Exercise 1. Let $k$ be a field. Let $Z$ be an integral scheme. Let $i: Z \to \mathbb{P}^2_k$ be a closed immersion. Show that one of the following statements holds

1. $Z \cong \text{Spec} l$ for $l$ a finite field extension of $k$
2. $Z \cong \text{Proj} k[X,Y,Z]/(F)$ for an irreducible homogeneous $F \in k[X,Y,Z]$
3. $Z \cong \mathbb{P}^2_k$.

Exercise 2. Let $k$ be a field. Let $f: X \to \mathbb{A}^1_k$ be a morphism of finite type. Assume $f^{-1}(y) \neq \emptyset$ for infinitely many $y \in \mathbb{A}^1_k$. Show that $f^{-1}(\eta) \neq \emptyset$, with $\eta$ the generic point of $\mathbb{A}^1_k$.

(Hint: first reduce to $X$ affine and integral.)

Exercise 3. Let $X$ be a separated scheme. Let $R_1 \to R_2$ be an injective ring homomorphism. Show that the map $X(R_1) \to X(R_2)$ is injective. Give an example of a non-separated $X$ and an injective ring homomorphism $R_1 \to R_2$ such that $X(R_1) \to X(R_2)$ is not injective.

Exercise 4. Let $R$ be a commutative ring and $f \in R[X]$ a monic polynomial of degree $\geq 1$.

1. Show that $\text{Spec} R[X]/(f) \to \text{Spec} R$ is surjective,
2. Show that $\text{Spec} R[X]/(f) \to \text{Spec} R$ is closed,
3. Show that $\text{Spec} R[X]/(f) \to \text{Spec} R$ is proper.

(Hint for (2): Let $Z \subset \text{Spec} R[X]/(f)$ be a closed subset corresponding to a prime ideal $p$. Let $q = R \cap p$. Consider the map $\text{Spec}(R/q)[X]/(f) \to \text{Spec} R/q$.)