvious that $\bigcup_{i=1}^n T_i \cap S_0 = \emptyset = \bigcup_{i=1}^n T_i' \cap S_1$, which implies that ζ is weakly normal.

(ii) \Rightarrow (i). See De Groot and Aarts [4, Lemma 9] or Verbeek [12, Theorem II. 2.3].

(ii) \Rightarrow (iii). Choose $S_0^+, S_1^+ \in \zeta^+$ such that $S_0^+ \cap S_1^+ = \emptyset$. As $S_0 \cap S_1 = \emptyset$, there exist $T_i \in \zeta$ and $T_i' \in \zeta$ ($i \in \{1, 2, \dots, n\}$) such that

$$(i) S_0 \cap \bigcup_{i=1}^n T_i' = \emptyset = S_1 \cap \bigcup_{i=1}^n T_i.$$

$$(ii) \bigcup_{i=1}^n T_i' \cup \bigcup_{i=1}^n T_i = X.$$

It then follows that $S_0^+ \cap \bigcup_{i=1}^n T_i'^+ = \emptyset = S_1^+ \cap \bigcup_{i=1}^n T_i^+$ and that $X \subset \beta_\zeta(X) \subset \bigcup_{i=1}^n T_i' \cup \bigcup_{i=1}^n T_i$; therefore

$$\beta_\zeta(X) = \bigcup_{i=1}^n (T_i'^+ \cap \beta_\zeta(X)) \cup \bigcup_{i=1}^n (T_i^+ \cap \beta_\zeta(X)).$$

(iii) \Rightarrow (ii). This can be proved in a similar way.

THEOREM 2.2: *A Hausdorff compactification αX of X is a GA compactification if and only if αX possesses a weakly normal closed T_1 -subbase \mathcal{T} such that for all $T_0, T_1 \in \mathcal{T}$ with $T_0 \cap T_1 \neq \emptyset$ we have $T_0 \cap T_1 \cap X \neq \emptyset$.*

Proof. (\Rightarrow) This follows from Lemma 2.1 and from the trivial observation that if $\alpha X = \beta_\zeta(X)$, then $\{S^+ \cap \beta_\zeta(X) \mid S \in \zeta\}$ is a closed T_1 -subbase for $\beta_\zeta(X)$.

(\Leftarrow) Suppose that αX possesses a weakly normal closed T_1 -subbase \mathcal{T} such that for all $T_0, T_1 \in \mathcal{T}$ with $T_0 \cap T_1 \neq \emptyset$ we have $T_0 \cap T_1 \cap X \neq \emptyset$. Define $\mathcal{T}|X = \{T \cap X \mid T \in \mathcal{T}\}$. We will show that $\alpha X = \beta_{\mathcal{T}|X}(X)$.

(A) Let $x \in \alpha X$ and define $\mathcal{U}(x) = \{T \cap X \mid T \in \mathcal{T} \text{ and } x \in T\}$. We claim that $\mathcal{U}(x)$ is a maximal linked system (in $\mathcal{T}|X$). That $\mathcal{U}(x)$ is a linked system is evident. Suppose that $\mathcal{U}(x)$ is not maximal linked. Then there exists a $T \in \mathcal{T}$ such that $\mathcal{U}(x) \cup \{(T \cap X)\}$ is linked and $T \cap X \notin \mathcal{U}(x)$. Then $x \notin T$ and since \mathcal{T} is a closed T_1 -subbase, there exists a $T_0 \in \mathcal{T}$ such that $x \in T_0$ and $T_0 \cap T = \emptyset$. Then $T_0 \cap X \in \mathcal{U}(x)$ and $(T_0 \cap X) \cap (T \cap X) = \emptyset$, which is a contradiction.

Define a map $f : \alpha X \rightarrow \lambda_{\mathcal{T}|X}(X)$ by $f(x) = \mathcal{U}(x)$. We show the following.

(B) f is continuous. Choose $T \cap X \in \mathcal{T}|X$. Then $x \in f^{-1}((T \cap X)^+) \Leftrightarrow f(x) \in (T \cap X)^+ \Leftrightarrow T \cap X \in f(x) = \mathcal{U}(x) \Leftrightarrow x \in T$, which shows that f is continuous.

(C) f is one to one. Choose $x, y \in \alpha X$ such that $x \neq y$. Then there exist $T_0, T_1 \in \mathcal{T}$ such that $x \in T_0$ and $y \in T_1$ and $T_0 \cap T_1 = \emptyset$. Now $T_0 \cap X \in \mathcal{U}(x)$ and $T_1 \cap X \in \mathcal{U}(y)$ and therefore, as $(T_0 \cap X) \cap (T_1 \cap X) = \emptyset$ we have $f(x) = \mathcal{U}(x) \neq \mathcal{U}(y) = f(y)$.

