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A DOWKER GROUP

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Abstract: We construct, in ZFC, a normal topological group, whose product with the circle group is not normal.

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0. Introduction. The purpose of this note is to give an example of a Dowker group: i.e. a normal topological group whose product with the circle group is not normal. We construct our example in ZFC alone, applying the B(X)-construction from [HavM] to a minor modification of M.E. Rudin's Dowker space [Ru]. The paper is organized as follows: Section 1 contains some definitions and preliminaries. In Section 2 we repeat the construction of B(X) and give some generalizations of the results from [HavM] in order to be able to show that for the modified Dowker space X of Section 4 B(X) is a topological group. In Section 3 we describe the Rudin's Dowker space R and show that under $\neg$CH B(R) is not a topological group.

Our construction shows once more the usefulness of Rudin's example: In [DovM] R was used to construct an extremally disconnected Dowker space.

1. Definitions and preliminaries. For topology see [En], for set theory see [Ku].

1.0. Free Boolean groups. Recall that a Boolean group is a group in which every element has order at most 2. Such groups are always Abelian.

For a set X we define the free Boolean group $B(X)$ of X to be the unique (up to isomorphism) Boolean group containing X such that every function from X to a Boolean group extends to a unique homomorphism from $B(X)$ to that group. For example $B(X) = \{\underline{x} \in {}^X2 : |x^{\leftarrow}(1)| < \omega\}$ as a subgroup of ${}^X2$. We shall write the elements of $B(X)$ as formal Boolean sums of elements of X. For every $n \in \mathbb{N}$ define $\varphi_n : X^n \longrightarrow B(X)$ by $\varphi_n(x) = x_1 + \ldots + x_n$ and let $X_n = \varphi_n[X^n]$.

1.1. $P_\varkappa$-spaces. Let X be a topological space. We call X a $P_\varkappa$-space, where $\varkappa$ is a cardinal, iff whenever $\mathcal{U}$ is a collection of fewer than $\varkappa$ open subsets of X, $\bigcap \mathcal{U}$ is open.

1.2. $k(X)$. For a space X we let

$k(X) = \min\{\varkappa \geq \omega$: Every open cover of X has a subcover of cardinality less than $\varkappa\}$.

Observe that $k(X) = \omega$ iff X is compact. Thus $k(X)$ might be called the compactness number of X.

From now on we assume that all spaces are Hausdorff. Observe that if X is a P_ω-space with $k(X) = \omega$ then X is simply a compact space.

For regular $\varkappa$, $P_\varkappa$-spaces with compactness number $\varkappa$ behave like compact spaces.

1.3. Proposition. Let X be a $P_\varkappa$-space with $k(X) = \varkappa$, $\varkappa$ regular. Then

(i) For all $n \in \mathbb{N}$ X^n is a $P_\varkappa$-space and $k(X^n) = \varkappa$.

(ii) If $f:X \to Y$ is continuous where Y is a $P_\varkappa$-space (and Hausdorff) then f is closed.

(iii) X is normal.

Proof: Imitate the proof for $\varkappa = \omega$. Note that only (i) needs regularity of $\varkappa$.

2. $B(X)$ revisited. We begin this section by repeating the construction of a topology for $B(X)$ given in [HavM].

2.0. Construction. Let X be a topological space. We defiee a topology on $B(X)$ as follows:
First for each n let τ_n be the quotient topology on X_n determined by X^n and φ_n. We then define

$$\tau = \{U \subseteq B(X): U \cap X_n \in \tau_n \text{ for all } n\},$$

i.e. τ is the topology on $B(X)$ determined by the spaces $\langle X_n, \tau_n \rangle$, $n \in \mathbb{N}$. Henceforth we will always assume that $B(X)$ carries this topology.

We now list some properties of $B(X)$, proved in [HavM]. Remember that all spaces are assumed to be Hausdorff.

2.1. Properties of $B(X)$.

(o) Both E and O are clopen in $B(X)$.

(i) Translations are continuous, hence $B(X)$ is homogeneous.

(ii) For each n $\langle X_n, \tau_n \rangle$ is a closed subspace of $\langle X_{n+2}, \tau_{n+2} \rangle$, and consequently each $\langle X_n, \tau_n \rangle$ is a closed subspace of $B(X)$.

