Exercise 1. Let B be a cyclic group (finite or infinite).
(a) Give an injective resolution of B in the category Ab of abelian groups.
Let A be an abelian group. Recall that the right derived functors of the left exact functor $\mathrm{Hom}(A, -)$ are denoted by $\mathrm{Ext}^i(A, -)$. Now let A and B be cyclic groups (finite or infinite).
(b) Compute $\mathrm{Ext}^i(A, B)$ for all $i \geq 0$.

Exercise 2. Let X be a regular integral scheme separated and of finite type over a field k. Assume that $\dim X = 1$. Consider the exact sequence

$$(*) \qquad 0 \to \mathcal{O}_X \to \mathcal{K}_X \to \mathcal{K}_X/\mathcal{O}_X \to 0$$

in $\mathrm{Mod}(\mathcal{O}_X)$, where $\mathcal{K}_X$ is the constant sheaf associated to the function field K_X of X.
(a) Let $|X|$ denote the set of closed points of X. Show that there is a natural isomorphism

$$\mathcal{K}_X/\mathcal{O}_X \xrightarrow{\sim} \bigoplus_{P \in |X|} i_*(K_X/\mathcal{O}_P)$$

where we consider $K_X/\mathcal{O}_P$ as $\mathcal{O}_P$-module, and $i\colon \{P\} \to X$ is the inclusion map.
(b) Show that the exact sequence (*) is a flasque resolution of $\mathcal{O}_X$.
(c) Show that there is a natural isomorphism of k-vector spaces

$$\mathrm{H}^1(X, \mathcal{O}_X) \xrightarrow{\sim} \mathrm{Coker}(K \to (\mathcal{K}_X/\mathcal{O}_X)(X))\,.$$

(d) Show (without using Grothendieck's vanishing theorem) that $\mathrm{H}^i(X, \mathcal{O}_X) = (0)$ for $i > 1$.
(e) In the case $X = \mathbf{P}^1_k$, show that $\mathrm{H}^1(X, \mathcal{O}_X) = (0)$.