Proportionality in Complex Domains

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What are these complex domains?

- **Multiwinner Voting:** A job panel must produce a shortlist of k candidates to continue to the next interview stage.
- **Participatory Budgeting:** Citizens must decide on the public projects, each coming with a cost, that are to be implemented by the local municipality, subject to a budget.

We look at *other* complex domains.

Talk Outline

- ⋆ Proportionality in Multwinner Voting (MWV).
- ⋆ MWV with Weighted Seats.
- ⋆ Judgment Aggregation.

(Approval-based) MWV Model

- Candidates $C = \{a, b, c, \dots\}$.
- Agents $N = \{1, \dots, n\}$.
- Each agent submits an *approval ballot* $A_i \subseteq C$.
- Outcome is a committee $W \subseteq C$ of size k.

Proportionality in MWV

Definition (ℓ-cohesiveness)

For an integer $\ell \in \{1, \ldots, k\}$, a group of agents $N' \subseteq N$ is ℓ-cohesive if $|N'| \geqslant n \cdot \frac{\ell}{k}$ and $|\bigcap_{i \in N'} A_i| \geqslant \ell$.

Definition (Proportional Justified Representation (PJR))

A committee W provides PJR if for every ℓ-cohesive group N', it holds that $|W \cap (\bigcup_{i \in N'} A_i)| \geqslant \ell$.

Definition (Extended Justified Representation (EJR))

A committee W provides EJR if for every ℓ-cohesive group N', there exists an agent $i \in N'$ such that $|W \cap A_i| \geqslant \ell$.

Multiwinner Voting with Weighted Seats

Joint work with Ulle Endriss, Ronald de Haan, Adrian Haret and Jan Maly.

MWV with Weighted Seats

Example

Each seat represents a role and some roles are more valuable than others.

- The committee has 5 seats with the following roles: (chair, treasurer, secretary, member, member).

Example

Each seat has an associated budget that is available for the seat's elected candidate to spend.

- The committee has 5 seats with the following budgets: (\$3278, \$1400, \$560, \$100, \$4).

Model

- $\star$ Candidates $C = \{a, b, c, \ldots\}$.
- $\star$ Agents $N = \{1, \ldots, n\}$.
- $\star$ Each agent submits an approval ballot $A_i \subseteq C$.
- $\star$ A weight vector $\boldsymbol{w} = (w_1, \ldots, w_k)$ with a weight for each of the k seats.
- $\star$ W is the sum of all the weights.
- $\star$ Outcome is a committee $\boldsymbol{c} = (c_1, \ldots, c_k)$.
- $\star$ For any set of candidates $A \subseteq C$, the satisfaction from a committee $\boldsymbol{c}$ is $\mathrm{sat}(A, \boldsymbol{c}) = \sum_{j=1}^{k} \mathbb{1}_{c_j \in A} \cdot w_j$.

Proportionality

For weight vector $\boldsymbol{w}$, the set of all *possible* satisfaction values is $\text{SAT}(\boldsymbol{w})$.

Example

If $\boldsymbol{w} = (5, 3, 1)$, then $\text{SAT}(\boldsymbol{w}) = \{1, 3, 4, 5, 6, 8, 9\}$.

Definition (ℓ-WS-cohesiveness)

For an integer $\ell \in \text{SAT}(\boldsymbol{w})$, a group of agents N' is ℓ-WS-cohesive if $|N'| \geqslant n \cdot \frac{\ell}{W}$ and there exists a $C' \subseteq \bigcap_{i \in N'} A_i$ with $|C'| = t$ such that there exists a committee $\boldsymbol{c}$ where $\mathrm{sat}(C', \boldsymbol{c}) \geqslant \ell$, and $|N'| \geqslant n \cdot \frac{t}{k}$.

Definition (ℓ-WSJR)

A committee $\boldsymbol{c}$ provides ℓ-WSJR if for every ℓ-WS-cohesive group N', there exists an agent $i \in N'$ such that $\mathrm{sat}(A_i, \boldsymbol{c}) \geqslant \ell$.

ℓ-WSJR

Unfortunately, ℓ-WSJR is not always satisfiable.

Example

- Candidates $C = \{a, b, c\}$.
- Agents $N = \{1, 2, 3\}$.
- Weight vector $\boldsymbol{w} = (3, 2, 1)$.
- Approval ballots are $A_1 = \{a\}$, $A_2 = \{b\}$ and $A_3 = \{c\}$.

Another negative result:

⋆ It is computationally hard to determine whether such a committee even exists.

Weakening ℓ-WSJR

Intuition: some cohesive group member is just one 'swap' away from the deserved satisfaction?

$I_{\boldsymbol{c}}(A)$ is the vector of positions within the committee $\boldsymbol{c}$ of candidates in A.