(iii) For each n, if X^n is normal then X_n is normal and consequently if each X^n is normal then $B(X)$ is normal. For in the latter case $B(X)$ is dominated by a countable collection of closed normal subspaces and hence normal.

(iv) If X is compact then $B(X)$ is a topological group.

(v) If for each $n \in \mathbb{N}$ X^n is normal and $\beta(X^n) = (\beta X)^n$ then $B(X)$ is a subspace of $B(\beta X)$ and hence a topological group.

We shall need some slight generalizations of 2.1 (iv),(v), in order to be able to show that for the space X from Section 4, $B(X)$ is a topological group. The proofs are almost identical to the ones in [HavM], but for the readers´ convenience we shall give rough sketches. First we generalize 2.1 (iv).

2.2. <u>Theorem.</u> Let X be a $P_{\varkappa}$-space with $k(X) = \varkappa$, $\varkappa$ a regular cardinal. Then $B(X)$ is a topological group.

Proof. The case $\varkappa = \omega$ is covered by 2.1 (iv), also $B(X)$ is Boolean, so it suffices to show that the addition is continuous. We assume that $\varkappa > \omega$.

As a quotient of a $P_{\varkappa}$-space each X_n is a $P_{\varkappa}$-space.

From this it follows that $B(X)$ - and hence $B(X) \times B(X)$ - is a $P_{\varkappa}$-space, too.

Because $\varkappa > \omega$, the sequence $\{X_n \times X_n\}_{n \in \mathbb{N}}$ dominates the space $B(X) \times B(X)$.

Thus, it suffices to show that for every $n \in \mathbb{N}$ $+ : X_n \times X_n \longrightarrow X_{2n}$ is continuous.

By 1.3(iii) and 2.1(iii) X^n and X_n are normal,in particular X_n is Hausdorff.So by 1.3(ii) $\varphi_n \times \varphi_n : X^n \times X^n \longrightarrow X_n \times X_n$ is closed. But now if $F \subseteq X_{2n}$ is closed then $+^{\leftarrow}[F] = (\varphi_n \times \varphi_n)[h^{\leftarrow}\varphi_{2n}^{\leftarrow}[F]]$ is closed,where $h : X^n \times X^n \longrightarrow X^{2n}$ is the obvious homomorphism.

Next we generalize 2.1(v).

2.3. <u>Lemma</u>. Let Y be a dense subspace of X and $n \in \mathbb{N}$. Assume that Y_n is completely regular and Y^n is C^*-embedded in X^n.

Then Y_n is a C^*-embedded subspace of X_n.

Proof. Consider the following diagram:

$$\begin{tikzcd} Y^n \arrow[r, hook, "i"] \arrow[d, "\varphi_n^Y"'] & X^n \arrow[d, "\varphi_n^X"] \\ Y_n \arrow[r, hook, "j"] & X_n \end{tikzcd}$$

where i and j are the inclusion maps.

$\varphi_n^X \circ i$ is continuous, $\varphi_n^X \circ i = j \circ \varphi_n^Y$ and φ_n^Y is quotient, so j is continuous.

Let $f:Y_n \to [0,1]$ be continuous. We shall find a continuous $g:X_n \to [0,1]$ with $g \circ j = f$. Let $\bar{f} = f \circ \varphi_n^Y$ and let $\bar{g}:X^n \to [0,1]$ be the (unique) extension of $\bar{f}$.

From the fact that $\bar{f}$ is constant on the fibers of φ_n^Y it is easy to deduce that $\bar{g}$ is constant on the fibers of φ_n^X. Thus, $\bar{g}$ induces a function $g:X_n \to [0,1]$ with $g \circ \varphi_n^X = \bar{g}$ and g is continuous because $\bar{g}$ is continuous and φ_n^X is quotient.

These two facts plus the complete regularity of Y_n establish that Y_n is a C^*-embedded subspace of X_n.

2.4. <u>Theorem</u>. Let Y be a dense subspace of X such that B(Y) is completely regular and Y^n is C^*-embedded in X^n for all $n \in \mathbb{N}$. Then B(Y) is a C^*-embedded subspace of B(X).

Proof.

