

AAG, WEEK 11: EXERCISE

Exercise 1.

- (1) Give an example of a quasi-isomorphism that is not a homotopy-equivalence.
- (2) Show that the relation ‘there exists a quasi-isomorphism $A^\bullet \rightarrow B^\bullet$ ’ is not an equivalence relation on the collection of objects of $\text{Kom}(\text{Ab})$.

Exercise 2. Now let \mathcal{A} be an abelian category and assume that for every $X \in \mathcal{A}$ there is a short exact sequence $0 \rightarrow X \rightarrow I \rightarrow J \rightarrow 0$ in \mathcal{A} with I and J injective.

- (1) Show that $\mathcal{A} = \text{Ab}$ satisfies this hypothesis.

Let qis be the equivalence relation on the collection of objects of $\text{Kom}(\mathcal{A})$ generated by the relation ‘there exists a quasi-isomorphism $A^\bullet \rightarrow B^\bullet$ ’.

- (2) Show that $A^\bullet \text{qis} B^\bullet$ if and only if $H^n(A) \cong H^n(B)$ for all n .