Exercise 1. Let \((X, O)\) be a locally ringed space, and \(f \in O(X)\) a global section. Let the distinguished set \(D(f) \subset X\) be defined by \(D(f) = \{x \in X \mid f \notin m_x\}\).

(i) Show that \(D(f)\) is open in \(X\).

(ii) Show that under the natural map \(O(X) \to O(D(f))\) the element \(f\) maps to a unit in \(O(D(f))\).

Exercise 2. Let \((R, \mathfrak{m})\) be a local ring and \(X\) a scheme.

(i) If \(f: \text{Spec} R \to X\) is a morphism, then any open set in \(X\) containing \(f(\mathfrak{m})\) contains the image of \(f\).

(ii) Let \(x\) be a point in \(X\). The set of morphisms \(f: \text{Spec} R \to X\) such that \(f(\mathfrak{m}) = x\) is in bijection with the set of local homomorphisms \(O_{X,x} \to R\).

Exercise 3. Let \(R\) be a ring. Then every irreducible closed subset of \(\text{Spec} R\) equals \(V(p)\) for some prime ideal \(p\) of \(R\), and \(p\) is its unique generic point. (For generic point, see [HAG], Exercise II.2.7.)

Exercise 4. Give an example of a local ring \((R, \mathfrak{m})\) such that \(\text{Spec} R\) is reduced but not irreducible. Also give an example of a local ring \((R, \mathfrak{m})\) such that \(\text{Spec} R\) is irreducible but not reduced.

Exercise 5. Let \(k\) be an algebraically closed field. Let \(X = (t(V), O_t(V))\) be the scheme associated to a variety \(V\) over \(k\) (cf. [HAG], Proposition II.2.6). Let \(x \in X\) be a point. The following statements are equivalent:

(i) \(x\) is closed in \(X\);

(ii) \(x\) is closed in \(U\) for all open affine neighborhoods \(U \subset X\) of \(x\);

(iii) \(x\) is closed in \(U\) for some open affine neighborhood \(U \subset X\) of \(x\).

Exercise 6. Give an example of: a scheme \(X\), an affine open subset \(U \subset X\), and a point \(x \in U\) such that \(x\) is closed in \(U\) but \(x\) is not closed in \(X\).