AAG, WEEK 3: EXERCISE

Let $k$ be a field. Let $U = \mathbb{A}^2_k \setminus \{0\}$ be the open subscheme of $\mathbb{A}^2_k$, where $0 \in \mathbb{A}^2_k$ is the closed point corresponding to maximal ideal $(x,y)$ of $k[x,y]$. (Warning: $U$ is not an affine scheme).

1. Show $U = D(x) \cup D(y)$, where $D(x), D(y)$ are the standard opens in $\mathbb{A}^2_k$;
2. Let $R$ be a $k$-algebra. Let $(s,t) \in R^2$, and $f: \text{Spec } R \to \mathbb{A}^2_k$ the morphism of $k$-schemes corresponding to $k[x,y] \to R$ with $x \mapsto s$ and $y \mapsto t$. Show that $\varphi$ factors over $U \hookrightarrow \mathbb{A}^2_k$ if and only if $\text{Spec } R = D(s) \cup D(t)$;
3. Show $U(R) = \{(s,t) \in R^2 \mid Rs + Rt = R\}$;
4. Show $U(K) = \{(s,t) \in K^2 \mid (s,t) \neq (0,0)\}$ for every field extension $k \subset K$.

In (3) and (4) the notation $U(R)$ stands for $\text{Hom}(\text{Spec } R, U)$ in the category of schemes over $k$. 