If $U \subseteq B(X)$ is open then for each $n \in \mathbb{N}$ $U \cap B(Y) \cap Y_n = U \cap Y_n = U \cap X_n \cap Y_n$ is open in Y_n, so $U \cap B(Y)$ is open in B(Y).

If $f:B(Y) \to [0,1]$ is continuous, then for each $n \in \mathbb{N}$ we obtain a (unique) extension $g_n:X_n \to$ [0,1 of $f \restriction Y_n$. It is easy to check that the g_n's are compatible and that $g = \bigcup_{n \in \mathbb{N}} g_n$ is a continuous extension of f.

2.5. Corollary. If X and Y are as in 2.4, then B(Y) is a topological group if B(X) is.

3. Dowker spaces. We describe Rudin's Dowker space and give some variations.

3.0.Construction. Let $\varkappa_0$ be a cardinal and for $n \in \mathbb{N}$ let $\varkappa_n$ be the n^{th} successor of $\varkappa_0$. Let $P = \Box_{n\in\mathbb{N}} \varkappa_n + 1$ i.e. the box product (see e.g.[Wi]) of the ordinal spaces $\varkappa_1 + 1$, $\varkappa_2 + 1, \dots$. Let $X' = \{f \in P: \forall n \in \mathbb{N} \ \mathrm{cf}(f(n)) > \varkappa_0\}$ and

$X = \{f \in X': \exists i \in \mathbb{N} \ \ \forall n \in \mathbb{N} \ \ \mathrm{cf}(f(n)) \leq \varkappa_i\}$.

Then X is always a Dowker space. We shall briefly indicate why and refer to [Ru] for full proofs.

3.1. X is not countably paracompact [Ru,II]. For $n \in \mathbb{N}$ let $D_n = \{f \in X: \exists\, i \geq n \ f(i) = \varkappa_i\}$. Then $\{D_n : n \in \mathbb{N}\}$ witnesses that X is not countably paracompact.

3.2. X is dense in X'.

3.3. If A and B are closed and disjoint in X then their closures are disjoint in X' ([Ru] Lemmas 5 and 6). Lemma 5 says that X' is a P_{ω_1}-space and Lemma 6 establishes that $\overline{A}_n \cap \overline{B}_n = \emptyset$ for all n where $A_n = \{f \in A: \forall i \in \mathbb{N} \ \ \mathrm{cf}(f(i)) \leq \varkappa_n\}$ (closures in X').

In Section 4 we shall reprove that X' is paracompact, thereby establishing (collectionwise) normality of X.
For the rest of this section we let $\varkappa_0 = \omega_0$ so that $\varkappa_i = \omega_i$ for $i \in \mathbb{N}$. Moreover we shall call this Dowker space R.
We shall show that if $2^{\omega} \geq \omega_2$ then B(R) is not a topological group.

3.4. Let H be a topological group which is also a P_{ω_1}-space then H has a local base at the identity consisting of open subgroups. For let $U_0 \ni e$ be open. Inductively find open $U_n \ni e$ for $n \in \mathbb{N}$ such that always $U_n = U_n^{-1}$ and $U_{n+1}^2 \subseteq U_n$. Then $\mathbb{N} = \bigcap_{n\in\mathbb{N}} U_n$ is an open subgroup contained in U_0.

3.5. Let G be an open subgroup of $B(R)$. For $x \in R$ let $G_x = \{y : x + y \in G\}$, then $\{G_x : x \in R\}$ is an open partition of R. Note that G_x is the intersection of R and the coset $x + G$.

3.6. Let $f \in P$ be such that for all $n \in \mathbb{N}$ $0 < f(n) < \omega_n$ and $f(n) < f(n+1)$ and $\sup_{m\in\mathbb{N}} f(n) = \omega_\omega$.
For $A \in [\mathbb{N}]^\omega$ let $C_A = \{h \in R : n \in A \longleftrightarrow h(n) \leq f(n)\}$. Then $\mathcal{C} = \{C_A : A \in [\mathbb{N}]^\omega\}$ is a clopen partition of R of size 2^ω.
For each A find $x_{A,1}, x_{A,2} \in C_A$ such that
- for some $n \in \mathbb{N}$ $\operatorname{cf}(x_{A,1}(n)) = \omega_1$ and $x_{A,1}(n)$ is not isolated in $\{\alpha \in \varkappa_n : \operatorname{cf}(\alpha) > \omega_0\}$
- for some $n \in \mathbb{N}$ $\operatorname{cf}(x_{A,2}(n)) = \omega_2$.