(D) f is the identity on X . Choose $x \in X$. Then $f(x) = \mathcal{U}(x) = \{T \cap X \mid T \in \mathcal{T} \text{ and } x \in T\} = x$.

(E) $f(\alpha X) = \beta_{\mathcal{T}|X}(\alpha X)$. The weak normality of \mathcal{T} , together with the property that for all $T_0, T_1 \in \mathcal{T}$ with $T_0 \cap T_1 \neq \emptyset$ we have $T_0 \cap T_1 \cap X \neq \emptyset$, imply that $\mathcal{T}|X$ also is weakly normal and consequently $\beta_{\mathcal{T}|X}(\alpha X)$ is Hausdorff (Lemma 2.1). Therefore $f(\alpha X) = \beta_{\mathcal{T}|X}(\alpha X)$, since X is dense in αX .

It now follows that f is a homeomorphism, which on X is the identity. This completes the proof.

Using an analogous characterization of Wallman compactifications, given by Steiner [11], we immediately obtain the following remarkable corollary:

COROLLARY 2.3. *If a Hausdorff compactification αX of X is a Wallman compactification, then it is a GA compactification.*

Many compactifications are Wallman compactifications; therefore it follows that many compactifications are GA compactifications.

In the following section we will use Theorem 2.2 to obtain our main result.

3. Every Hausdorff compactification of a locally compact separable space is a GA compactification.

THEOREM 3.1: *Every Hausdorff compactification αX of a locally compact space X such that $\text{weight}(\alpha X) \leq 2^{\aleph_0}$ is a GA compactification.*

Proof. Let \mathcal{B} be an open basis for αX with $\text{card}(\mathcal{B}) \leq 2^{\aleph_0}$. Without loss of generality we may assume that \mathcal{B} is closed under finite intersections and finite unions. Define

$$\mathcal{C} = \{(\text{cl}_{\alpha X}(B_0), \text{cl}_{\alpha X}(B_1)) \mid B_i \in \mathcal{B} (i = 0, 1) \text{ and} \\ \text{cl}_{\alpha X}(B_0) \cap \text{cl}_{\alpha X}(B_1) = \emptyset\}.$$

For each pair $(\text{cl}_{\alpha X}(B_0), \text{cl}_{\alpha X}(B_1)) \in \mathcal{C}$ choose an $f \in C(\alpha X, I)$ such that $f(\text{cl}_{\alpha X}(B_0)) = 0$ and $f(\text{cl}_{\alpha X}(B_1)) = 1$. Let \mathcal{F} denote the set of mappings obtained in this way, and assume that \mathcal{F} is most economically well-ordered (denote the order by $<$). Note that $\text{card}(\mathcal{F}) \leq 2^{\aleph_0}$. By transfinite induction we will construct for each $f \in \mathcal{F}$ a $\delta_f \in (0, 1)$ such that

$$(*) \quad \text{cl}_{\alpha X}(f^{-1}[0, \delta_f]) \cap \text{cl}_{\alpha X}(g^{-1}[0, \delta_g]) \neq \emptyset \\ \Rightarrow \text{cl}_{\alpha X}(f^{-1}[0, \delta_f]) \cap \text{cl}_{\alpha X}(g^{-1}[0, \delta_g]) \cap X \neq \emptyset$$

for all $g < f$ ($g \in \mathcal{F}$).

Let $f \in \mathcal{F}$ and define $\mathcal{M} = \{f^{-1}(p) \setminus X \mid p \in f(\alpha X \setminus X)\} \cup \{\{x\} \mid x \in X\}$. Then \mathcal{M} is an upper semicontinuous decomposition of αX such that the decomposition space $\alpha_f(X)$ is a Hausdorff compactification of X with $f(\alpha X \setminus X)$ as remainder. Let P_f denote the projection map. Then P_f is the identity on X .

Moreover define $h_f : \alpha_f(X) \rightarrow I$ by $h_f = f \circ P_f^{-1}$. Then h_f is continuous and the diagram

$$\begin{array}{ccc} \alpha X & \xrightarrow{P_f} & \alpha_f(X) \\ \downarrow f & & \swarrow h_f \\ & & I \end{array}$$

commutes. Notice that h_f restricted to $\alpha_f(X) \setminus X$ is a homeomorphism (we will identify $\alpha_f(X) \setminus X$ and $h_f(\alpha_f(X) \setminus X)$).