Definition (ℓ-WSJR-1)

A committee $\boldsymbol{c}$ provides ℓ-WSJR-1 if for every ℓ-WS-cohesive group N', there exists an agent $i \in N'$ and some $j \in I_{\boldsymbol{c}}(C \setminus A_i)$ such that either (i), we have $w_j + \mathrm{sat}(A_i, \boldsymbol{c}) \geqslant \ell$ if there exists some candidate $c \in A_i$ with $c \notin \boldsymbol{c}$, or (ii), for some $h \in I_{\boldsymbol{c}}(A_i)$, it holds that $w_j + \mathrm{sat}(A_i, \boldsymbol{c}) - w_h \geqslant \ell$.

Can ℓ-WSJR-1 always be satisfied?

w-MES

The rule works in k rounds where agents pay to assign candidates to weights from $\boldsymbol{w} = (w_1, \ldots, w_k)$:

- $\star$ In round $r \in \{1, \ldots, k\}$, agents consider assignments to weight w_r.
- $\star$ $b_i(r)$ is agent i's budget to start round r, and in round 1, we set $b_i(1) = \frac{W}{n}$.
- $\star$ In round r, we say a pair (c, w_r) is *q-affordable* for some $q \in \mathbb{R}_{\geqslant 0}$, with c currently unelected, if:

$$\sum_{i \in N(c)} \min(q, b_i(r)) \geqslant w_r.$$

- $\star$ If no pair is q-affordable then go to the next round, otherwise, for a q-affordable pair (c, w_r) for a minimum q, assign c to w_r and continue to the next round.

w-MES and ℓ-WSJR-1

Good news in the following *restricted setting*.

Party-list elections: An election where for every pair of agents $i, j \in N$, it holds that either $A_i = A_j$, or $A_i \cap A_j = \emptyset$, and for every agent i, we have $|A_i| \geqslant k$.

Theorem

w-MES satisfies ℓ-WSJR-1 on party-list elections.

Judgment Aggregation

Joint work with Ulle Endriss and Ronald de Haan.

Judgment Aggregation (JA)

Work done in the general JA framework.

Julian Chingoma, Ulle Endriss, and Ronald de Haan (May 2022). "Simulating Multiwinner Voting Rules in Judgment Aggregation". In: Proceedings of the 21st International Conference on Autonomous Agents and Multiagent Systems (AAMAS-2022). IFAAMAS

Interpretation: MWV with a variable number of winners (VMWV), and with logical constraints.

Example

- The candidates are $\{a, b, c, d, e\}$.
- A constraint may be: $\neg(a \wedge b \wedge c) \wedge (d \rightarrow \neg e)$.

Model (VMWV with logical constraints)

- Candidates $C = \{a, b, c, \ldots\}$.
- Agents $N = \{1, \ldots, n\}$.
- A logical constraint Γ.
- Each agent submits an *approval ballot* $A_i \subseteq C$ that respects Γ.
- $\mathrm{Mod}(\Gamma)$ is the set of all committees respecting Γ.
- Outcome is a committee $W \in \mathrm{Mod}(\Gamma)$.

Proportionality

Definition ((W, Γ, ℓ)-cohesiveness)

For an integer $\ell \in \{1, \ldots, |W|\}$ for a committee W, we say a group of agents N' is (W, Γ, ℓ)-cohesive if $|N'| \geqslant n \cdot \frac{\ell}{|W|}$ and $|\{c \in \bigcap_{i \in N'} A_i \mid c \text{ is logically independent of } C \setminus \{c\}\}| \geqslant \ell$.

Adapt PJR instead of EJR.

Definition (ℓ-JA-PJR)

Given a constraint Γ, we say that a committee W provides ℓ-JA-PJR, if for every (W, Γ, ℓ)-cohesive group of agents N', it is the case that $|W \cap (\bigcup_{i \in N'} A_i)| \geqslant \ell$.

Aggregation Rules

- ⋆ Use scoring functions $\boldsymbol{a}$ and $\boldsymbol{d}$, for approvals and disapprovals (with $\boldsymbol{a}(0) = \boldsymbol{d}(0) = 0$).

$$\underset{W \in \mathrm{Mod}(\Gamma)}{\operatorname{argmax}} \sum_{i \in N} \boldsymbol{a}(|W \cap A_i|) - \boldsymbol{d}(|W \cap C \setminus A_i|)$$

Definition (PAV-JA)

PAV-JA uses $\boldsymbol{a}(t) = t$ and $\boldsymbol{d}(t) = \sum_{j=m}^{t} \frac{1}{j}$.

Definition (CC-JA)

CC-JA uses $\boldsymbol{a}(t) = 1$ when $t \geqslant 1$, and $\boldsymbol{d}(t) = 1$ if $t \geqslant \lceil \frac{m}{2} \rceil + 1$, otherwise, $\boldsymbol{d}(t) = 0$.

Rules and ℓ-JA-PJR

Theorem

PAV-JA satisfies ℓ-JA-PJR for every value $\ell \geqslant \frac{|W|}{m-|W|+1}$.

Theorem

Assuming logical independence between all candidates, CC-JA satisfies ℓ-JA-PJR for $\ell = 1$ and fails it for every $\ell > 1$.