Now using $2^\omega \geq \omega_2$ we extract from $\mathcal{C}$ a clopen partition $\{V_\alpha : \alpha \in \omega_2\}$ of R together with points $\{x_\alpha : \alpha \in \omega_2\}$ such that
(i) $x_\alpha \in V_\alpha$ for each α.
(ii) If $\alpha \in \omega_1$ then there is a decreasing sequence $\{C_{\alpha\beta} : \beta \in \omega_2\}$ of clopen sets with $x_\alpha \in \bigcap_{\beta\in\omega_2} C_{\alpha\beta}$ but $x_\alpha \notin \operatorname{Int}(\bigcap_{\beta\in\alpha} C_{\alpha\beta})$
(iii) if $\alpha \in \omega_2 \setminus \omega_1$, a similar sequence $\{C_{\alpha\beta} : \beta \in \omega_1\}$ of length ω_1.

3.7. For $\alpha \in \omega_2$ define $\mathcal{D}_\alpha$ as follows:

if $\alpha \in \omega_1$ $\mathcal{D}_\alpha = \{V_\beta : \beta \in \omega_1 \wedge \beta \neq \alpha\} \cup$
$\cup \{C_{\gamma,\alpha} : \gamma \in \omega_2 \setminus \omega_1\} \cup \{V_\gamma \setminus C_{\gamma,\alpha} : \gamma \in \omega_1 \setminus \omega_0\}$

if $\alpha \in \omega_2 \setminus \omega_1$, $\mathcal{D}_\alpha = \{V_\beta : \beta \in \omega_2 \setminus \omega_1 \wedge \beta \neq \alpha\} \cup$
$\cup \{C_{\gamma,\alpha} : \gamma \in \omega_1\} \cup \{V_\gamma \setminus C_{\gamma,\alpha} : \gamma \in \omega_1\}$.
For each $\alpha \in \omega_2$ $\mathcal{D}_\alpha \cup \{V_\alpha\}$ is a clopen partition of R.

3.8. We define an open set $O \subseteq X^4$ as follows:
$O = \bigcup_{\alpha \in \omega_2} V_\alpha^4 \cup \bigcup_{\alpha \in \omega_2} \bigcup_{W \in \mathcal{D}_\alpha} \bigcup_{\sigma \in S_4} \sigma[V_\alpha^2 \times W^2]$
(S_4 acts on X^4 in the obvious way $\sigma(x_1, \dots, x_4) = (x_{\sigma(1)}, \dots, {}_{\sigma(4)})$.
Then $O = \varphi_4^{\leftarrow}[\varphi_4[O]]$ so that $\varphi_4[O]$ is a neighborhood of 0 in X_4 (the verification is straightforward).

3.9. Now suppose that G is an open subgroup of B(R) such tat $G \cap X_4 \subseteq \varphi_4[O]$; we shall show that this gives a contradiction.

The partition $\{G_x : x \in R\}$ has the following property:
if $\{a,b,c,d\} \cap G_x$ has 0, 2 or 4 elements for each $x \in R$ then $a + b + c + d \in G$.
Any partition refining $\{G_x : x \in R\}$ also has this property, so $\mathcal{W}$, the common refinement of $\{G_x : x \in R\}$ and $\{V_\alpha : \alpha \in \omega_2\}$ also has this property.

Fix for each $\alpha \in \omega_2$ $W_\alpha \in \mathcal{W}$ with $x_\alpha \in W_\alpha$, then $W_\alpha \subseteq V_\alpha$ of course.

For each $\alpha \in \omega_2$ let

$$\beta_\alpha = \min\{\beta : W_\alpha \nsubseteq C_{\alpha\beta}\}.$$

Find $\gamma_0 \in \omega_2 \setminus \omega_1$, $\gamma_1 \in \omega_1$ and $S \subseteq \omega_2 \setminus \omega_1$ unbounded such that

for $\alpha \in \omega_1$ $\qquad \beta_\alpha < \gamma_0$ and
for $\alpha \in S$ $\qquad \beta_\alpha = \gamma_1$.