Let f_0 be the first element of \mathcal{F} and define $\delta_f = \frac{1}{2}$. Assume that all δ_g have been constructed for all $g < f$ ($g \in \mathcal{F}$) such that (*) is satisfied.

(A) Let U be an open subset of $\alpha_f(X)$ and let

$$A = \{\delta \in (0, 1) \mid \overline{h_f^{-1}[0, \delta]} \cap \bar{U} \cap (\alpha_f(X) \setminus X) \neq \overline{h_f^{-1}[0, \delta]} \cap \bar{U} \cap (\alpha_f(X) \setminus X)\}.$$

(if $B \subset \alpha_f(X)$, then \bar{B} denotes the closure of B in $\alpha_f(X)$). Then A is a subset of $f(\alpha X \setminus X)$ while moreover A is countable. Indeed, choose $\delta \in I \setminus f(\alpha X \setminus X)$. Then

$$\begin{aligned} \overline{h_f^{-1}[0, \delta]} \cap \bar{U} \cap (\alpha_f(X) \setminus X) &\subset h_f^{-1}[0, \delta] \cap \bar{U} \cap (\alpha_f(X) \setminus X) = \\ &h_f^{-1}[0, \delta] \cap \bar{U} \cap (\alpha_f(X) \setminus X) \subset \overline{h_f^{-1}[0, \delta]} \cap \bar{U} \cap (\alpha_f(X) \setminus X), \end{aligned}$$

since $f^{-1}(\delta) \cap (\alpha X \setminus X) = \emptyset$.

To show that A is countable, assume that A were uncountable. Then as A is an uncountable subset of the real numbers it must contain a condensation point. What is more, it is obvious that there even exists a condensation point δ_0 which is a limit point from below. Now, let O be an open neighborhood of δ_0 in $\alpha_f(X)$. Then there exists a $\delta_1 \in O \cap A$ such that $\delta_1 < \delta_0$ and consequently

$$\delta_1 \in h_f^{-1}[0, \delta_0] \cap O \cap \bar{U} \subset O \cap \overline{h_f^{-1}[0, \delta_0]} \cap \bar{U}.$$

Therefore, it follows that $\delta_0 \in \overline{h_f^{-1}[0, \delta_0]} \cap \bar{U} \cap (\alpha_f(X) \setminus X)$, which is a contradiction.

(B) There exists a $\delta_0 \in (0, 1)$ such that

$$\begin{aligned} \overline{g^{-1}[0, \delta_0] \cap X \cap \overline{h_f^{-1}[0, \delta_0]} \cap (\alpha_f(X) \setminus X)} \\ = \overline{g^{-1}[0, \delta_0] \cap h_f^{-1}[0, \delta_0] \cap X \cap (\alpha_f(X) \setminus X)} \end{aligned}$$

for all $g < f$ ($g \in \mathcal{F}$). As $\text{card} \{g \in \mathcal{F} \mid g < f\} < 2^{\aleph_0}$, since the well-ordering is most economical, and as X (being locally compact) is open in $\alpha_f(X)$, we conclude from (A) that

$$\begin{aligned} \text{card} \left(\bigcup_{g < f} \left\{ \delta \in (0, 1) \mid \overline{g^{-1}[0, \delta_0] \cap X \cap \overline{h_f^{-1}[0, \delta]} \cap (\alpha_f(X) \setminus X)} \right. \right. \\ \left. \left. \neq \overline{g^{-1}[0, \delta_0] \cap h_f^{-1}[0, \delta] \cap X \cap (\alpha_f(X) \setminus X)} \right\} \right) < \aleph_0 \cdot 2^{\aleph_0} = 2^{\aleph_0}, \end{aligned}$$

and therefore such a choice for δ_0 is possible.

(C) Define $\delta_f = \delta_0$. We claim that (*) is satisfied. Take $g \in \mathcal{F}$ such that $g < f$ and assume that $\text{cl}_{\alpha X}(f^{-1}[0, \delta_f]) \cap \text{cl}_{\alpha X}(g^{-1}[0, \delta_g]) \neq \emptyset$. Then $P_f(\text{cl}_{\alpha X}(f^{-1}[0, \delta_f])) \cap P_f(\text{cl}_{\alpha X}(g^{-1}[0, \delta_g])) \neq \emptyset$ and consequently

$$\overline{P_f f^{-1}[0, \delta_f]} \cap \overline{g^{-1}[0, \delta_g]} \cap \overline{X} \neq \emptyset,$$

since it is easily seen that $P_f(\text{cl}_{\alpha X}(U)) = \overline{U \cap X}$ for each open $U \subset \alpha X$. Therefore

$$\overline{h_f^{-1}[0, \delta_f]} \cap \overline{g^{-1}[0, \delta_g]} \cap \overline{X} \neq \emptyset.$$