Now pick $\gamma_2 \in S$ $\gamma_2 > \gamma_0$ and pick $y_1 \in W_{\gamma_1} \setminus C_{\gamma_1,\gamma_2}$ and $y_2 \in W_{\gamma_2} \setminus C_{\gamma_2,\gamma_1}$.
Consider $F = \{x_{\gamma_1}, y_1, x_{\gamma_2}, y_2\}$.
Then $x_{\gamma_1} + y_1 + x_{\gamma_2} + y_2 \in G$ because $|F \cap W_{\gamma_1}| = |F \cap W_{\gamma_2}| = 2$ and

$F \cap W = \emptyset$, $W \neq W_{\gamma_1}, W_{\gamma_2}$. On the other hand $x_{\gamma_1} + y_1 + x_{\gamma_2} + y_2 \notin \notin \varphi_4[0]$ because $(x = \langle x_{\gamma_1}, y_1, x_{\gamma_2}, y_2 \rangle)$:

- for no α $F \subseteq V_\alpha$ so $x \notin \bigcup_{\alpha \in \omega_2} V_\alpha^4$
- if $x \in \sigma[V_\alpha^2 \times V^2]$ for some $V \in \mathfrak{D}_\alpha$ then $F \cap V_\alpha \neq \emptyset$ so $\alpha = \gamma_1$ or $\alpha = \gamma_2$. If $\alpha = \gamma_1$, then, since $\langle x_{\gamma_2}, y_2 \rangle \subseteq V_{\gamma_2}$, either $V = = C_{\gamma_2,\gamma_1}$ or $V = V_{\gamma_2} \setminus C_{\gamma_2,\gamma_1}$; but both are impossible since $x_{\gamma_2} \in C_{\gamma_2,\gamma_1} \not\ni y_2$. Likewise $\alpha = \gamma_2$ is impossible.

Thus, combining 3.6 and 3.9, we find that B(R) is not a topological group, assume $2^\omega \geq \omega_2$. This leaves open what will happen if $2^\omega = \omega_1$.

3.10. Question. Is B(R) a topological group under CH ?

4. <u>A good Dowker space.</u> In this section we let $\varkappa_0 = 2^\omega$ and we let X be the Dowker space constructed in 3.0. We shall show that B(X) is a topological group, and in fact a Dowker group.

To begin we quote from [Ha] the following fact

4.0. For each $n \in \mathbb{N}$ X' is homeomorphic with $(X')^n$ and the homeomorphism can be chosen to map X onto X^n.

Furthermore we need the following

4.1. X' is paracompact and $k(X') = \varkappa_1$

Proof. We fix some notation: for $f, g \in P$ we say $f < g$ iff $f(n) < g(n)$ for all n and $f \leq g$ iff $f(n) \leq g(n)$ for all n. For $f, g \in \in P$ with $f < g$ we put

$$U_{f,g} = X' \cap \prod_{n \in \mathbb{N}} (f(n), g(n)] = \{h \in X' : f < h \leq g\} .$$

For $U = U_{f,g}$ put $t_U(n) = \sup\{h(n) : h \in U\}$ $(n \in \mathbb{N})$. Then $U_{f,g} \cap X = = U_{f,t_U} \cap X'$ and $t_U(n)$ is always a limit ordinal.

Let $\mathcal{O}$ be an open cover of X'. We find a disjoint open refinement $\mathcal{U}$ of $\mathcal{O}$ of size $\leq 2^\omega = \varkappa_0$. We define a sequence

$\{\mathcal{U}_\alpha\}_{\alpha\in\omega_1}$ of disjoint basic open covers of X' such that

(i) $\alpha \in \beta \in \omega_1 \longrightarrow \mathcal{U}_\beta$ refines $\mathcal{U}_\alpha$

(ii) $\alpha \in \omega_1 \quad\longrightarrow |\mathcal{U}_\alpha| \leq 2^\omega$

(iii) $\alpha \in \omega_1 \wedge U \in \mathcal{U}_\alpha \longrightarrow \{V \in \mathcal{U}_{\alpha+1} : V \subseteq U\} = \{U\}$ iff $U \subseteq O$ for some $O \in \mathcal{O}$.