Now assume that

$$\overline{h_f^{-1}[0, \delta_f]} \cap \overline{g^{-1}[0, \delta_g]} \cap X \cap X = \emptyset.$$

It then follows that

$$\begin{aligned} \overline{h_f^{-1}[0, \delta_f]} \cap \overline{g^{-1}[0, \delta_g]} \cap \overline{X} \cap (\alpha_f(X) \setminus X) \\ = \overline{h_f^{-1}[0, \delta_f]} \cap \overline{g^{-1}[0, \delta_g]} \cap \overline{X} \cap (\alpha_f(X) \setminus X) \neq \emptyset, \end{aligned} \quad (\text{B})$$

which implies that $\overline{h_f^{-1}[0, \delta_f]} \cap \overline{g^{-1}[0, \delta_g]} \cap X \neq \emptyset$, which is a contradiction. Therefore $\overline{h_f^{-1}[0, \delta_f]} \cap \overline{g^{-1}[0, \delta_g]} \cap \overline{X} \cap X \neq \emptyset$. Now choose

$$x \in \overline{h_f^{-1}[0, \delta_f]} \cap \overline{g^{-1}[0, \delta_g]} \cap \overline{X} \cap X;$$

then $x \in \text{cl}_{\alpha X}(f^{-1}[0, \delta_f]) \cap \text{cl}_{\alpha X}(g^{-1}[0, \delta_g])$. Thus (*) holds indeed for δ_f ; this completes the construction of the $\delta_f (f \in \mathcal{F})$.

(D) Define $\mathcal{A} = \{\text{cl}_{\alpha X}(f^{-1}[0, \delta_f]) \mid f \in \mathcal{F}\}$. It is easy to see that \mathcal{A} is a closed base for αX . Moreover it is clear that for all $A_0, A_1 \in \mathcal{A}$ with $A_0 \cap A_1 \neq \emptyset$ it follows that $A_0 \cap A_1 \cap X \neq \emptyset$; hence, by Theorem 2.2, it suffices to show that \mathcal{A} is weakly normal and T_1 to prove the theorem.

Take $A_0, A_1 \in \mathcal{A}$ such that $A_0 \cap A_1 = \emptyset$. Then, using the fact that αX is compact Hausdorff, there exist closed sets G_0 and G_1 such that $A_0 \cap G_1 = \emptyset = G_0 \cap A_1$ and $G_0 \cup G_1 = \alpha X$. Now, since \mathcal{B} is closed under finite intersections and finite unions there exist $B_0, B_1 \in \mathcal{B}$ such that $A_0 \subset B_0 \subset \text{cl}_{\alpha X}(B_0)$ and $G_1 \subset B_1 \subset \text{cl}_{\alpha X}(B_1)$ and $\text{cl}_{\alpha X}(B_0) \cap \text{cl}_{\alpha X}(B_1) = \emptyset$. Now choose $A_1' \in \mathcal{A}$ such that $\text{cl}_{\alpha X}(B_1) \subset A_1'$ and $A_1' \cap \text{cl}_{\alpha X}(B_0) = \emptyset$. In the same way we can find an $A_0' \in \mathcal{A}$ such that $G_0 \subset A_0'$ and $A_0' \cap A_1 = \emptyset$. Therefore $A \cap A_1' = \emptyset = A' \cap A_1$ and $A_0' \cup A_1' = \alpha X$. Therefore \mathcal{A} is a normal closed base. That \mathcal{A} is also T_1 can be proved in the same way.

COROLLARY 3.2. *Every Hausdorff compactification of a locally compact separable space is a GA compactification.*

Proof. Let αX be a Hausdorff compactification of a locally compact separable space X . Then αX is also separable and consequently $\text{weight}(\alpha X) \leq 2^{\aleph_0}$ (Juhász [8]).

Remark. In the proof of Theorem 3.1 we, in fact, implicitly made use of a concept called *strongly \aleph_1 compact*, which is a very useful concept in compacti-

fication theory. It was introduced in the theory of Wallman compactifications by E. S. Berney [2]. Parts (A) and (B) of the proof of 3.1 are modifications of a technique also due to Berney [2].

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