Let $\mathcal{U}_0 = \{X'\}$.

For $x \in X'$ and $\alpha \in \omega_1$ $U_{x,\alpha}$ is always the unique element of $\mathcal{U}_\alpha$ containing x. If α is a limit, put $U_{x,\alpha} = \bigcap\{U_{x,\beta} : \beta \in \alpha\}$ and $\mathcal{U}_\alpha = \{U_{x,\alpha} : x \in X'\}$. If $\mathcal{U}_\alpha$ is found make $\mathcal{U}_{\alpha+1}$ as follows. Let $U \in \mathcal{U}_\alpha$ if $U \subseteq$ some $O \in \mathcal{O}$, put $S(U) = \{U\}$. Otherwise consider two cases.

a) For some n $\mu = \mathrm{cf}\,(t_U(n)) \leq 2^\omega$ (i.e. $t_U \notin X'$). Let $\langle \lambda_\xi : \xi \in \mu\rangle$ be a strictly increasing, continuous and cofinal sequence in $t_U(n)$ with $\lambda_0 = o$ and $\mathrm{cf}\,(\lambda_\xi) < 2^\omega$ for all ξ. Put $U_\xi = \{f \in U : \lambda_\xi < f(n) \leq \lambda_{\xi+1}\}$ ($\xi \in \mu$) and let $S(U) = \{U_\xi : \xi \in \mu\}$.

b) For all n $\mathrm{cf}\,(t_U(n)) > 2^\omega$ (i.e. $t_U \in X'$); pick $O \in \mathcal{O}$ with $t_U \in O$ and $f < t_U$ such that $U_{f,t_U} \subseteq O$. For $A \subseteq \mathbb{N}$ let

$U_A = \{h \in U : n \in A \rightarrow h(n) \leq f(n),\ n \notin A \rightarrow h(n) > f(n)\}$,

and set $S(U) = \{U_A : A \subseteq \mathbb{N}\}$.

Now let $\mathcal{U}_{\alpha+1} = \bigcup\{S(U) : U \in \mathcal{U}_\alpha\}$. It follows that always $|S(U)| \leq 2^\omega$ and hence inductively that $|\mathcal{U}_\alpha| \leq 2^\omega$ for $\alpha \in \omega_1$. Let $\mathcal{U} = \{U \in \bigcup_{\alpha\in\omega_1}\mathcal{U}_\alpha : S(U) = \{U\}\}$. Then, as in [Ru], $\mathcal{U}$ is a disjoint open refinement of $\mathcal{O}$ and by construction $|\mathcal{U}| \leq 2^\omega$.

The above argument is from [Ru] but we included it because we need to know that the refinement is not too big.

We now collect everything together in.

4.2. Theorem. $B(X)$ is a Dowker group.

Proof. (i) $X = X_1$ is a closed subspace of $B(X)$, so $B(X)$ is not countably paracompact.

(ii) From 3.3, 4.0 and 4.1 it follows that for all n X^n is normal and C^*-embedded in $(X')^n$, hence $B(X)$ is normal by 2.1 (iii) and a C^*-embedded subspace of $B(X')$ by 2.4.

(iii) X' is a $P_{\varkappa_1}$-space and $k(X') = \varkappa_1$ hence $B(X')$ is a topological group.

(iv) By 2.5 $B(X)$ is a topological group.

4.3. Remark. Actually, the method of Section 3 and this section yield the following result:

If X is the space constructed in 3.0 then

(i) if $2^{\omega} \leq \varkappa_0$ then $B(X)$ is a topological group,

(ii) if $2^{\omega} \geq \varkappa_2$ then $B(X)$ is not a topological group.

This leaves open a generalization of the question 3.10:

Is $B(X)$ a topological group if $2^{\omega} = \varkappa_1$?

If we specialize by setting $\varkappa_0 = \omega_1$ then we obtain a space X for which $B(X)$ is a topological group if $2^{\omega} = \omega_1$, not a topological group if $2^{\omega} \geq \omega_3$ and maybe (not) a topological group if $2^{\omega} = \omega_2$.

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